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EECE 491 – Discrete-time Signal Processing

Practice Exam

<u>Problem 1</u>: [**Ideal LPF**]

a) Determine the impulse response $h[n]$ of an ideal discrete-time low-pass filter with cutoff frequency $\omega_c = 0.4\pi$.

b) If we allow a group delay of 2 time units, what is the corresponding impulse response?

<u>Problem 2</u>: [**LTI Systems**] The input/output relationship for three different systems is given below:

- System I: $x[n] = (1/3)^n$, $y[n] = 2(1/3)^n$
- System II: $x[n] = (1/2)^n$, $y[n] = (1/4)^n$
- System III: $x[n] = (2/3)^n u[n]$, $y[n] = 4(2/3)^n u[n] - 3(1/2)^n u[n]$

Using the Eigen-function property of LTI systems, which of the following statements most accurately describes each of the systems above? If you choose (iii), specify $h(n)$ and $H(e^{j\omega})$.

i. The system cannot possibly be LTI
ii. The system must be LTI
iii. The system can be LTI, and there is only one LTI system that satisfies this I/O relationship
iv. The system can be LTI, and cannot be uniquely determined from the information in this I/O relationship

<u>Problem 3</u>: [**LTI Systems**] Consider the system shown below in Figure 1, where $|\alpha| < 1$:

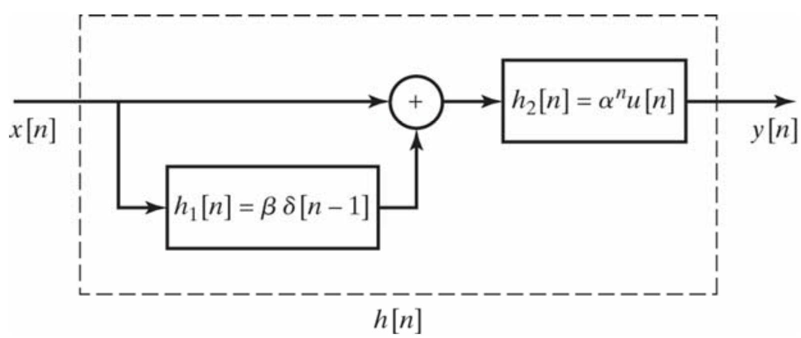

Figure 1

a) Determine the impulse response $h[n]$ and frequency response $H(e^{j\omega})$ of the overall system.

b) Is this system causal?

Problem 4: [**LTI Systems**] For the system shown in Figure 2, determine the output $y[n]$ when the input $x[n]$ is $\delta[n]$ and $H\left(e^{j\omega}\right)$ is

$$H\left(e^{j\omega}\right)=\begin{cases}1 & |\omega|<0.5\pi \\ 0 & 0.5\pi<|\omega|<\pi\end{cases}$$

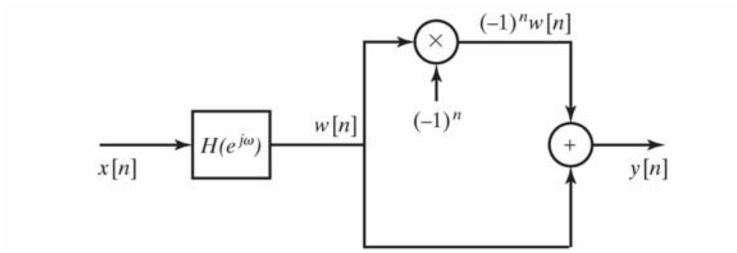

Figure 2

Problem 5: [**Pole-Zero Systems**]

a) The pole-zero plot shown in Figure 3 corresponds to the z-transform $X(z)$ of a causal sequence $x[n]$. Sketch the pole-zero plot of $Y(z)$, where $y[n]=x[-n+3]$ and determine the ROC of $Y(z)$.

