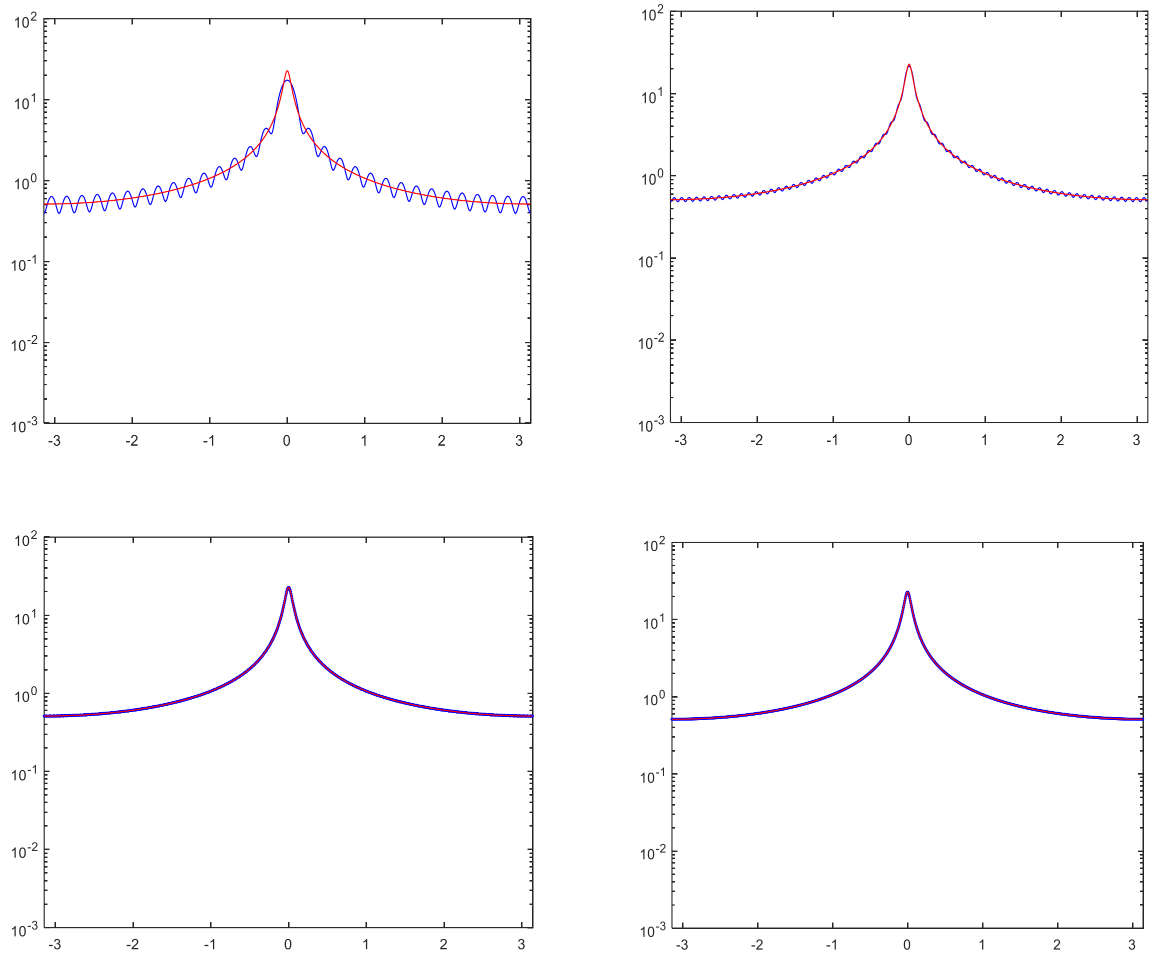
Prof. M. Mansour **American University of Beirut**

