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EECE 491 – Discrete-time Signal Processing

Problem Set: FIR filter design

Problem 1: We wish to approximate the following *causal* filter by using a rectangular window.

$$H\left(e^{j\omega}\right) = \frac{1}{1-\alpha e^{-j\omega}}, \quad \alpha = 0.956$$

Use rectangular windows of lengths L = 32, 64, 128, and 256. Plot each in a separate plot and plot the "desired" response in each plot using a dashed line. Comment on the effect of the length of the window. Comment on the presence or absence of Gibbs' phenomenon. Note: For each plot, please use a log scale for the magnitude. For each plot, a log-scale for the magnitude.

Problem 2: For this problem, we will examine an FIR band-pass filter.

a) An FIR bandpass filter with lower and upper cutoff frequencies at π/4 and 3π/4 respectively, can be obtained with the MATLAB code below:

```
n = 35;
c = [0.25,0.75];
a = fir1(n,c);
```

Here, the filter length is (n + 1), and its normalized cutoff frequencies in [0, π] are at $\omega_c = \pi c$. The filter coefficients are returned in *a*.

b) Plot the magnitude and phase of your filter's frequency response using the following code:

```
[h,w] = freqz(a,1,512);
mag = abs(h);
phase = angle(h);
figure; plot(w,mag); grid on
figure; plot(w(175:340),mag(175:340)); grid on
figure; semilogy(w,mag); grid on
figure; plot(w,phase); grid on
```

Read more in Matlab on how use the **`freqz()`** function.

Here, **`semilogy();`** produces a plot with a logarithmic vertical scale. What is the role of the following statement: **`plot(w(175:340), mag(175:340));`**?

Interpret the appearance of the magnitude and phase plots. In particular:

i. What are the passband ripple and stopband attenuations of your filter in dB, what is the transition bandwidth, and how accurate are the positions of the band edges relative to the design values?
ii. How many zeros does the filter have? How many of them are on the unit-circle?
iii. How many poles does the filter have, and where?
iv. Explain the significance of the slope of the phase and the location and height of all jumps.

Problem 3: Design an FIR LPF filter satisfying the specs below by applying a window $w(n)$ to the impulse response $h_d(n)$ of an ideal LPF with cutoff $\omega_c = 0.3\pi$. Referring to the lecture slides, which window (rectangle, Bartlett, Hann, Hamming, and Blackman) can be used to meet these specs? Give the corresponding window length M+1 for each window that satisfies the specs. Use Matlab and read help on hamming, hann, blackman, etc. commands.

$$0.95 < H(\omega) < 1.05, \quad 0 \le |\omega| \le 0.25\pi$$

$$-0.1 < H(\omega) < 0.1, \quad 0.35\pi \le |\omega| \le \pi$$

Problem 4: Design an FIR filter satisfying the specs below by applying a Kaiser window $w(n)$ to the impulse response $h_d(n)$ of an ideal LPF with cutoff $\omega_c = 0.64\pi$. Find the required values of β and M required to meet the specs.

$$0.98 < H(\omega) < 1.02, \quad 0 \le |\omega| \le 0.63\pi$$
$$-0.15 < H(\omega) < 0.15, \quad 0.65\pi \le |\omega| \le \pi$$

Problem 5: Assume you are given an ideal LPF with frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| < \pi/4 \\ 0 & \pi/4 < |\omega| \le \pi \end{cases}$$

We wish to derive new filters from this LPF by manipulations of the impulse response $h(n)$.

a) Plot the frequency response $H_1(\omega)$ for the system whose impulse response is $h_1(n) = h(2n)$.

b) Plot the frequency response $H_2(\omega)$ for the system whose impulse response is $h_2(n) = h(n/2)$ for $n = 0, \pm 2, \pm 4, \cdots$ and zero otherwise.

c) Plot the frequency response $H_3(\omega)$ for the system whose impulse response is $h_3(n) = e^{j\pi n} h(n) = (-1)^n h(n)$.

Problem 6: Design an FIR filter satisfying the specs below:

- Passband edge $\omega_p = 0.01\pi$
- Stopband edge $\omega_s = 0.02\pi$
- Maximum stopband gain: $\delta_s \le -60$ dB relative to passband

a) Assume you use the Kaiser window method, find the required values of β and M parameters to meet the specs.

b) Next, the filter turns out to be inappropriate because of the large number of computations required. A suggestion was given to design the filter as a cascade of two stages: $h'(n) = p(n) * q(n)$. To design $p(n)$, first design a filter, $g(n)$, with passband edge $\omega_p' = 10\omega_p$, stopband $\omega_s' = 10\omega_s$, and stopband gain $\delta_s' = \delta_s$. The filter $p(n)$ can then be obtained by expanding $g(n)$ by a factor of 10:

$$p(n) = \begin{cases} g(n/10) & \text{when } n/10 \text{ is integer} \\ 0 & \text{otherwise} \end{cases}$$

What are the required values of β' and M' parameters to meet the specs for $g(n)$?

