**United Nations
Educational, Scientific and
Cultural Organization**

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Water Resource Systems Planning and Management
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Exercises

Content

Chapter 1 Water Resources Planning and Management: an overview

1.1 How would you define 'Integrated Water Resources Management" and what distinguishes it from "Sustainable Water Resources Management"?

1.2 Can you identify some common water management issues that are found in many parts of the world?

1.3 Comment on the common practice of governments giving aid to those in drought or flood areas without any incentives to alter land use management practices in anticipation of the next flood or drought.

1.4 What tools are available for integrated water resources planning and management?

1.5 What structural and non-structural measures can be taken to better manage water resources?

1.6 Find the following statistics:

• % freshwater resources worldwide available for drinking:	
• Number of people who die each year from diseases associated with unsafe drinking water:	
• % freshwater resources in polar regions:	
• U.S. per capita annual withdrawal of cubic meters of freshwater:	
• World per capita annual withdrawal of cubic meters of freshwater:	
• Tons of pollutants entering U.S. lakes and rivers daily:	
• Average number of gallons of water consumed by humans in a lifetime:	
• Average number of kilometers per day a woman in a developing country must walk to fetch fresh water:	

1.7 Briefly describe the 6 greatest rivers in the world.

1.8 Identify some of the major water resource management issues in the region where you live. What management alternatives might effectively reduce some of the problems or provide additional economic, environmental, or social benefits.

1.9 Describe some water resource systems consisting of various interdependent components. What are the inputs to the systems and what are their outputs?   How did you decide what to include in the system and what not to include?   How did you decide on the level of spatial and temporal detail to be included?

1.10 Sustainability is a concept applied to renewable resource management. In your words define what that means and how it can be used in a changing and uncertain environment both with respect to water supplies and demands. Over what space and time scales is it applicable, and how can one decide whether or not some plan or management policy will be sustainable? How does this concept relate to the adaptive management concept?

1.11 Identify and discuss briefly some of the major issues and challenges facing water managers today.

Chapter 2 Water Resource Systems Modelling: its role in planning and management

2.1 What is a system?

2.2 What is systems analysis?

2.3 What is a mathematical model?

2.4 Why develop and use models?

2.5 What is a decision support system?

2.6 What is shared vision modeling and planning?

2.7 What characteristics of water resources planning or management problems make them suitable for analysis using quantitative systems analysis techniques?

2.8 Identify some specific water resource systems planning problems and for each problem specify in words possible objectives, the unknown decision variables whose values need to be determined, and the constraints or that must be met by any solution of the problem.

2.9 From a review of the recent issues of various journals pertaining to water resources and the appropriate areas of engineering, economics, planning and operations research, identify those journals that contain articles on water resources systems planning and analysis, and the topics or problems currently being discussed.

2.10 Many water resource systems planning problems involve considerations that are very difficult if not impossible to quantify, and hence they cannot easily be incorporated into any mathematical model for defining and evaluating various alternative solutions. Briefly discuss what value these admittedly incomplete quantitative models may have in the planning process when non-quantifiable aspects are also important. Can you identify some planning problems that have such intangible objectives?

2.11 Define integrated water management and what that entails as distinct from just water management.

2.12 Water resource systems serve many purposes and can satisfy many objectives. What is the difference between purposes and objectives?

2.13 How would you characterize the steps of a planning process aimed at solving a particular problem?

2.14 Suppose you live in an area where the only source of water (at a reasonable cost) is from an aquifer that receives no recharge. Briefly discuss how you might develop a plan for its use over time.
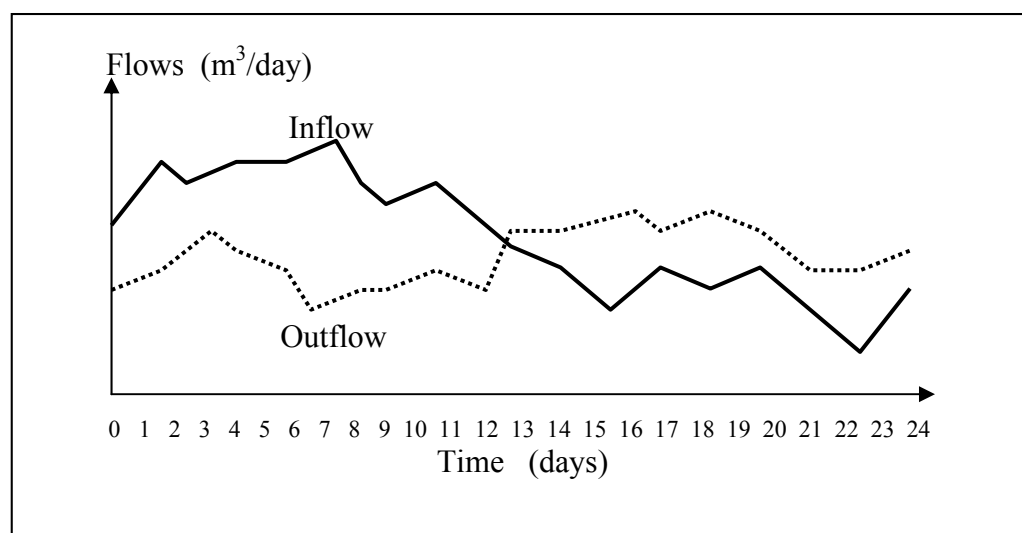
Chapter 3 Modelling methods for Evaluating Alternatives

3.1 Briefly outline why multiple disciplines are needed to efficiently and effectively manage water resources in major river basins, or even in local watersheds.

3.2   Describe in a page or two what some of the issues are in the region where you live.

3.3   Define adaptive management, shared vision modeling, and sustainability.

3.4   Distinguish what a manager does from what an analyst (modeler) does.

3.5 Identify some typical or common water resources planning or management problems that are suitable for analysis using quantitative systems analysis techniques.

3.6 Consider the following five alternatives for the production of energy (10^3 kwh/day) and irrigation supplies (10^6 m^3/month):

Alternative	Energy Production	Irrigation Supply
A	22	20
B	10	35
C	20	32
D	12	21
E	6	25

Which alternative would be the best in your opinion and why?   Why might a decision maker select alternative E even realizing other alternatives exist that can give more hydropower energy and irrigation supply?

3.7 Define a model similar to Equations 3.1 to 3.3 for finding the dimensions of a cylindrical tank that minimizes the total cost of storing a specified volume of water. What are the unknown decision variables?   What are the model parameters?   Develop an iterative approach for solving this model.

3.8 Briefly distinguish between simulation and optimization.

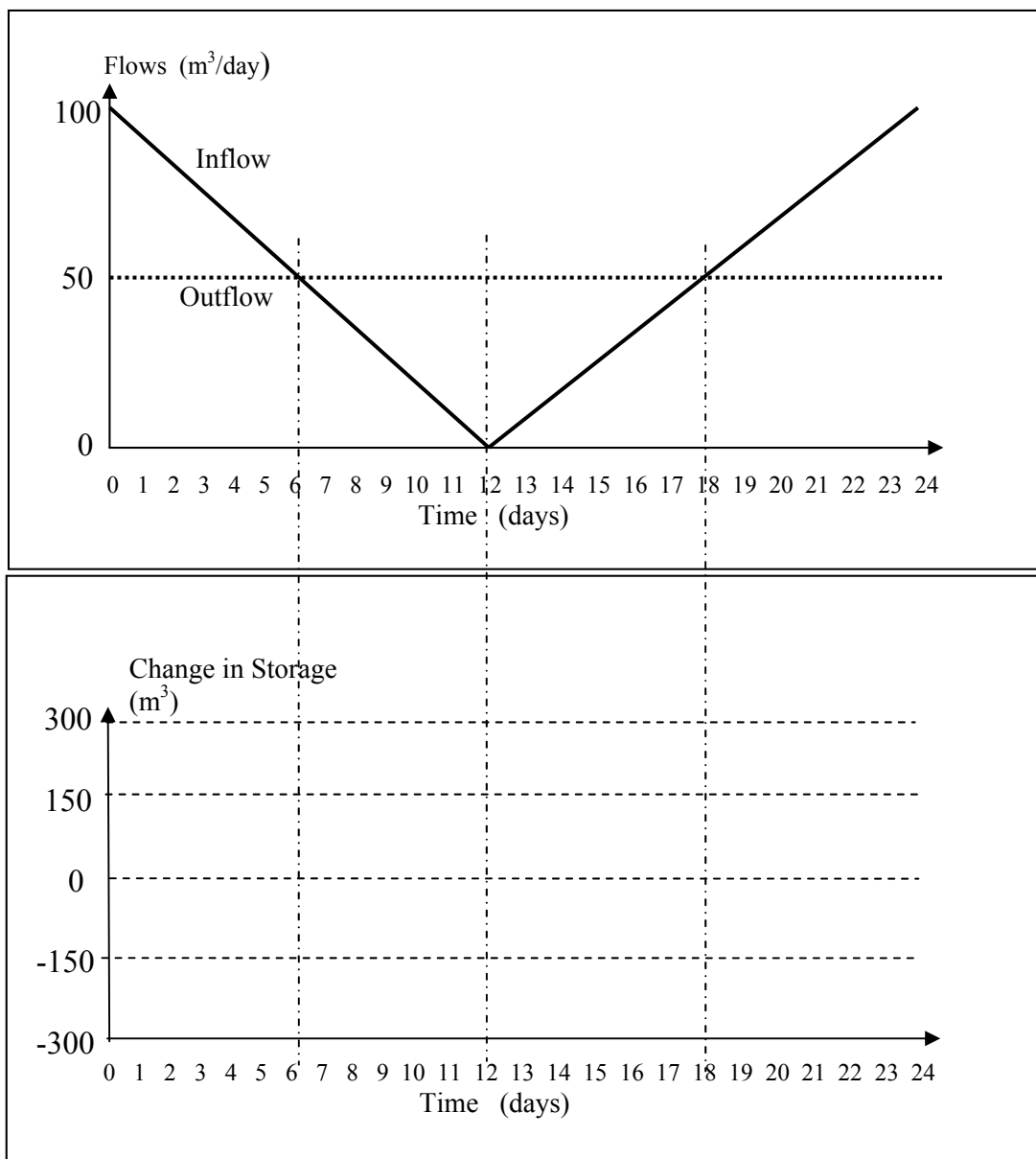3.9   Consider a tank, a lake or reservoir or an aquifer having inflows and outflows as shown in the graph below.

