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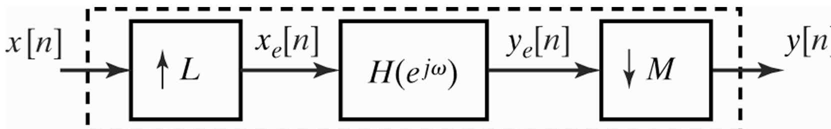
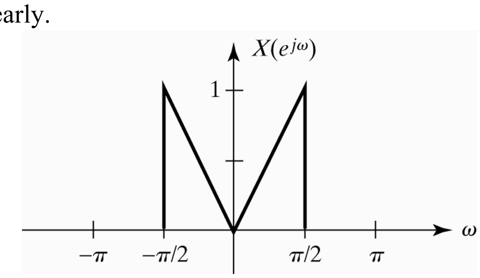
EECE 491 – Discrete-time Signal Processing

Problem Set: Sample Rate Conversion

Problem 1: Consider the discrete-time system shown below, where L and M are positive integers. The relationships between $x_e[n]$ and $x[n]$, as well as between $y_e[n]$ and $y[n]$ are as discussed in class. The discrete-time filter $H(e^{j\omega})$ is a low-pass filter with cutoff frequency $\pi/4$ and gain M.

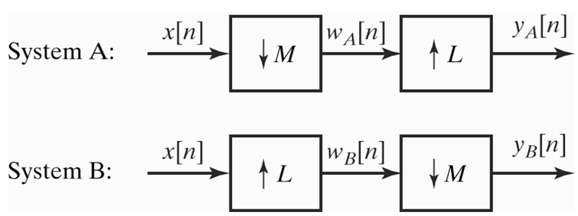


a) Assume the DTFT of the input $X(e^{j\omega})$ is as shown below. Let L=2 and M=4. Sketch $X_e(e^{j\omega}), Y_e(e^{j\omega}), Y(e^{j\omega})$ as a function of ω. Label all your plots clearly.



b) Now assume L = 2 and M = 8. Determine $y[n]$.

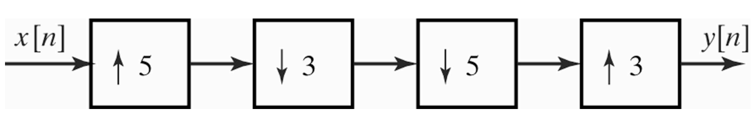
Problem 2: Consider the following two discrete-time systems:



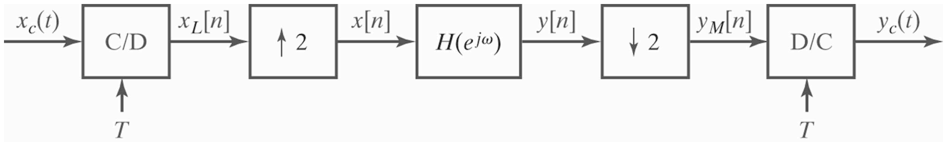
a) For M = 2 and L = 3, and arbitrary input $x[n]$, will $y_A[n] = y_B[n]$?

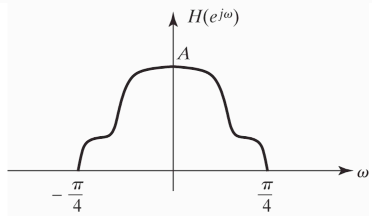
b) Determine a general condition on M and L to guarantee that $y_A[n] = y_B[n]$ for arbitrary $x[n]$.

Problem 3: For the system shown below, determine $y[n]$ in terms of $x[n]$. Simplify your answer as much as possible.



Problem 4: For the system below, assume $X_c(j\Omega)$ is bandlimited to $2\pi(1000)$ and $H(e^{j\omega})$ is as shown below.





a) Determine the most general condition on T so that the overall system from $x_c(t)$ to $y_c(t)$ is LTI. **Hint: Use the Noble identities to simplify your analysis**.

b) Sketch and clearly label the overall effective continuous-time frequency response $H_{eff}(j\Omega)$ when the condition in part a) holds.

c) Now assume that $X_c(j\Omega)$ is bandlimited to avoid aliasing, i.e., $X_c(j\Omega)=0$ for $|\Omega|\geq \pi/T$. For a general sampling period T, we would like to choose the DT system $H(e^{j\omega})$ so that the overall CT system from $x_c(t)$ to $y_c(t)$ is LTI for any input $x_c(t)$ bandlimited as above. Determine the most general condition on $H(e^{j\omega})$ so that the overall system from $x_c(t)$ to $y_c(t)$ is LTI. Assuming that these conditions hold, specify also the overall equivalent CT frequency response $H_{eff}(j\Omega)$ in terms of $H(e^{j\omega})$.