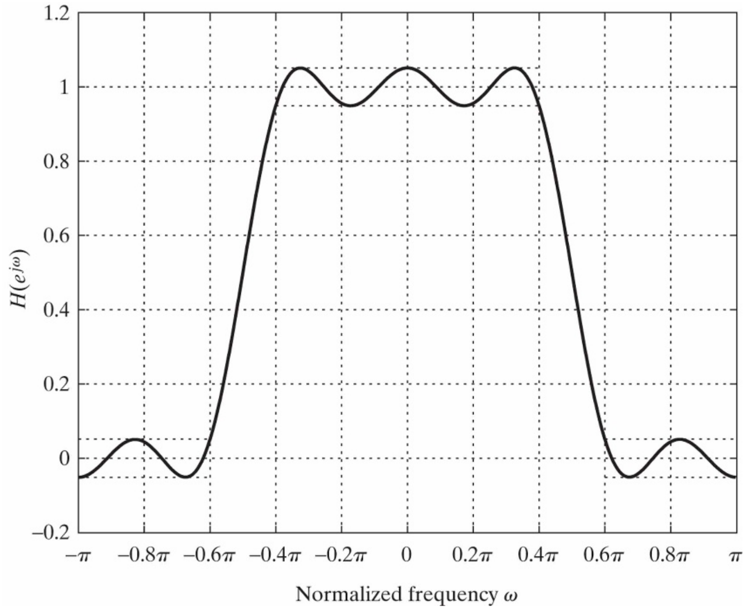
c) Draw $P(\omega)$ from $\omega = 0$ to $\omega = \pi/4$.

d) What specs should be used in designing $q(n)$ to guarantee that $h'(n) = p(n) * q(n)$ meets or exceeds the original specs? Specify the passband edge ω_p'', stopband edge ω_s'', and stopband attenuation δ_p'', for $q(n)$.

e) What are the required values of β'' and M'' parameters to meet the specs for $q(n)$? How many non-zero samples will $h'(n) = p(n) * q(n)$ have?

f) Now comparing the complexities of the filter $h(n)$ designed in part (a) with $h'(n)$ designed in parts (b)-(f) as the convolution $h'(n) = p(n) * q(n)$, which filter requires fewer non-trivial multiplications?

Problem 7: A zero-phase FIR filter $h(n)$ has an associated frequency response as shown below. The filter was designed using the Parks-McClellan (PM) algorithm with the input parameters shown below.



- Passband edge $\omega_p = 0.4\pi$
- Stopband edge $\omega_s = 0.6\pi$
- Error weighting function: $W(\omega) = 1$
- Ideal gain in passband: $G_p = 1$
- Ideal gain in stopband: $G_s = 0$

Also, the length of the filter is $N + 1 = 2L + 1$ where $h(n) = 0$ for $|n| > L$. The value of L is unknown.

It is claimed that there are two filters, each with frequency response identical to that shown in the figure above, and each having been designed by the PM algorithm with different values for the input parameter L:

- Filter 1: $L = L_1$ and length $N_1 + 1 = 2L_1 + 1$
- Filter 2: $L = L_2 > L_1$ and length $N_2 + 1 = 2L_2 + 1$

Both filters were designed exactly using the same PM algorithm and input parameters, except for the value of L.

a) What are the possible values for L_1?
b) What are the possible values for $L_2 > L_1$?
c) Are the impulse responses $h_1(n)$ and $h_2(n)$ of the two filters identical?
d) The alternation theorem guarantees uniqueness of the r^{th}-order polynomial. If your answer to (c) is yes, explain why the alternation theorem is not violated. If your answer is no, show how the two filters $h_1(n)$ and $h_2(n)$ are related.

Problem 8: We wish to design a band-pass filter using Parks-McClellan (PM) algorithm. The specifications are:

$\omega_{s1} = 0.16\pi$
$\omega_{p1} = 0.32\pi$
$\omega_{p2} = 0.67\pi$
$\omega_{s2} = 0.83\pi$
$\delta_p = \delta_{s1} = \delta_{s2} = 0.01$.

Find the smallest filter order N that meets this specification. Plot the impulse response of this filter. For any value of N, the following MATLAB code will be helpful. The function `firpm()` implements the PM algorithm. Type help `firpm` in Matlab to learn how to use the function.

```
f = [0 0.16 0.32 0.67 0.83 1];
m = [0 0 1 1 0 0];
w = [1 1 1];
a = firpm(N,f,m,w);
```

The vector `f` corresponds to $[0, \omega_{s1}/\pi, \omega_{p1}/\pi, \omega_{p2}/\pi, \omega_{s2}/\pi, \pi/\pi]$. The vector `m` contains the desired values of the magnitude at the frequencies in `f`, and the vector `w` is the error weighting in the passband and stopbands. You need to plot the resulting magnitude response and check if the conditions on the errors are met in the pass-band and the two stop-bands.