       a) When was the inflow its maximum and minimum values?
       b) When was the outflow its minimum value?
       c) When was the storage volume its maximum value?
       d) When was the storage volume its minimum value?
       e) Write a mass balance equation for the time series of storage volumes assuming constant inflows and outflows during each time period.

3.10 Describe, using words and a flow diagram, how you might simulate the operation of a storage reservoir over time. To simulate a reservoir, what data do you need to have or know?

3.11 Identify and discuss a water resources planning situation that illustrates the need for a combined optimization-simulation study in order to identify the best alternative solutions and their impacts.

3.12 Given the changing inflows and constant outflow from a tank or reservoir, as shown in the graph below, sketch a plot of the storage volumes over the same period of time. Show how to determine the value of the slope of the storage volume plot at any time from the inflow and outflow graph below.

3.13 Write a flow chart/computer simulation program for computing the maximum yield of water that can be obtained given any value of active reservoir storage capacity, *K,* using.

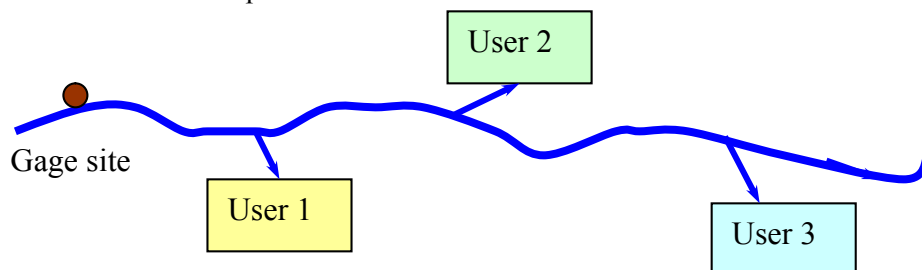| Year y	Flow Q_v	Year y	Flow Q_v
1	5	9	3
2	7	10	6
3	8	11	8
4	4	12	9
5	3	13	3
6	3	14	4
7	2	15	9
8	1		

Find the values of the storage capacity *K* required for yields of 2, 3, 3.5, 4, 4.5 and 5.

3.14 How many different simulations of a water resource system would be required to ensure that there is at least a 95% chance that the best solution obtained is within the better 5% of all possible solutions that could be obtained?   What assumptions must be made in order for your answer to be valid? Can any statement be made comparing the value of the best solution obtained from the all the simulations to the value of the truly optimal solution?

3.15 Assume in a particular river basin 20 development projects are being proposed. Assume each project has a fixed capacity and operating policy and it is only a question of which of the 20 projects would maximize the net benefits to the region. Assuming 5 minutes of computer time is required to simulate and evaluate each combination of projects, show that it would require 36 days of computer time even if 99% of the alternative combinations could be discarded using "good judgment." What does this suggest about the use of simulation for regional interdependent multiproject water resources planning?

3.16 Assume you wish to determine the allocation of water X_j to three different users *j*, who obtain benefits $R_j(X_j)$. The total water available is *Q*. Write a flow chart showing how you can find the allocation to each user that results in the highest total benefits.

3.17   Consider the allocation problem illustrated below.

The allocation priority in each simulation period t is:

> First 10 units of streamflow at the gage remain in the stream.
> Next 20 units go to User 3.
> Next 60 units are equally shared by Users 1 and 2.
> Next 10 units go to User 2.
> Remainder goes downstream.

a) Assume no incremental flow along the stream and no return flow from users. Define the allocation policy at each site. This will be a graph of allocation as a function of the flow at the allocation site.

b) Simulate this allocation policy using any river basin simulation model such as RIBASIM, WEAP, Modsim, or other selected model (see CD) for any specified inflow series ranging from 0 to 130 units.

Chapter 4 Optimization Methods

Engineering economics:

4.1 Consider two alternative water resource projects, A and B. Project A will cost $2,533,000 and will return $1,000,000 at the end of 5 years and $4,000,000 at the end of 10 years. Project B will cost $4,000,000 and will return $2,000,000 at the end of 5 and 15 years, and another $3,000,000 at the end of 10 years. Project A has a life of 10 years, and B has a life of 15 years. Assuming an interest rate of 0.1 (10%) per year:

      (a) What is the present value of each project?
      (b) What is each project's annual net benefit?
      (c) Would the preferred project differ if the interest rates were 0.05?
      (d) Assuming that each of these projects would be replaced with a similar project having the same time stream of costs and returns, show that by extending each series of projects to a common terminal year (e.g., 30 years), the annual net benefits of each series of projects would be will be same as found in part (b).

4.2 Show that $\displaystyle\sum_{t=1}^{T}(1+r)^{-t} = \frac{(1+r)^{T}-1}{r(1+r)^{T}}$.

4.3 a) Show that if compounding occurs at the end of m equal length periods within a year in which the nominal interest rate is r, then the *effective* annual interest rate, r', is equal to

$$r' = \left(1+\frac{r}{m}\right)^{m} - 1$$

b) Show that when compounding is continuous (i.e., when the number of periods $m \to \infty$), the compound interest factor required to convert a present value to a future value in year T is e^{rT}. [*Hint*: Use the fact that $\lim_{k \to \infty}(1 + 1/k)^{k} = e$, the base of natural logarithms.]

4.4 The term "capitalized cost" refers to the present value PV of an infinite series of end-of-year equal payments, A. Assuming an interest rate of r, show that as the terminal period $T \to \infty$, PV = A/r.

4.5 The *internal rate of return* of any project is or plan is the interest rate that equals the present value of all receipts or income with the present value of all costs. Show that the internal rate of return of projects A and B in Exercise 4.1 are approximately 8 and 6%, respectively. These are the interest rates r, for each project, that essentially satisfy the equation

$$\sum_{t=0}^{T}(R_{t} - C_{t})(1+r)^{-t} = 0$$

4.6 In Exercise 4.1, the maximum annual benefits were used as an economic criterion for plan selection. The maximum benefit-cost ratio, or annual benefits divided by annual costs, is another criterion. Benefit-cost ratios should be no less than one if the annual benefits are to exceed the annual costs. Consider two projects, I and II:

	Project	
	I	*II*
Annual benefits	20	2
Annual costs	18	1.5
Annual net benefits	2	0.5
Benefit-cost ratio	1.11	1.3

What additional information is needed before one can determine which project is the most economical project?

4.7 Bonds are often sold to raise money for water resources project investments. Each bond is a promise to pay a specified amount of interest, usually semiannually, and to pay the face value of the bond at some specified future date. The selling price of a bond may differ from its face value. Since the interest payments are specified in advance, the current market interest rates dictate the purchase price of the bond.

