Prof. M. Mansour
American University of Beirut
Department of Electrical and Computer Engineering
EECE 491 – Discrete-time Signal Processing

Problem Set: The z-Transform

Problem 1: An LTI system is characterized by the system function

$$H(z) = \frac{\left(1 - \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}, \quad |z| > \frac{1}{2}$$

a) Determine the impulse response of the system.

b) Determine the difference equation relating the system input $x[n]$ and the system output $y[n]$.

Problem 2: Consider an LTI system that is stable and for which the z-transform of the impulse response, is given by

$$H(z) = \frac{3 - 7z^{-1} + 5z^{-2}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

Suppose $x[n]$, the input to the system, is a unit step sequence.

a) Find the output $y[n]$ by evaluating the discrete convolution of $x[n]$ and $h[n]$.

b) Find the output $y[n]$ by computing the inverse z-transform of $Y(z)$.

Problem 3: Consider a stable linear time-invariant system. The z-transform of the impulse response is

$$H(z) = \frac{z^{-1} + z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}$$

Suppose $x[n]$, the input to the system, is $2u[n]$. Determine $y[n]$ at $n = 1$. (Read Section 3.4 in Proakis and example 3.4.1.)

Problem 4: Suppose the z-transform of $x[n]$, is given by

$$X(z) = \frac{z^{10}}{\left(z - \frac{1}{2}\right)\left(z - \frac{3}{2}\right)^{10}\left(z + \frac{3}{2}\right)^{2}\left(z + \frac{5}{2}\right)\left(z + \frac{7}{2}\right)}$$

It is also know that $x[n]$ is a stable sequence. (Read Section 3.4 in Proakis and example 3.4.1.)

a) Determine the region of convergence of $X(z)$.

b) Determine $x[n]$ at $n = -8$

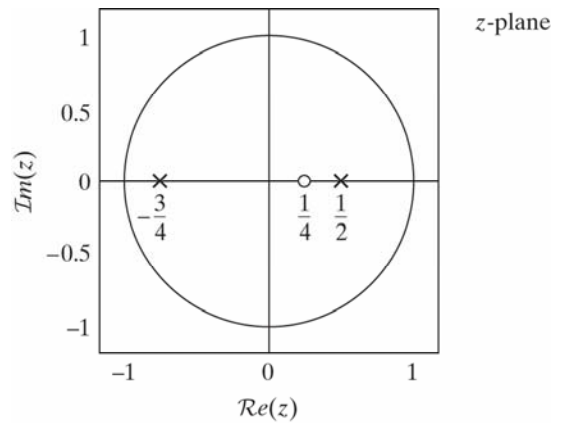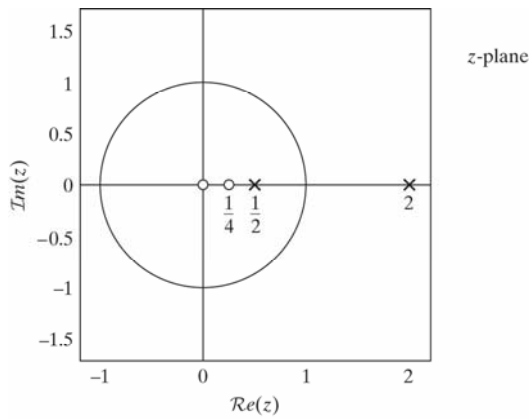**Problem 5:** When the input to a causal LTI system is

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^{n} u[n] - \frac{4}{3} 2^{n} u[-n-1]$$

The z-transform of the output is

$$Y(z) = \frac{1+z^{-1}}{\left(1-z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

a) Find the z-transform of $x[n]$.

b) Determine the region of convergence of $Y(z)$.

c) Find the impulse response of the system.

d) Is the system stable?

Problem 6: The signal $y[n]$ is the output of an LTI system with impulse response $h[n]$ for a given input $x[n]$. Assume in the following that $y[n]$ is stable and has z-transform $Y(z)$ with the pole-zero plot diagram shown below on the left. The signal $x[n]$ is stable and the pole-zero plot diagram shown below on the right.

a) Determine the region of convergence of $Y(z)$.
b) Is $y[n]$ left sided, right sided, or two sided?
c) Determine the region of convergence of $X(z)$.
d) Is $x[n]$ a causal sequence?
e) What is $x[0]$?
f) Draw the pole-zero plot of $H(z)$ and specify its ROC.
g) Is $h[n]$ anticausal?