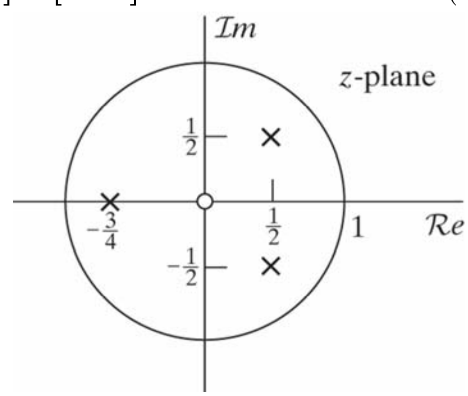

Figure 3

b) Let $x[n]$ be the sequence with the pole-zero plot shown in Figure 4. Sketch the pole-zero plot or $v[n]=\cos\left(\frac{\pi n}{2}\right)x[n]$.

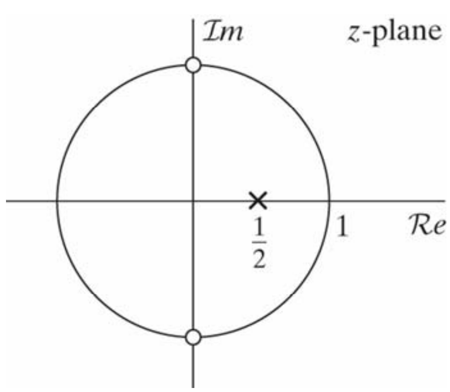

Figure 4

Problem 6: The pole-zero plots shown in Figure 5 describe six different causal LTI systems. Answer the following questions for each case. Only indicate the entries corresponding to a "YES" in the table below.