Consider a bond having a face value of $10,000, paying $500 annually for 10 years. The bond or "coupon" interest rate based on its face value is 500/10,000, or 5%. If the bond is purchased for $10,000, the actual interest rate paid to the owner will equal the bond or "coupon" rate. But suppose that one can invest money in similar quality (equal risk) bonds or notes and receive 10% interest. As long as this is possible, the $10,000, 5% bond will not sell in a competitive market. In order to sell it, its purchase price has to be such that the actual interest rate paid to the owner will be 10%. In this case, show that the purchase price will be $6927.

The interest paid by the some bonds, especially municipal bonds, may be exempt from state and federal income taxes. If an investor is in the 30% income tax bracket, for example, a 5% municipal tax-exempt bond is equivalent to about a 7 % taxable bond. This tax exemption helps reduce local taxes needed to pay the interest on municipal bonds, as well as providing attractive investment opportunities to individuals in high tax brackets.

Lagrange Multipliers

4.8 What is the meaning of the Lagrange multiplier associated with the constraint of the following model?

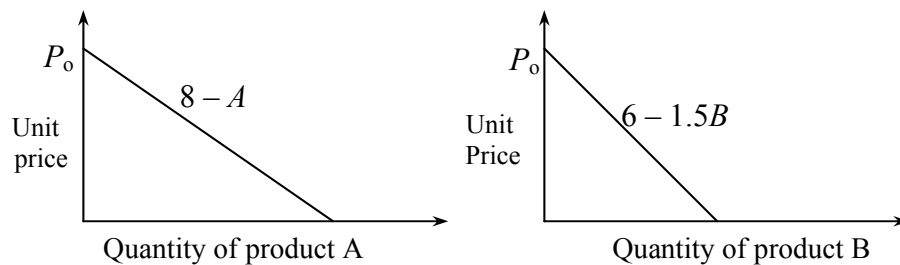Maximize   Benefit(X) – Cost(X)

Subject to: X ≤ 23

4.9 Assume water can be allocated to three users. The allocation, x_j, to each use j provides the following returns: $R(x_1) = (12x_1 - x_1^2)$, $R(x_2) = (8x_2 - x_2^2)$ and $R(x_3) = (18x_3 - 3x_3^2)$. Assume that the objective is to maximize the total return, $F(X)$, from all three allocations and that the sum of all allocations cannot exceed 10. a) How much would each use like to have? b) Show that at the maximum total return solution the marginal values, $\partial(R(x_j))/\partial x_j$, are each equal to the shadow price or Lagrange multiplier (dual variable)   associated with the constraint on the amount of water available. c) Finally, without resolving a Lagrange multiplier problem, what would the solution be if 15 units of water were available to allocate to the three users and what would be the value of the Lagrange multiplier?

4.10 In Exercise 4.9, how would the Lagrange multiplier procedure differ if the objective function, $F(X)$, were to be minimized?

4.11 Assume that the objective was to minimize the sum of squared deviations of the actual allocations x_j from some desired or known target allocations T_j. Given a supply of water Q less than the sum of all target allocations T_j, structure a planning model and its corresponding Lagrangian. Will a global minimum be obtained from solving the partial differential equations derived from the Lagrangian? Why?

4.12 Using Lagrange multipliers, prove that the least-cost design of a cylindrical storage tank of any volume $V > 0$ has one-third of its cost in its base and top and two-thirds of its cost in its side, regardless of the cost per unit area of its base or side. (It is these types of rules that end up in handbooks in engineering design.)

4.13 An industrial firm makes two products, A and B. These products require water and other resources. Water is the scarce resource-they have plenty of other needed resources. The products they make are unique, and hence they can set the unit price of each product at any value they want to. However experience tells them that the higher the unit price for a product, the less amount of that product they will sell. The relationship between unit price and quantity that can be sold is given by the following two *demand* functions.

(a) What are the amounts of A and B, and their unit prices, that maximize the total revenue obtained?

(b) Suppose the total amount of A and B could not exceed some amount T^{max}. What are the amounts of A and B, and their unit prices, that maximize total revenue, if

   i) $T^{max} = 10$

   ii) $T^{max} = 5$

Water is needed to make each unit of A and B. The production functions relating the amount of water X_A needed to make A, and the amount of water X_B needed to make B are $A = 0.5\,X_A$, and $B = 0.25\,X_B$, respectively.

(c) Find the amounts of A and B and their unit prices that maximize total revenue assuming the total amount of water available is 10 units.

(d) What is the value of the dual variable, or shadow price, associated with the 10 units of available water?

Dynamic programming

4.14 Solve for the optimal integer allocations x_1, x_2, and x_3 for the problem defined by Exercise 4.9 assuming the total available water is 3 and 4. Also solve for the optimal allocation policy if the total water available is 7 and each x_j must not exceed 4.

4.15   Consider a three-season reservoir operation problem. The inflows are 10, 50 and 20 in seasons 1, 2 and 3 respectively. Find the operating policy that minimizes the sum of total squared deviations from a constant storage target of 20 and a constant release target of 25 in each of the three seasons. Develop a discrete dynamic programming model that considers only 4 discrete storage values: 0, 10, 20 and 30. Assume the releases cannot be less than 10 or greater than 40. Show how the model's recursive equations change depending on whether the decisions are the releases or the final storage volumes. Verify the optimal operating policy is the same regardless of whether the decision variables are the releases or the final storage volumes in each period. Which model do you think is easier to solve? How would each model change if more importance were given to the desired releases than to the desired storage volumes?

4.16 Show that the constraint limiting a reservoir release, r_t, to be no greater than the initial storage volume, s_t, plus inflow, i_t, is redundant to the continuity equation

$$s_t + i_t - r_t = s_{t+1}.$$

4.17 Develop a general recursive equation for a forward-moving dynamic programming solution procedure for a single reservoir operating problem. Define all variables and functions used. Why is this not a very useful approach to finding a reservoir operating policy?

4.18 The following table provides estimates for the recent values of the costs of additional wastewater treatment plant capacity needed at the end of each 5-year period for the next 20 years. Find the capacity expansion schedule that minimizes the present values of the total future costs. If there is more than one least cost solution, indicate which one you think is better, and why?

		DISCOUNTED COST OF ADDITIONAL CAPACITY: UNITS OF ADDITIONAL CAPACITY					*Additional Required Capacity at End of Period[a]*
Period	Years	2	4	6	8	10	
1	1-5	12	15	18	23	26	2
2	6-10	8	11	13	15		6
3	11-15	6	8				8
4	16-20	4					10

[a]This is the total required capacity that must exist at the end of period.

The cost in each period t must be paid at the beginning of the period. What was the discount factor used to convert the costs at the beginning of each period t to present value costs shown above. In other words how would a cost at the beginning of period t be discounted to the beginning of period 1, given an annual interest rate of r? (Only the algebraic expression of the discount factor is asked, not the numerical value of r.)

4.19 Consider a wastewater treatment plant in which it is possible to include five different treatment processes in series. These treatment processes must together remove at least 90% of the 100 units of influent waste. Assuming the R_i is the amount of waste removed by process i, the following conditions must hold:

$$20 \le R_1 \le 30$$
$$0 \le R_2 \le 30$$
$$0 \le R_3 \le 10$$
$$0 \le R_4 \le 20$$
$$0 \le R_5 \le 30$$

(a) Write the constrained optimization-planning model for finding the least-cost combination of the removals R_i that together will remove 90% of the influent waste. The cost of the various discrete sizes of each unit process i depend upon the waste entering the process i as well as the amount of waste removed, as indicated in the table below.

PROCESS i:		1	2	3	4	5
Influent, I_i	Removal, R_i		Annual Cost $= C_i(I_i, R_i)$			
100	20	5				
100	30	10				
80	10		3	3	1	
80	20		9		2	
80	30		13			
70	10		4	5	2	
70	20		10		3	
70	30		15			
60	10			6	2	3
60	20				4	6
60	30					9
50	10			7	3	4
50	20				5	8
50	30					10
40	10			8	5	5
40	20				7	12
40	30					18
30	10				8	8
30	20				10	12
20	10					8

(b) Draw the dynamic programming network and solve this problem by dynamic programming. Indicate on the network the calculations required to find the least-cost path from state 100 at stage 1 to state 10 at stage 6 using both forward- and backward-moving dynamic programming solution procedures.

(c) Could the following conditions be included in the original dynamic programming model and still be solved without requiring R_4 to be 0 in the first case and R_3 to be 0 in the second case?
(i) $R_4 = 0$ if $R_3 = 0$, or
(ii) $R_3 = 0$ if $R_2 \leq 20$.

4.20 The city of Eutro Falls is under a court order to reduce the amount of phosphorus that which it discharges in its sewage to Lake Algae. The city presently has three wastewater treatment plants. Each plant i currently discharges P_t kg/day of phosphorus into the lake. Some or all plants must reduce their discharges so that the *total* for the three plants does not exceed P kg/day.

Let X_t be the *percent* of the phosphorus by additional treatment at plant i, and the $C_i(X_i)$ the cost of such treatment ($/year) at each plant i.

a) Structure a planning model to determine the least cost (i.e., a cost effective) treatment plant for the city.
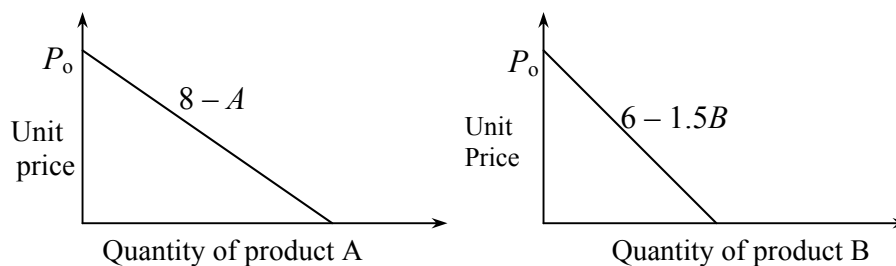
   b) Restructure the model for the solution by dynamic programming. Define the stages, states, decision variables, and the recursive equation for each stage.

   c) Now assume $P_1 = 20$; $P_2 = 15$; $P_3 = 25$; and $P = 20$. Make up some cost data and check the model if it works.

4.21 Find (draw) a *rule curve* for operating a single reservoir that maximizes the sum of the benefits for flood control, recreation, water supply and hydropower. Assume the average inflows in four seasons of a year are 40, 80 60, 20, and the active reservoir capacity is 100. For an average storage S and for a release of R in a season, the hydropower benefits are 2 times the square root of the product of S and R, $2(SR)^{0.5}$ and the water supply benefits are $3R^{0.7}$ in each season. The recreation benefits are $40-(70-S)^2$ in the third season. The flood control benefits are $20 - (40 - S)^2$ in the second season. Specify the dynamic programming recursion equations you are using to solve the problem.

4.22 How would the model defined in Exercise 4.21 change if there were a water user upstream of this reservoir and you were to find the best water allocation policy for that user, assuming known benefits associated with these allocations that are to be included in the overall maximum benefits objective function?

4.23 Suppose there are four water users along a river who benefit from receiving water. Each has a water target, i.e., each expects and plans for a specified amount. These known water targets are W(1), W(2), W(3), and W(4) for the four users respectively. Show how dynamic programming can be used to find two allocation policies. One is to be based on minimizing the maximum deficit deviation from any target allocation. The other is to be based on minimizing the maximum percentage deficit from any target allocation.

4-24 An industrial firm makes two products, A and B. These products require water and other resources. Water is the scarce resource-they have plenty of other needed resources. The products they make are unique, and hence they can set the unit price of each product at any value they want to. However experience tells them that the higher the unit price for a product, the less amount of that product they will sell. The relationship between unit price and quantity that can be sold is given by the following two *demand* functions.

   (a) What are the amounts of A and B, and their unit prices, that maximize the total revenue that can be obtained?   (You can use calculus to solve this problem if you wish.)

   (b) Suppose the total amount of A and B could not exceed some amount T^{max}. What are the amounts of A and B, and their unit prices, that maximize total revenue, if
        iii) $T^{max} = 10$
        iv) $T^{max} = 5$

Water is needed to make each unit of A and B. The production functions relating the amount of water X_A needed to make A, and the amount of water X_B needed to make B are $A = 0.5\,X_A$, and $B = 0.25\,X_B$, respectively.

(c) Find the amounts of A and B and their unit prices that maximize total revenue assuming the total amount of water available is 10 units. Use discrete dynamic programming, both forward- and backward-moving algorithms. You can assume integer values of each water allocation X for this exercise. Show your work on a network. For the backward moving algorithm, also show your work using tables showing the state, S_i, the possible decision variables X_A and X_B and their values, the best decision, and the best value, $F_i(S_i)$, associated with the best decision.

Gradient "Hill-climbing" methods

4.25   Solve Exercise 4.24(b) using hill-climbing techniques and assuming discrete integer values and $T^{max} = 5$. For example, which product would you produce if you could make only 1 unit of either A or B? If you could make another unit of A or B, which would you make? Continue this process up to 5 units of products A and/or B.

Linear and non-linear programming

4.26 Consider the industrial firm that makes two products A and B as described in Exercise 4.24(b). Using *Lingo* (or any other program you wish):
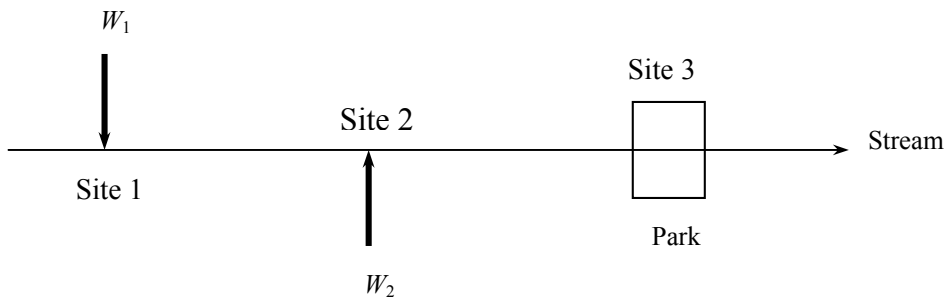
(a) Find the amounts of A and B and their unit prices that maximize total revenue assuming the total amount of water available is 10 units.

(b) What is the value of the dual variable, or shadow price, associated with the 10 units of available water?

(c) Suppose the demand functions are not really certain. How sensitive are the allocations of water to the parameter values in those functions? How sensitive are the allocations to the parameter values 0.5 and 0.25 in the production functions?

4.27 Assume that there are m industries or municipalities adjacent to a river, which discharge their wastes into the river. Denote the discharge sites by the subscript i and let W_i be the kg of waste discharged into the river each day at those sites i. To improve the quality downstream, wastewater treatment plants may be required at each site i. Let x_i be the fraction of waste removed by treatment at each site i. Develop a model for estimating how much waste is removal is required at each site to maintain acceptable water quality in the river at a minimum total cost. Use the following additional notation:

a_{ij} = decrease in quality at site j per unit of waste discharged at site i
q_j = quality at site j that would result if all controlled upstream discharges were eliminated (i.e., $W_1 = W_2 = 0$)
Q_j = minimum acceptable quality at site j
C_i = cost per unit (fraction) of waste removed at site i

4.28 Assume that there are two sites along a stream, $i = 1, 2$, at which waste (BOD) is discharged. Currently, without any wastewater treatment, the quality (DO), q_2 and q_3, at each of sites 2 and 3 is less than the minimum desired, Q_2 and Q_3, respectively. For each unit of

waste removed at site i upstream of site j, the quality improves by A_{ij}. How much treatment is required at sites 1 and 2 that meets the standards at a minimum total cost?
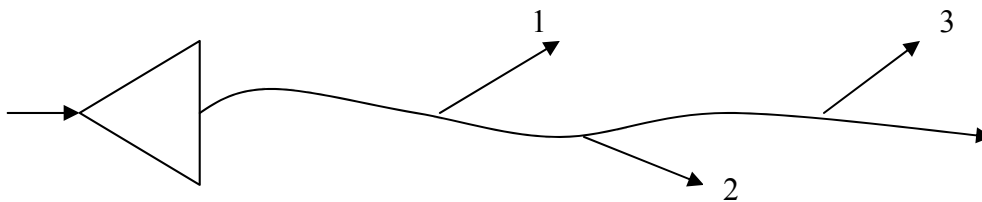

W_1

Site 3

Site 2

Stream

Site 1

Park

W_2

Following are the necessary data:

$C_i =$ cost per unit fraction of waste treatment at site i (both C_1 and C_2 are unknown but for the same amount of treatment, whatever that amount, $C_1 > C_2$)

$R_i =$ decision variables, unknown waste removal fractions at sites $i = 1, 2$

$A_{12} = 1/20$	$W_1 = 100$	$Q_2 = 6$
$A_{13} = 1/40$	$W_2 = 75 Q_3 = 4$	
$A_{23} = 1/30$	$q_2 = 3$	$q_3 = 1$

4.29   Define a linear programming model for finding the tradeoff between active storage capacity and the maximum percentage deviation from a known target storage volume and a known target release in each period. How could the solution of the model be use to define a reservoir policy?

4.30   Consider the possibility of building a reservoir upstream of three demand sites along a river.

1

3

2

The net benefits derived from each use depend on the reliable amounts of water allocated to each use. Letting x_{it} be the allocation to use i in period t, the net benefits for each period t equal

1. $6x_{1t} - x_{1t}^2$
2. $7x_{2t} - 1.5 x_{2t}^2$
3. $8x_{3t} - 0.5 x_{3t}^2$

Assume the average inflows to the reservoir in each of four seasons of the year equal 10, 2, 8, 12.

a) Find the tradeoff between the yield (the 'reliable' release that can be guaranteed in each season) and the reservoir capacity.

b) Find the tradeoff between the yield and the maximum total net benefits that can be obtained from allocating that yield among the three users.

c) Find the tradeoff between the reservoir capacity and the total net benefits one can obtain from allocating the total releases, not just the reliable yield, to the downstream users.

d)   Assuming a reservoir capacity of 5, and dividing the release into integer increments of 2 (i.e., 2, 4, 6 and 8), using linear programming, find the optimal operating policy. Assume the maximum release cannot exceed 8, and the minimum release cannot be less than 2. How does this solution differ from that obtained using DP?

e) If you were maximizing the total net benefit obtained from the three users and if the water available to allocate to the three users were 15 in a particular time period, what would be the value of the Lagrange multiplier or dual variable associated with the constraint that you cannot allocate more than 15 to the three uses?

f)   There is the possibility of obtaining recreational benefits in seasons 2 and 3 from reservoir storage. No recreational benefits can occur in seasons 1 and 4. To obtain these benefits facilities must be built, and the question is at what elevation (storage volume) should they be built. This is called the recreational storage volume target. Recreational benefits in each recreation season equal 8 per unit of storage target if the actual storage equals the storage target. If the actual storage is less than the target the losses are 12 per unit deficit – the difference between the target and actual storage volumes. If the actual storage volume is greater than the target volume the losses are 4 per unit excess. What is the reservoir capacity and recreation storage target that maximizes the annual total net benefits obtained from downstream allocations and recreation in the reservoir less the annual cost of the reservoir, $3K^{1.2}$, where K is the reservoir capacity?

g) In (f) above, suppose the allocation benefits and net recreation benefits were given weights indicating their relative importance. What happens to the relationship between capacity K and recreation target Ts as the total allocation benefits are given a greater weight in comparison to recreation net benefits?

4.31   Using the network representation of the wastewater treatment plant design problem defined in Exercise 4.19, write a linear programming model for defining the least-cost sequence of unit treatment process (i.e., the least-cost path through the network). [*Hint*: Let each decision variable x_{ij} indicate whether or not the link between nodes (or states) i and j connecting two successive stages is on the least-cost or optimal path. The constraints for each node must ensure that what enters the node must also leave the node.]

4.32   Two types of crops can be grown in particular irrigation area each year. Each unit quantity of crop A can be sold for a price P_A and requires W_A units of water, L_A units of land, F_A units of fertilizer, and H_A units of labor. Similarly, crop B can be sold at a unit price of P_B and requires W_B, L_B, F_B and H_B units of water, land, fertilizer, and labor, respectively, per unit of crop. Assume that the available quantities of water, land, fertilizer, and labor are known, and equal W, L, F, and H, respectively.

(a) Structure a linear programming model for estimating the quantities of each of the two crops that should be produced in order to maximize total income.

(b) Solve the problem graphically, using the following data:

	REQUIREMENTS PER UNIT OF:		Maximum Available
Resource	*Crop A*	*Crop B*	*Resources*
Water	2	3	60
Land	5	2	80
Fertilizer	3	2	60
Labor	1	2	40
Unit Price	30	25	

(c) Define the meaning of the dual variables, and their values, associated with each constraint.

(d) Write the dual model of this problem and interpret its objective and constraints.

(e) Solve the primal and dual models using an existing computer program, and indicate the meaning of all output data.

(f) Assume that one could purchase additional water, land, fertilizer, and labor with capital that could be borrowed from a bank at an annual interest rate r. How would this opportunity alter the linear programming model? The objective continues to be a maximization of net income. Assume there is a maximum limit on the amount of money that can be borrowed from the bank.

(g) Assume that the unit price P_j of crop j were a decreasing linear function $(P_j^o - b_j x_j)$ of the quantity, x_j, produced. How could the linear model be restructured also as to identify not only how much of each crop to produce, but also the unit price at which each crop should be sold in order to maximize total income?

4.33 Using linear programming model, derive an annual storage-yield function for a reservoir at a site having the following record of annual flows:

Year y	Flow Q_y	Year y	Flow Q_y
1	5	9	3
2	7	10	6
3	8	11	8
4	4	12	9
5	3	13	3
6	3	14	4
7	2	15	9
8	1		

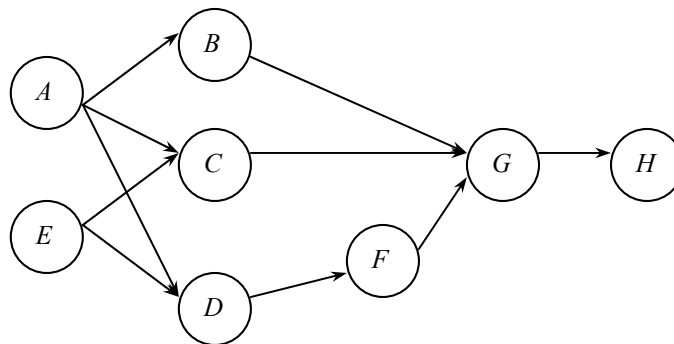a) Find the values of the storage capacity required for yields of 2, 3, 3.5, 4, 4.5, and 5.

b) Develop a flow chart defining a procedure for finding the yields for various increasing values of K.

4.34 Water resources planning usually involves a set of separate tasks. Let the index i denote each task, and H_i the set of tasks that must precede task i. The duration of each task i is estimated to be d_i.

a) Develop a linear programming model to identify the starting times of tasks that maximizes the time, T, required to complete the total planning project.

b) Apply the general model to the following planning project:

Task A: Determine planning objectives and stakeholder interests. Duration: 4 months

Task B: Determine structural and non-structural alternatives that will influence objectives. Duration: 1 month.

Task C: Develop an optimization model for preliminary screening of alternatives and for estimating tradeoffs among objectives. Duration: 1 month.

Task D: Identify data requirements and collect data. Duration: 2 months.

Task E: Develop a data management system for the project. Duration: 3 months.

Task F: Develop an interactive shared vision simulation model with the stakeholders.
Duration: 2 Months.

Task G: Work with stakeholders in an effort to come to a consensus (a shared vision) of the best plan. Duration: 4 months.

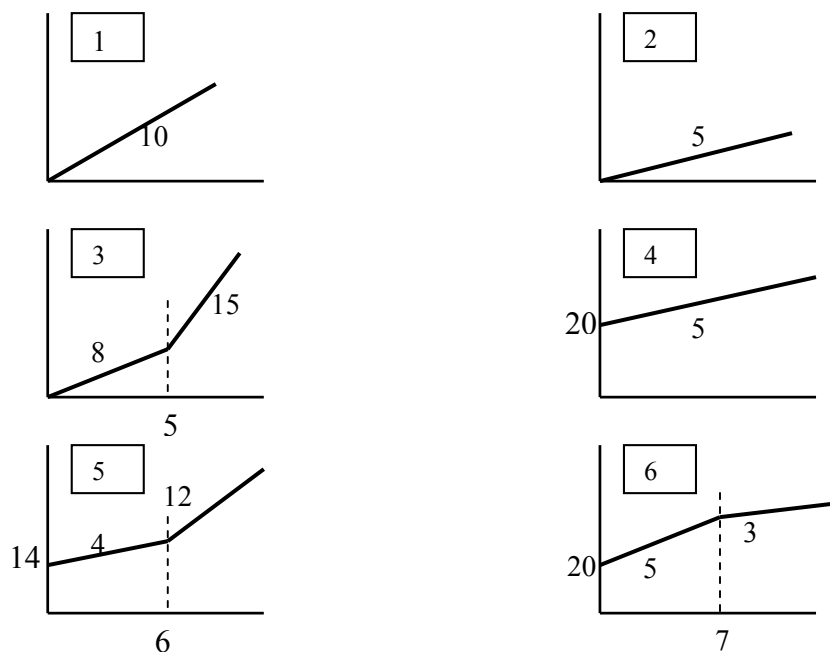Task H: Prepare, present and submit a report. Duration: 2 months.

4.35 In Exercise 4.34 suppose the project is penalized if its completion time exceeds a target T. The difference between 14 months and T months is Δ, and the penalty is $P(\Delta)$. You could reduce the time it takes to complete task E by one month at a cost of $200, and by two months at a cost of $500. Similarly, suppose the cost of task A could be reduced by a month at a cost of $600 and two months at a cost of $1400. Construct a model to find the most economical project completion time. Next modify the linear programming model to find the minimum total added cost if the total project time is to be reduced by 1 or 2 months. What is that added cost and for which tasks?