Department of Electrical and Computer Engineering

EECE 491 – Digital Signal Processing

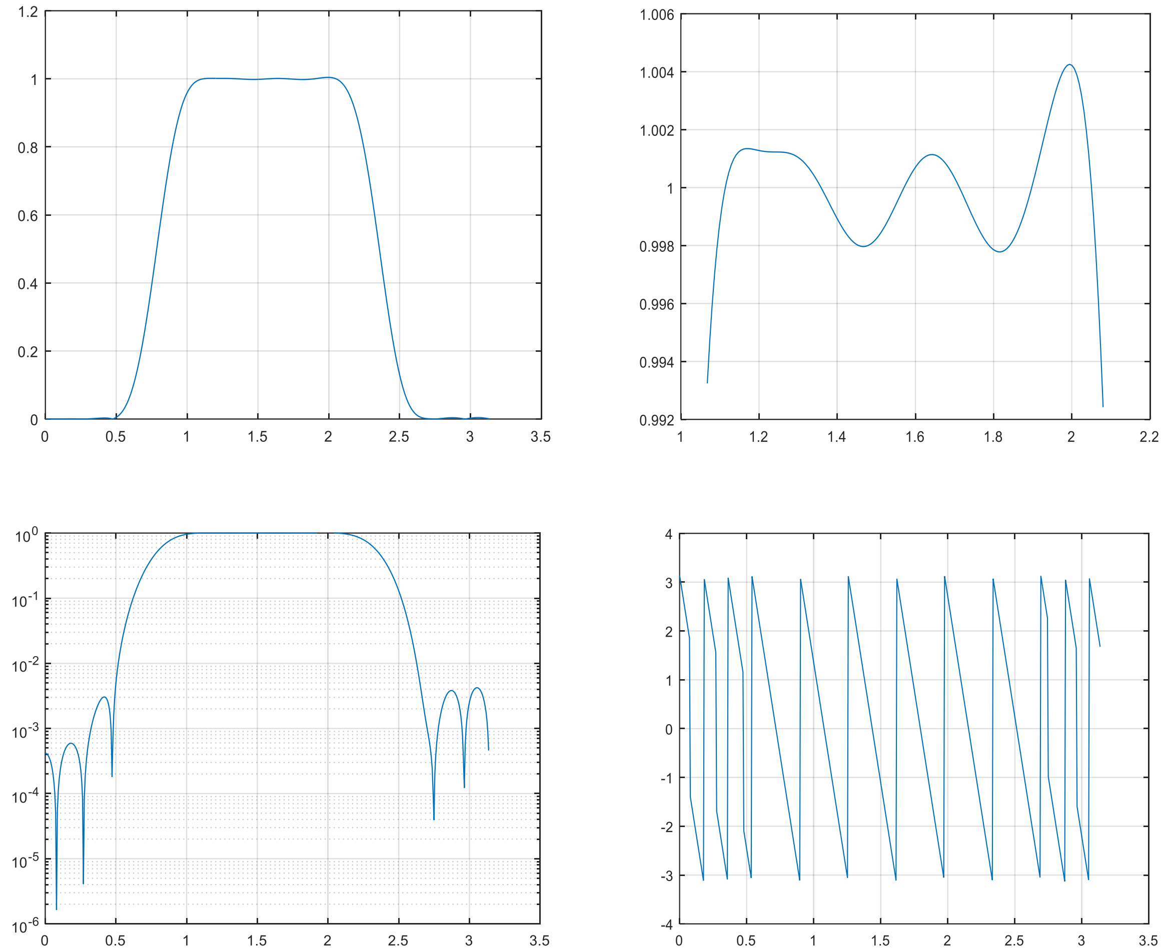
Problem Set 4 - Solution: FIR Filter Design

Problem 1:



We can see that as the window length increases, the generated windowed response converges to approximate the desired response, until we get to L = 256 where we see a complete convergence and we can hardly tell the difference. As for the Gibb's effect, we can say that it is absent for a large enough window length L.

Problem 2:



- plot(w(175:340), mag(175:340)): This statement plots the magnitude of the frequency response of the FIR filter within the passband of the filter. It permits us to visualize the passband ripples
- Passband ripple = max(mag)-1 = 0.0043 = 0.037 dB
- Stopband attenuation = 0.004813 = -46.35 dB
- $w_{s1} = 0.16\,\pi$, $w_{p1} = 0.342\,\pi$, $w_{p2} = 0.664\,\pi$, $w_{s2} = 0.84\,\pi$
- The first transition bandwidth is $w_{p1}-w_{s1} = 0.182\,\pi$ and the second is $w_{s2}-w_{p2} = 0.176\,\pi$
- We can see that $(w_{s1}+w_{p1})/2 = 0.25\,\pi$ and $(w_{s2}+w_{p2})/2 = 0.75\,\pi$, thus the positions are accurate.
- The filter is a FIR filter with 35 taps thus it has 35 zeros, 11 out of which are on the unit circle. This can be seen by either plotting the pole-zero plot or by checking the log(mag) plot.
- It has 35 poles at the origin (pole of order 35)
- The slope is negative which implies that we have a positive group delay. As for the jumps, we have 2 types. The first is when $-\pi$ wraps to π which is of height $2\,\pi$, and the second is the one occurring at the zeros which is of height π.

