Prof. M. Mansour **American University of Beirut**
Department of Electrical and Computer Engineering
EECE 491 – Discrete-time Signal Processing

Problem Set: Frequency Analysis of LTI Systems

Problem 1: A simple causal LTI system characterized by the following unit sample response:

$$h[0]=2$$
$$h[1]=3$$
$$h[2]=2$$
$$h[n]=0, \quad \forall n>2$$

a) What is the frequency response, $H\left(e^{j\omega}\right)$, of this LTI system?

b) What is the magnitude of $H\left(e^{j\omega}\right)$ at $\omega=0,\pi/2,\pi$?

c) If this LTI system is used as a filter, what is the set of frequencies that are removed?

Problem 2: Let $x[n]$ be a causal, N-point sequence that is zero outside the range $0\le n\le N-1$. When $x[n]$ is the input to the causal LTI system represented by the difference equation:

$$y[n]-\frac{1}{4}y[n-2]=x[n-2]-\frac{1}{4}x[n]$$

the output is $y[n]$, also a causal, N-point sequence.

a) Show that the causal LTI system described by this difference equation represents an all-pass filter.

b) Given that

$$\sum_{n=0}^{N-1}\left|x[n]\right|^2=5$$

determine the value of

$$\sum_{n=0}^{N-1}\left|y[n]\right|^2$$

Problem 3: Suppose a causal linear time invariant (LTI) system with frequency response $H\left(e^{j\omega}\right)$ is described by the following difference equation relating input $x[n]$ to output $y[n]$:

$$y[n]=x[n]+\alpha x[n-1]+\beta x[n-2]+\gamma x[n-3]$$

Here, α, β, and γ are constants independent of ω.

a) Determine the values of α, β and γ so that the frequency response of system H is $H\left(e^{j\omega}\right)=1-0.5e^{-j2\omega}\cos\omega$.

b) Suppose that $y[\cdot]$, the output of the LTI system with frequency response $H\left(e^{j\omega}\right)$, is used as the input to a second causal LTI system with frequency response $G\left(e^{j\omega}\right)$, producing $W\left(e^{j\omega}\right)$. If $H\left(e^{j\omega}\right)=1-0.5e^{-j2\omega}\cos\omega$, what should the frequency response, $G\left(e^{j\omega}\right)$, be so that $w[n]=x[n]$ for all n? Here $W\left(e^{j\omega}\right)=\text{DTFT}\{w[n]\}$.

c) How would you choose the coefficients α, β, and γ so that $H\left(e^{j\omega}\right)$ has linear phase?

Problem 4: An LTI system with impulse response $h_1[n]$ is an ideal lowpass filter with cutoff frequency $\omega_c = \pi/2$. The frequency response of the system is $H_1(e^{j\omega})$. Suppose a new LTI system with impulse response $h_2[n]$ is obtained from $h_1[n]$ by $h_2[n] = (-1)^n h_1[n]$. Sketch the frequency response $H_2(e^{j\omega})$.

Problem 5: A sequence $x[n]$ is the output of a linear time-invariant system whose input is $s[n]$. This system is described by the difference equation

$$x[n] = s[n] - e^{-8\alpha} s[n-8], \qquad (1.1)$$

where $\alpha > 0$.

a) Find the system function

$$H(z) = \frac{X(z)}{S(z)}$$

and plot its poles and zeros in the z-plane. Indicate the region of convergence.

b) We wish to recover $s[n]$ from $x[n]$ with a linear time-invariant system. Find the system function

$$H_2(z) = \frac{Y(z)}{X(z)}$$

such that $y[n] = s[n]$. Find all possible regions of convergence for $H_2(z)$, and for each, tell whether or not the system is causal and/or stable.

c) Find all possible choices for the impulse response $h_2[n]$ such that

$$y[n] = h_2[n] * x[n] = s[n]$$

d) For all choices determined in Part (c), demonstrate, by explicitly evaluating the convolution in Eq.(1.1), that when $s[n] = \delta[n]$, then $y[n] = \delta[n]$.

Problem 6: Consider an LTI system with input $x[n]$ and output $y[n]$. When the input to the system is

$$x[n] = 5\frac{\sin(0.4\pi n)}{\pi n} + 10\cos(0.5\pi n)$$

the corresponding output is

$$y[n] = 10\frac{\sin(0.3\pi(n-10))}{\pi(n-10)}$$

Determine the frequency response $H(e^{j\omega})$ and the impulse response $h[n]$ for the LTI system.

<u>Problem 7</u>: Figure 1 shows the pole–zero plots for three different causal LTI systems with real impulse responses. Indicate which of the following properties apply to each of the systems pictured: stable, IIR, FIR, minimum phase, all-pass, linear phase, positive group delay at all ω :

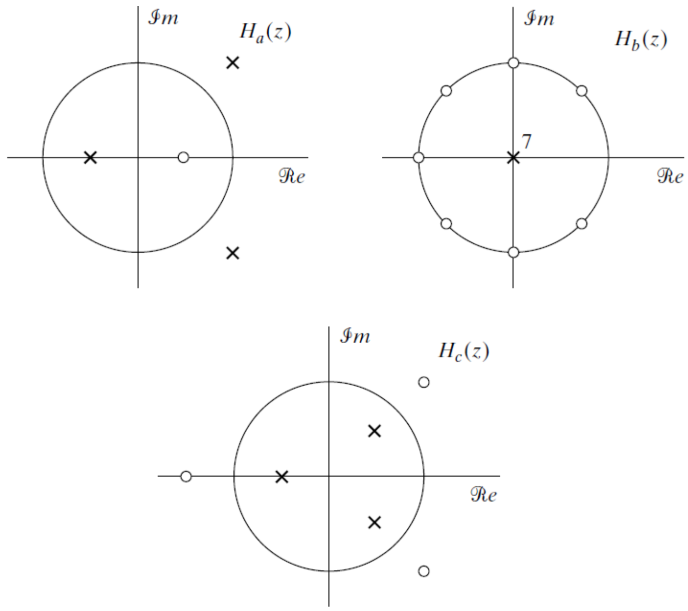

Figure 1: Pole-Zero plots