4.36   Solve the reservoir operation problem described in Exercise 4.15 using linear programming. If the reservoir capacity is unknown, show how a cost function (that includes fixed costs and economies of scale) for the reservoir capacity could be included in the linear programming model.

4.37   An upstream reservoir could be built to serve two downstream users. Each user has a constant water demand target. The first user's target is 30; the second user's target is 50. These targets apply to each of 6 within-year seasons. Find the tradeoff between the required reservoir capacity and maximum deficit to any user at any time, for an average year. The

average flows into the reservoir in each of the six successive seasons are: 40, 80, 100, 130, 70, 50.

4.38   Two groundwater well fields can be used to meet the water demands of a single user. The maximum capacity of the A well field is 15 units of water per period, and the maximum capacity of the B well field is 10 units of water per period. The annual cost of building and operating each well field, each period, is a function of the amount of water pumped and transported from that well field. Three sets of cost functions are shown below: Construct a LP model and use it to define and then plot the total least-cost function and the associated individual well field capacities required to meet demands from 0 to 25, assuming cost functions 1 and 2 apply to well fields A and B respectfully. Next define another least-cost function and associated capacities assuming cost functions 3 and 4 apply to A and B respectively. Finally define a least-cost function and associated capacities assuming well field cost functions 5 and 6 apply. You can check your model results just using common sense – the least-cost functions should be obvious, even without using optimization.

4.39 Referring to Exercise 4.38 above, assume cost functions 5 and 6 represent the cost of adding additional capacity to well fields A and B respectively in any of the next 5 five-year construction periods, i.e., in the next 25 years. Identify and plot the least-cost capacity expansion schedule (one that minimizes the total present value of current and future expansions, assuming demands of 5, 10, 15, 20 and 25 are to be met at the end of years 5, 10, 15, 20 and 25 respectfully. Costs, including fixed costs, of capacity expansion in each construction period have to be paid at the beginning of the construction period.

4.40 Consider a crop production problem involving three types of crops. How many hectares of each crop should be planted to maximize total income?

Resources:	Max Limits		Resource requirements			
		Crops:	Corn	Wheat	Oats	
Water	1000/week		3.0	1.0	1.5	units/week/ha
Labor	300/week		0.8	0.2	0.3	person hrs/week/ha
Land	625 hectares					
Yield $/ha			400	200	250	

Show a two-dimensional graph that defines the optimal solution(s) among Corn, Wheat and
Oats.

4.41 Releases from a reservoir are used for water supply or for hydropower. The benefit per
unit of water allocated to hydropower is BH and the benefit per unit of water allocated to
water supply is BW. For any given release the difference between the allocations to the two
uses cannot exceed 50% of the total amount of water available. Show graphically how to
determine the most profitable allocation of the water for some assumed values of Bh and Bw.
From the graph identify which constraints are binding and what their "dual prices" mean (in
words).

4.42 Suppose there are four water users along a river who benefit from receiving water. Each
has a known water target, i.e., each expects and plans for a specified amount. These known
water targets are W_1, W_2, W_3, and W_4 for the four users respectively. Find two allocation
policies. One is to be based on minimizing the maximum deficit deviation from any target
allocation. The other is to be based on minimizing the maximum percentage deficit from any
target allocation.

Deficit allocations are allocations that are less than the target allocation. For example if a
target allocation is 30 and the actual allocation is 20, the deficit is 10. Water in excess of the
targets can remain in the river. The policies are to indicate what the allocations should be for
any particular river flow Q. The policies can be expressed on a graph showing the amount of
Q on the horizontal axis, and each user's allocation on the vertical axis.

Create the two optimization models that can be used to find the two policies and indicate how
they would be used to define the policies. What are the unknown variables and what are the
known variables? Specify the model in words as well as mathematically.

4.43 In Indonesia there exists a wet season followed by a dry season each year. In one area of
Indonesia all farmers within an irrigation district plant and grow rice during the wet season.
This crop brings the farmer the largest income per hectare; thus they would all prefer to
continue growing rice during the dry season. However, there is insufficient water during the
dry season to irrigate all 5000 hectares of available irrigable land for rice production. Assume
an available irrigation water supply of 32×10^6 m^3 at the beginning of each dry season, and a
minimum requirement of 7000 m^3/ha for rice and 1800 m^3/ha for the second crop.

    (a) What proportion of the 5000 hectares should the irrigation district manager
        allocate for rice during the dry season each year, provided that all available
        hectares must be given sufficient water for rice or the second crop?

    (b) Suppose that crop production functions are available for the two crops, indicating
        the increase in yield per hectare per m^3 of additional water, up to 10, 000 m^3/ha
        for the second crop. Develop a model in which the water allocation per hectare,
        as well as the hectares allocated to each crop, is to be determined, assuming a
        specified price or return per unit of yield of each crop. Under what conditions
        would the solution of this model be the same as in part (a)?

4.44 Along the Nile River in Egypt, irrigation farming is practiced for the production of
cotton, maize, rice, sorghum, full and short berseem for animal production, wheat, barley,
horsebeans, and winter and summer tomatoes. Cattle and buffalo are also produced, and
together with the crops that require labor, water. Fertilizer, and land area (feddans). Farm
types or management practices are fairly uniform, and hence in any analysis of irrigation
policies in this region this distinction need not be made. Given the accompanying data
develop a model for determining the tons of crops and numbers of animals to be grown that

will maximize (a) net economic benefits based on Egyptian prices, and (b) net economic benefits based on international prices. Identify all variables used in the model.

Known parameters:

C_i	= miscellaneous cost of land preparation per feddan
P_i^E	= Egyptian price per 1000 tons of crop i
P_i^I	= international price per 1000 tons of crop i
v	= value of meat and dairy production per animal
g	= annual labor cost per worker
f^P	= cost of P fertilizer per ton
f^N	= cost of N fertilizer per ton
Y_i	= yield of crop i, tons/feddan
α	= feddans serviced per animal
β	= tons straw equivalent per ton of berseem carryover from winter to summer
r^w	= berseem requirements per animal in winter
s^{wh}	= straw yield from wheat, tons per feddan
s^{ba}	= straw yield from barley, tons per feddan
r^s	= straw requirements per animal in summer
μ_i^N	= N fertilizer required per feddan of crop i
μ_i^P	= P fertilizer required per feddan of crop i
l_{im}	= labor requirements per feddan in month m, man-days
w_{im}	= water requirements per feddan in month m, 1000 m^3
h_{im}	= land requirements per month, fraction (1 = full month)

Required Constraints. (assume known resource limitations for labor, water, and land):
(a) Summer and winter fodder (berseem) requirements for the animals.
(b) Monthly labor limitations.
(c) Monthly water limitations.
(d) Land availability each month.
(e) Minimum number of animals required for cultivation.
(f) Upper bounds on summer and winter tomatoes (assume these are known).
(g) Lower bounds on cotton areas (assume this is known).

Other possible constraints:
(a) Crop balances.
(b) Fertilizer balances.
(c) Labor balance.
(d) Land balance.

4.45 In Algeria there are two distinct cropping intensities, depending upon the availability of water. Consider a single crop that can be grown under intensive rotation or extensive rotation on a total of A hectares. Assume that the annual water requirements for the intensive rotation policy are 16000 m^3 per hectare, and for the extensive rotation policy they 4000 m^3 per hectare. The annual net production returns are 4000 and 2000 dinars, respectively. If the total water available is 320,000 m^3, show that as the available land area A increases, the rotation policy that maximizes total net income changes from one that is totally intensive to one that is increasingly extensive.

Would the same conclusion hold if instead of fixed net incomes of 4000 and 2000 dinars per hectares of intensive and extensive rotation, the net income depended on the quantity of crop produced? Assuming that intensive rotation produces twice as much produced by extensive rotation, and that the net income per unit of crop Y is defined by the simple linear function 5 –

0.05Y, develop and solve a linear programming model to determine the optimal rotation policies if A equals 20, 50, and 80. Need this net income or price function be linear to be included in a linear programming model?
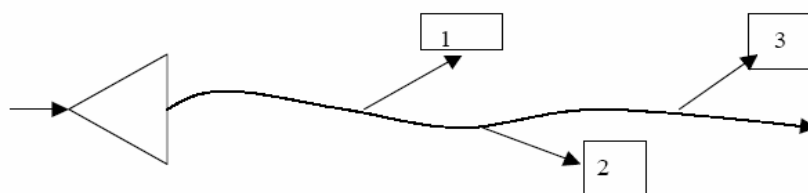
4.46 Current stream quality is below desired minimum levels throughout the stream in spite of treatment at each of the treatment plant and discharge sites shown below. Currently effluent standards are not being met, and minimum desired streamflow concentrations can be met by meeting effluent standards. All current wastewater discharges must undergo additional treatment. The issue is where additional treatment is to occur and how much.