Problem 3:

- Maximum passband error: $\delta_p = 0.05 = -26\ dB$
- Maximum stopband error: $\delta_s = 0.1 = -20\ dB$
- This requires a window with peak approximation error < -26 dB. From slides, Hann, Hamming, and Blackman meet this criterion.
- Next, we use the approximate width of main lobe which is equal to transition width to find filter length L=M+1
- Hann: $0.1\pi = 8\ \pi/\text{M}$ => M = 80
- Hamming: $0.1\ \pi = 8\ \pi/\text{M}$ => M = 80
- Blackman: $0.1\ \pi = 12\ \pi/\text{M}$ => M = 120

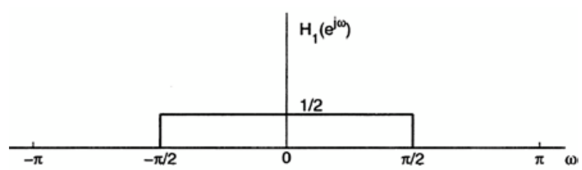
Problem 4:

- Since filters designed by window method must have $\delta_p = \delta_s$, we must choose the smaller value $\delta = 0.02$
  - ⇨ A = $-20\log_{10}(\delta) = 33.9794$
  - ⇨ $\beta = 0.5842(A-21)^{0.4} + 0.07886(A - 21) = 2.65$
  - ⇨ M = $\left\lceil \frac{A-8}{2.285\Delta w} \right\rceil = 181$

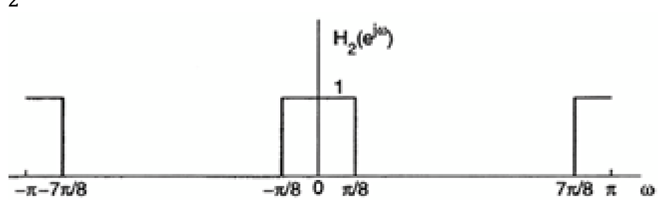
Problem 5:

a) $H_1(w) = \sum_{n=-\infty}^{\infty} h(2n)e^{-jwn} = \sum_{n\ even} h(n)e^{\frac{-jwn}{2}} = \sum_{n=-\infty}^{\infty} \frac{1}{2}[h(n) + (-1)^n h(n)]e^{-jwn/2}$

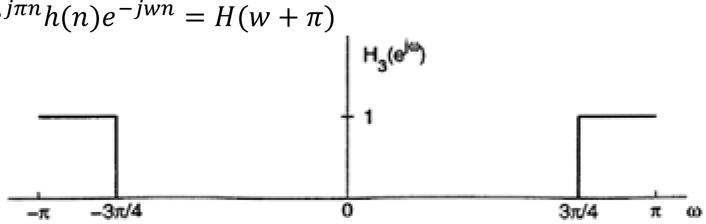
$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} h(n)e^{-jwn/2} + \frac{1}{2}\sum_{n=-\infty}^{\infty} h(n)e^{-j(\frac{w}{2}+\pi)n} = \frac{1}{2}H\left(\frac{w}{2}\right) + \frac{1}{2}H\left(\frac{w+2\pi}{2}\right)$$



b) $H_2(w) = \sum_{n\ even} h(\frac{n}{2})e^{-jwn} = \sum_{n=-\infty}^{\infty} h(n)e^{-j2wn} = H(2w)$



c) $H_3(w) = \sum_{n=-\infty}^{\infty} e^{j\pi n}h(n)e^{-jwn} = H(w+\pi)$



Problem 6:

a) A = 60

$\beta = \frac{A-8}{8}$ = 6.5

M = $\left\lceil \frac{A-8}{2.285*0.01\pi} \right\rceil$ = 743

b) $w'_p = \pi/10$

$w'_s = \pi/5$

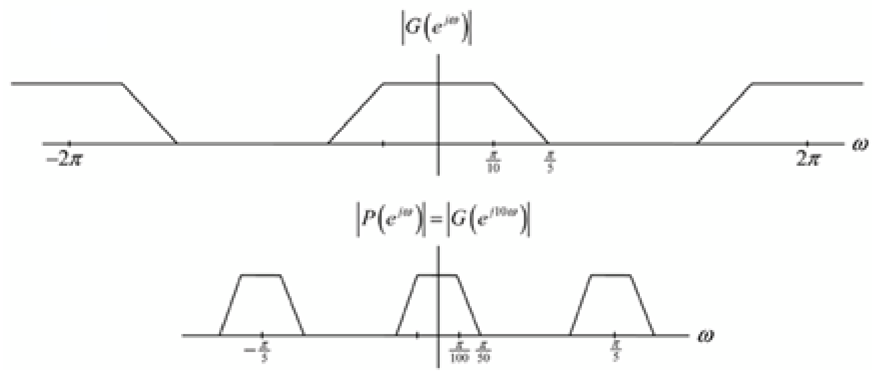
δ' = -60 dB

A' = 60

$\beta = \frac{A-8}{8}$ = 6.5

M = $\left\lceil \frac{A-8}{2.285*0.1\pi} \right\rceil$ = 74.3 ~= 74

c) Note that P(w) = G(10w)



d) From above plot, q(n) must filter the aliases of P(w). So

$W''_p = \pi/100$

$W''_s = \pi/5 - \pi/50 = 0.18\,\pi$

δ'' = -60 dB

e) A" = 60

$\beta'' = \frac{A-8}{8}$ = 6.5

M" = $\left\lceil \frac{A-8}{2.285*0.17\pi} \right\rceil$ = 43.7 ~= 44

The number of non-zero samples of h'(n) will be the length of the convolution of h'(n) = p(n)*q(n)

- Length of q(n) is L_q = M" + 1 = 44 + 1 = 45
- Length of g(n) is L_g = M' + 1 = 74 + 1 = 75
- Length of p(n):
  p(n) = g(n/10) when n/10 is integer and 0 otherwise.
  There are 75 samples of g(n). For each non-zero sample of g(n), p(n) has 9 zeros after it, except the last sample, so $L_p = (L_g - 1)*9 + L_g = 10L_g - 9$ = 10*75 -9 = 741.
  The length of the convolution will then be $L_{h'} = L_p + L_q - 1$ = 741 +45 – 1 = 785

f) - For the original filter h(n) of length $L_H = M + 1 = 743 + 1 = 744$, convolving the input x(n) with h(n) require 744 multiplications per output sample.

- For q(n), it has length 45. Convolving the input x(n) with q(n) first, 45 non-trivial multiplications per output sample are required. Nest, convolving the result with p(n) requires multiplying the result with the non-zero samples of p(n) which are 75. Hence the total multiplications are 45 + 75 = 120 per output sample, which are less than part (a)

Problem 7:

a) From the figure, $H(e^{jw})$ exhibits eight alternations of the error on the interval $0 \le w \le \pi$ as an approximation to an idea lowpass filter with the given parameters. Because a lowpass filter designed with the Parks-McClellan algorithm has either L+2 or L+3 alternations and because we are told that there is another filter out there that meets the specs with $N_2>N_1$, we should consider the L+3 case to find the smaller value of N.
With L+3 = 8 alternations, L+5. Then, since $(N_1-1)/2 = 5$, we have $N_1 = 11$ as the only possible value.
b) Since there are 8 alternations, L can be no greater than 6. Therefore $(N_2-1)/2 \le 6$, which implies $N_2 \le 13$. Since the only other possible value of N for a lowpass filter was found in A, we have $N_2 = 13$ as the only possible value.
c) Yes. Since both filters have identical frequency responses, they must have identical impulse responses.
d) While the alternation theorem states that for a given r there is a unique r th order polynomial that satisfies it, the theorem makes no claim about how this polynomial may or may not relate to a polynomial satisfying the alternation theorem for a different value of r. It turns out that in this case, the single 5th order polynomial satisfying the alternation theorem for $r_1 = L_1 = 5$ is identical to the single 6th order polynomial satisfying the alternation theorem for $r_2 = L_2 = 6$.

Problem 8:

The minimum N that will satisfy the given constraints is N = 27.

The corresponding plots are given below: