# All Tests to Determine an Infinite Series is Convergent or Divergent
# Math 201 - Fall 2013

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1. (**$n$-th Term Test**) If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is **Divergent**.

2. (**Geometric Series**) Suppose $a$ and $r$ are fixed numbers. If $|r| < 1$, then
$$\sum_{n=1}^{\infty} ar^n$$
is **Convergent** to
$$\frac{a}{1-r}.$$
Otherwise, it is **Divergent**.

3. (**Telescopic Series**) Assume that for all $n$, $a_n = b_n - b_{n+1}$, where $\{a_n\}$ and $\{b_n\}$ are two sequences. Then
$$\sum_{n=1}^{\infty} a_n = b_1 - \lim_{n\to\infty} b_n.$$

4. (**Integral Test**) Suppose that for all $n$, $a_n = f(n)$, where $f$ is a continuous, positive, decreasing function of $x$ for all $x \geq N$ ($N$ is a positive integer). Then

   $\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)dx$ **both converge** or **both diverge**.

5. (**Comparison Test**) Let $\sum a_n$ and $\sum b_n$ be series with nonnegative terms. Suppose that for some integer $N$, $a_n \leq b_n$ for all $n \geq N$. Then:
   - If $\sum b_n$ **converges**, then $\sum a_n$ **converges**.
   - If $\sum a_n$ **diverges**, then $\sum b_n$ **diverges**.

6. (**Limit Comparison Test**) Suppose that $a_n, b_n > 0$ for all $n \geq N$, where $N$ is a fixed positive integer. If

$$L = \lim_{n\to\infty} \frac{a_n}{b_n},$$

then

- If $0 \neq L \neq \infty$ (call it the **"Ideal Case"**), then $\sum a_n$ and $\sum b_n$ **both converge** or **both diverge**.
- If $L = 0$ and $\sum b_n$ **converges**, then $\sum a_n$ **converges**.
- If $L = \infty$ and $\sum b_n$ **diverges**, then $\sum a_n$ **diverges**.

7. ($n$**-th Root Test**) Suppose that $a_n > 0$ for all $n \geq N$, where $N$ is a fixed positive integer. If

$$L = \lim_{n\to\infty} \sqrt[n]{a_n},$$

then

- If $L < 1$, then $\sum a_n$ **converges**.
- If $L > 1$, then $\sum a_n$ **diverges**.
- If $L = 1$, then the test fails.

8. (**Ratio Test**) Suppose that $a_n > 0$ for all $n \geq N$, where $N$ is a fixed positive integer. If

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n},$$

then

- If $L < 1$, then $\sum a_n$ **converges**.
- If $L > 1$, then $\sum a_n$ **diverges**.
- If $L = 1$, then the test fails.