Develop a model to identify cost-effective options for meeting effluent standards where ever wastewater is discharged into the stream. The decisions variables include the amount of wastewater to treat at each site and then release to the river. Any wastewater at any site that is not undergoing additional treatment can be piped to other sites. Identify other issues that could affect the eventual decision.

Assume known current wastewater flows at site i = q_i.
Additional treatment to meet effluent standards cost = $a_i + b_i(D_i)^{c}i$
where D_i is the total wastewater flow undergoing additional treatment at site i and $c_i < 1$.
Pipeline and pumping for each pipeline segment costs approximately $\alpha_{ij} + \beta(q_{ij})^{\gamma}$
where q_{ij} is pipeline flow between adjacent sites i and j and $\gamma < 1$.

4.47 Consider the system shown below where a reservoir is upstream of three demand sites along a river.

The net benefits derived from each use depend on the reliable amounts of water allocated to each use. Letting x_{it} be the allocation to use i in period t, the net benefits for each period t equal

1. $6x_{1t} - x_{1t}^2$
2. $7x_{2t} - 1.5\, x_{2t}^2$
3. $8x_{3t} - 0.5\, x_{3t}^2$

Assume the average inflows to the reservoir in each of four seasons of the year equal 10, 2, 8, 12 units per season and that the reservoir capacity is 5 volume units.

a) Find the optimal operating policy for this reservoir that maximizes the total (four season) allocation benefits for the users.
b) Simulate the operation of the reservoir and the allocation policy and using RIBASIM or WEAP or other simulation program (see CD).

Chapter 5 Fuzzy Optimization

5.1   An upstream reservoir serves as a recreation site for swimmers, wind surfers and boaters. It also serves as a flood storage reservoir in the second period. The reservoir's releases can be diverted to an irrigation area. A wetland area further downstream receives the unallocated portion of the reservoir release plus the return flow from the irrigation area. The irrigation return flow contains a salinity concentration that can damage the ecosystem.

a)   Assume there exist recreation lake level targets, irrigation allocation targets, and wetland flow and salinity targets. The challenge is to determine the reservoir releases and irrigation allocations so as to 'best' meet these targets. This is the crisp' problem.

b) Assume that the targets used in a) above are really fairly fuzzy. Derive fuzzy membership functions for these targets and solve for the 'best' reservoir release and allocation policy based on these fuzzy membership functions. This is the 'fuzzy' problem.

Data:
Reservoir storage capacity: 30 mcm;
During period 2 the flood storage capacity is 5 mcm.
Irrigation return flow fraction: 0.3 (i.e., 30% of that diverted for irrigation);
Salinity concentration of reservoir water: 1 ppt;
Salinity concentration of irrigation return flow: 20 ppt;
Reservoir average inflows for four seasons, respectively: 5, 50, 20, 10 mcm;

Targets for part a):
Target maximum salinity concentration in wetland: 3 ppt;
Target storage target for all seasons: 20 mcm;
Minimum flow target in wetland in each season, respectively: 10, 20, 15, 15 mcm;
Maximum flow target in wetland in each season, respectively: 20, 30, 25, 25 mcm;
Target irrigation allocations: 0, 20, 15, 5 mcm;

a) Find the reservoir releases in each season that best meet the flow and salinity targets in the system. This is the crisp problem.

b) Next create fuzzy membership functions to replace the targets and solve the problem.

Chapter 6 Data-Based Models

6.1   Develop a flow chart showing how you would apply genetic algorithms to finding the parameters, a_{ij}, of a water quality prediction model, such as the one we have used to find the concentration downstream of an upstream discharge site. This will be based on observed values of mass inputs, W_i, and concentrations, C_j, and flows, Q_j, at site j.

$$C_j = \Sigma_i\ W_i\ a_{ij}\ )\ /\ Q_j$$

The objective to be used for fitness is to minimize the sum of the differences between the observed C_j and the computed C_j. To convert this to a maximization objective you could use something like the following:

Max $1\ /\ (1 + D)$

Where $D \geq (C_j$ obs $- C_j$ calculated)
$D \geq (C_j$ calculated $- C_j$ obs.)

6.2 Use the genetic algorithm program called GANLC to predict the parameter values asked for in problem 6.1, and then the artificial neural network ANN to obtain a predictor of downstream water quality based on the values of these parameters. Both GANLC and ANN are contained on the attached CD. You may use the model and data presented in Section 5.2 of Chapter 4 if you wish.

6.3 Using a genetic algorithm program (for example the one called GANLC contained on the CD) find the allocations X_i that maximize the total benefits to the three water users i along a stream, whose individual benefits are:

Use 1: $6\ X_1 - X_1^2$
Use 2: $7\ X_2 - X_2^2$
Use 3: $8\ X_3 - X_3^2$

Assume the available stream flow is some known value (ranging from 0 to 20).

Determine the effect of different genetic algorithm parameter values on the ability to find the best solution.

6.4 Consider the wastewater treatment problem illustrated in the drawing below.

The initial stream concentration just upstream of site 1 is 32. The maximum concentration of the pollutant just upstream of site 2 is 20 mg/l (g/m^3), and at site 3 it is 25 mg/l. Assume the marginal cost per fraction (or percentage) of the waste load removed at site 1 is no less than that cost at site 2, regardless of the amount removed.

Using the genetic algorithm program GANLC (contained on the CD), or other suitable genetic algorithm program, solve for the least cost wastewater treatment at sites 1 and 2 that will satisfy the quality constraints at sites 2 and 3 respectively.

Discuss the sensitivity of the GA parameter values in finding the best solution. You can get the exact solution using LINGO as discussed in Section 5.3 in Chapter 4.

6.5 Develop an artificial neural network (for example using ANN found in the CD) for flow routing given the following two sets of upstream and downstream flows. Use one set of 5-periods for .training. (finding the unknown weights and other variables) and the other set for validation of the calculated parameter values (weights and bias constants). Develop the simplest artificial neural network you can that does an adequate job of prediction.

Time period	Upstream flow	Downstream flow
1	450	366
2	685	593
3	830	755
4	580	636
5	200	325
1	550	439
2	255	304
3	830	678
4	680	679
5	470	534

[These outflows come from the following model, assuming an initial storage in period 1 of 50, the detention storage that will remain in the reach even if the inflows go to 0. For each period t:

$$\text{Outflow(t)} = 1.5(-50 + \text{initial storage(t)} + \text{inflow(t)})^{0.9} \ ]$$

where the outflow is the downstream flow and inflow is the upstream flow.

Chapter 7 Concepts in Probability, Statistics and Stochastic Modelling

7.1 Give an example of a water resources planning study with which you have some familiarity. Make a list of the basic information used in the study and the methods used transform that information into decisions, recommendations, and conclusions.

      (a) Indicate the major sources of uncertainty and possible error in the basic information and in the transformation of that information into decisions, recommendations, and conclusions.

      (b) In systems studies, sources of error and uncertainty are sometimes grouped into three categories:

        1. Uncertainty due to the natural variability of rainfall, temperature, and stream flows which affect a system's operation.
        2. Uncertainty due to errors made in the estimation of the models' parameters with a limited amount of data.
        3. Uncertainty or errors introduced into the analysis because conceptual and/or mathematical models do not reflect the true nature of the relationships being described.

      Indicate, if applicable, into which category each of the sources of error or uncertainty you have identified falls.

7.2 The following matrix displays the joint probabilities of different weather conditions and of different recreation benefit levels obtained from use of a reservoir in a state park:

	POSSIBLE RECREATION BENEFITS		
Weather	RB_1	RB_2	RB_2
Wet	0.10	0.20	0.10
Dry	0.10	0.30	0.20

      (a) Compute the probabilities of recreation levels RB_1, RB_2, RB_3, and of dry and wet weather.

      (b) Compute the conditional probabilities $P(\text{wet} \,|\, RB_1)$, $P(RB_3 \,|\, \text{dry})$, and $P(RB_2 \,|\, \text{wet})$.

7.3 In flood protection planning, the 100-year flood, which is an estimate of the quantile $x_{0.99}$, is often used as the design flow. Assuming that the floods in different years are independently distributed:

      (a) Show that the probability of at least one 100-year flood in a 5-year period is 0.049.

      (b) What is the probability of at least one 100-year flood in a 100-year period?

      (c) If floods at 1000 different sites occur independently, what is the probability of at least one 100-year flood at some site in any single year?