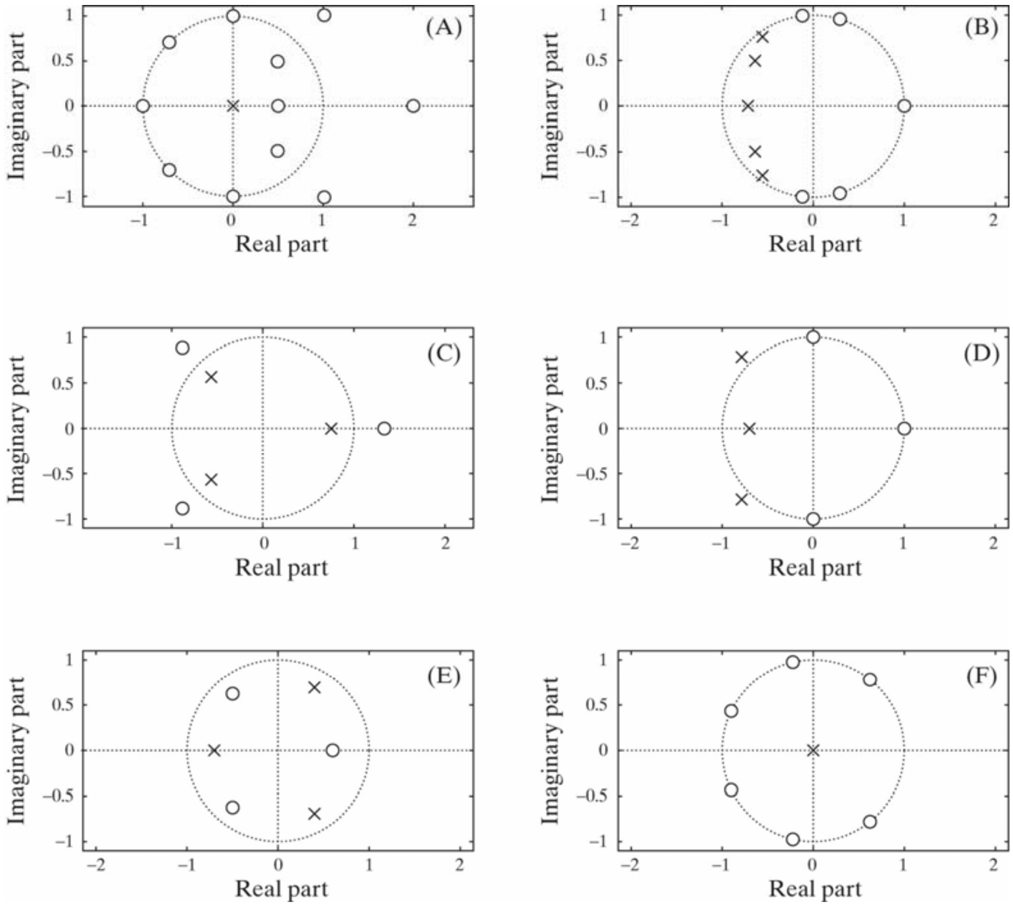

Figure 5

System	A	B	C	D	E	F
IIR?						
FIR?						
Stable?						
Minimum-phase?						
Linear phase?						
$\left\|H\left(e^{j\omega}\right)\right\|$ = constant?						
Has stable and causal inverse?						
Has shortest impulse response (least # of non-zero samples)?						
Has low-pass frequency response?						
Has minimum group delay?						

Problem 7: Consider an LTI system whose impulse response is given by

$$h[n] = a\cdot\delta[n] + b\cdot\delta[n-1] + c\cdot\delta[n-2] + d\cdot\delta[n-3]$$

where the coefficients a, b, c, d are complex numbers. Give three different non-trivial conditions on a, b, c, d so that the corresponding frequency response $H\left(e^{j\omega}\right)$ has linear phase. Determine the phase in each case.

Problem 8: Figure 6 shows the frequency response $A_e\left(e^{j\omega}\right)$ of a discrete –time FIR filter of length $M+1$.

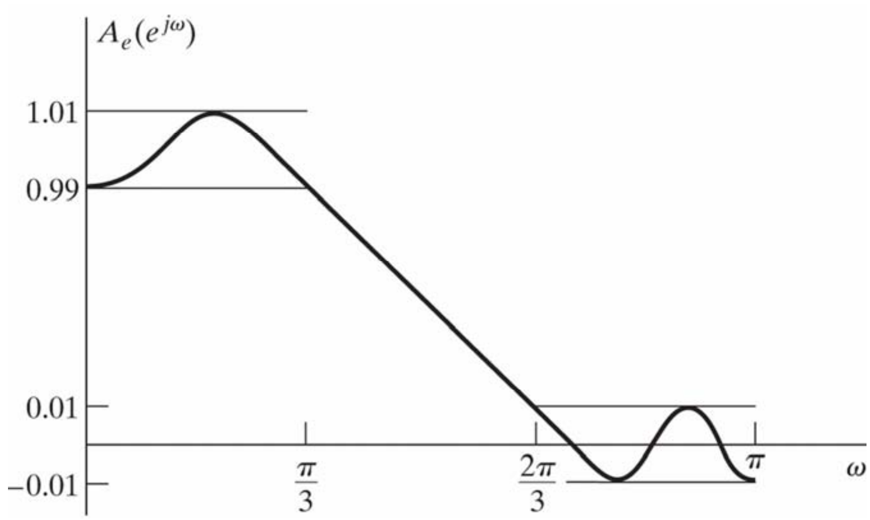

Figure 6

a) What type (I, II, III, or IV) of linear-phase system is this? Explain how you can tell. In case your answer results in $(M+1)$ being even, assume $M+1=2L$. In case your answer in $(M+1)$ being odd, assume $M+1=2L+1$.

   Type I: $(M+1)$ odd, symmetric
   Type II: $(M+1)$ even, symmetric
   Type III: $(M+1)$ odd, anti-symmetric
   Type IV: $(M+1)$ even, anti-symmetric

b) Show that $A_e\left(e^{j\omega}\right)$ **cannot correspond** to an FIR filter generated by the Parks-McClellan algorithm with a passband edge frequency of $\pi/3$, a stopband edge frequency of $2\pi/3$, and error-weighting function of unity in the passband and stopband. Explain your reasoning. *Hint*: The alternation theorem states that the best approximation is unique.

c) Based on Figure 6 and the statement that $A_e\left(e^{j\omega}\right)$ cannot correspond to an optimal filter, what can be concluded about the value of L for the filter in the figure?