American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2013-2014

quiz # 1

time: 75 min

Exercise 1 *(12 points)* (answer on page 1 of t he booklet)

Find the limit of the following **sequences**:

a) $\frac{1+(-1)^n}{2^n}$ b) $\lim_{n\to+\infty}\left(\frac{2n-1}{2n+1}\right)^n$ c) $\lim_{n\to+\infty}(n+\ln n)^{2/n}$

Exercise 2 *(24 points)* (answer on page 2 of the booklet)

Determine if the following **series** converges or diverges **Justify your answers**

a) $\sum_{n=1}^{+\infty}\frac{2e^{-n}}{n+\ln n}$ b) $\sum_{n=1}^{\infty}\left(\frac{1+\cos(1/2^n)}{3}\right)^n$ c) $\sum_{n=4}^{+\infty}\sin\left(\frac{1}{\ln^2 n}\right)$ d) $\sum_{n=2}^{\infty}\frac{n^2+3^n}{2^n+n^3}$

Exercise 3 *(15 points)* (answer on page 3 of the booklet)

Consider the function

$$F(x)=\int_0^x\frac{e^{-t}-1}{t}\,dt$$

a) Express $F(x)$ as a power series

b) Estimate the value of $F(0.1)$ with an error of magnitude less than 10^{-3}

Exercise 4 *(16 points)* (answer on page 4 of the booklet)

Find the values of x for which the series

$$\sum_{n=1}^{+\infty}(2^{1/n}-1)\,(x-1)^n$$

converges, distinguishing between absolute and conditional convergence

Exercise 5 *(18 points)* (answer on page 5 of the booklet)

a) Give the first four terms of the Maclaurin series of $f(x)=\sqrt{2+3x}$, then deduce $f^{(3)}(0)$

b) Find the values of r and s for which

$$\lim_{x\to0}\left(\frac{\sin 3x}{x^3}+\frac{r}{x^2}+s\right)=0$$

Exercise 6 *(15 points)* (answer on page 6 of the booklet)

If $a_n>0$ and $\sum a_n$ converges, what can be said about $\sum(a_n)^p$, where $p\in\mathbb{R}$?