7.4 The price to be charged for water by an irrigation district has yet to be determined. Currently it appears as if there is as 60% probability that the price will be $10 per unit of water and a 40% probability that the price will be $5 per unit. The demand for water is

uncertain. The estimated probabilities of different demands given alternative prices are as follows:

Price / Quantity:	30	55	80	100	120
		Prob. of Quantity Demanded given Price			
$ 5	0.00	0.15	0.30	0.35	0.20
$ 10	0.20	0.30	0.40	0.10	0.00

(a) What is the most likely value of future revenue from water sales?
(b) What are the mean and variance of future water sales?
(c) What is the median value and interquartile range of future water sales?
(d) What price will maximize the revenue from the sale of water?

7.5 Plot the following data on possible recreation losses and irrigated agricultural yields. Show that use of the expected storage level or expected allocation underestimates the expected value of the convex function describing reservoir losses while it overestimates the expected value of the concave function describing crop yield. A concave function $f(x)$ has the property that $f(x) \leq f(x_o) + f'(x_o)(x - x_o)$ for any x_o; prove that use of $f(E[X])$ will always overestimate the expected value of a concave function $f(X)$ when X is a random variable.

Irrigation Water Allocation	Crop Yield/Hectare	Probability of Allocation
10	6.5	0.20
20	10.0	0.30
30	12.0	0.30
40	11.0	0.20

Summer Storage Level	Decrease in Recreation Benefits	Probability of Storage level
200	5	0.10
250	2	0.20
300	0	0.40
350	1	0.20
400	4	0.10

7.6 Complications can be added to the economic evaluation of a project by uncertainty concerning the usefulness life of the project. For example, the time at which the useful life of a reservoir will end due to silting is never known with certainty when the reservoir is being planned. If the discount rate is high and the life is relatively long, the uncertainty may not very important. However, if the life of a reservoir, or of a wastewater treatment facility, or any other such project, relatively short, the practice of using the expected life to calculate present costs or benefits may be misleading.

In this problem, assume that a project results in $1000 of net benefits at the end of each year is expected to last between 10 and 30 years. The probability of ending at the end of each year within the range of 11 to 30 is the same. Given a discount rate of 10%:

(a) Compute the present value of net benefits NB_o, assuming a 20-year project life.
(b) Compare this with the expected present net benefits $E[NB_o]$ taking account of uncertainty in the project lifetime.
(c) Compute the probability that the actual present net benefits is at least $1000 less than NB_o, the benefit estimate based on a 20-year life.
(d) What is the chance of getting $1000 more than the original estimate NB_o?

7.7 A continuous random variable that could describe the proportion of fish or other animals in different large samples which have some distinctive features is the beta distribution whose density is (a > 0, b > 0):

$$f_X(x) = \begin{cases} cx^{\alpha-1}(1-x)^{\beta-1} & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Directly calculate the value of c and the mean and variance of X for $\alpha = \beta = 2$.

(b) In general, $c = \Gamma(\alpha + \beta)/\Gamma(\alpha)\Gamma(\beta)$, where $\Gamma(\alpha)$ is the gamma function equal to $(\alpha - 1)!$ for integer α. Using this information, derive the general expression for the mean and variance of X. To obtain a formula which gives the values of the integrals of interest, note that the expression for c must be such that the integral over (0, 1) of the density function is unity for any α and β.

7.8 The joint probability density of rainfall at two places on rainy days could be described by

$$f_{X,Y}(x, y) \begin{cases} 2/(x + y + 1)^3 & x, y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Calculate and graph:
(a) $F_{XY}(x, y)$, the joint distribution function of X and Y.
(b) $F_Y(y)$, the marginal cumulative distribution function of Y, and $f_Y(y)$, the density function of Y.
(c) $f_{Y|X}(y \mid x)$, the conditional density function of Y given that $X = x$, and $F_{Y|X}(y \mid x)$, the conditional cumulative distribution function of Y given that $X = x$ (the cumulative distribution function is obtained by integrating the density function).

Show that

$$F_{Y|X}(y \mid x = 0) > F_Y(y) \qquad \text{for } y > 0$$

Find a value of x_o and y_o for which

$$F_{Y|X}(y_o \mid x_o) < F_Y(y_o)$$

7.9 Let X and Y be two continuous independent random variables. Prove that

$$E[g(X)h(Y)] = E[g(X)]E[h(Y)]$$

for any two real-valued functions g and h. Then show that Cov$(X, Y) = 0$ if X and Y are independent.

7.10 A frequent problem is that observations (X, Y) are taken on such quantities as flow and concentration and then a derived quantity $g(X, Y)$ such as mass flux is calculated. Given that one has estimates of the standard deviations of the observations X and Y and their correlation, an estimate of the standard deviation of $g(X, Y)$ is needed. Using a second-order Taylor series expansion for the mean of $g(X, Y)$ as a function of its partial derivatives and of the means, variances, covariance of the X ad Y. Using a first-order approximation of $g(X, Y)$, obtained an estimates of the variances of $g(X, Y)$ as a function of its partial derivatives and the moments of X and Y. Note, the covariance of X and Y equals

$$E[(X - \mu_X)(Y - \mu_Y)] = \sigma^2{}_{XY}$$

7.11 A study of the behavior of water waves impinging upon and reflecting off a breakwater located on a sloping beach was conducted in a small tank. The height (crest-to-trough) of the waves was measured a short distance from the wave generator and at several points along the beach different distances from the breakwater were measured and their mean and standard error recorded.

Location	Mean Wave Height (cm)	Standard Error of Mean (cm)
Near wave generator	3.32	0.06
1.9 cm from breakwater	4.42	0.09
1.9 cm from breakwater	2.59	0.09
1.9 cm from breakwater	3.26	0.06

At which points were the wave heights significantly different from the height near wave generator assuming that errors were independent?

Of interest to the experimenter is the ratio of the wave heights near the breakwater to the initial wave heights in the deep water. Using the results in Exercise 7.10, estimate the standard error of this ratio at the three points assuming that errors made in measuring the height of waves at the three points and near the wave generator are independent. At which point does the ratio appear to be significantly different from 1.00?

Using the results of Exercise 7.10, show that the ratio of the mean wave heights is probably a biased estimate of the actual ratio. Does this bias appear to be important?

7.12 Derive Kirby's bound, Equation 7.45, on the estimate of the coefficient of skewness by computing the sample estimates of the skewness of the most skewed sample it would be possible to observe. Derive also the upper bound $(n - 1)^{1/2}$ for the estimate of the population coefficient of variation

$$\frac{\nu_x}{\bar{x}}$$

when all the observations must be nonnegative.

7.13 The errors in the predictions of water quality models are sometimes described by the double exponential distribution whose density is

$$f(x) = \frac{\alpha}{2}\exp(-\alpha|x - \beta|\,) \qquad\qquad -\infty < x < +\infty$$

What are the maximum likelihood estimates of α and β?   Note that

$$\frac{d}{d\beta}|x - \beta| \;=\; \begin{cases} -1 & x > \beta \\[2mm] +1 & x < \beta \end{cases}$$

Is there always a unique solution for β?

7.14 Derive the equations that one would need to solve to obtain maximum likelihood estimates of the two parameters α and β of the gamma distribution. Note an analytical expression for $d\Gamma(\alpha)/d\alpha$ is not available so that a closed form expression for maximum likelihood estimate of α is not available. What is the maximum likelihood estimate of β as a function of the maximum likelihood estimates of α?

7.15 The log-Pearson Type-III distribution is often used to model flood flows. If X has a log-Pearson Type-III distribution then

$$Y = ln(X) - m$$

has a two parameter gamma distribution where e^m is the lower bound of X if $\beta > 0$ and e^m is the upper bound of X if $\beta < 0$. The density function of Y can be written

$$f_Y(y)dy = \frac{(\beta y)^{\alpha-1}}{\Gamma(\alpha)}\exp(-\beta y)d(\beta y) \quad 0 < \beta y < +\infty$$

Calculate the mean and variance of X in terms of α, β and m. Note that

$$E[X^r] = E[(\exp(Y + m))^r] = \exp(rm)\,E[\exp(rY)]$$

To evaluate the required integrals remember that the constant terms in the definition of $f_Y(y)$ ensure that the integral of this density function over the range of by must be unity for any values of α and β so long as $\alpha > 0$ and $\beta y > 0$. For what values of r and β does the mean of X fail to exist? How do the values of m, α and β affect the shape and scale of the distribution of X?

7.16 When plotting observations to compare the empirical and fitted distributions of streamflows, or other variables, it is necessary to assign a cumulative probability to each observation. These are called *plotting positions*. As noted in the text, for the ith largest observation X_i,

$$E[F_X(X_i)] = i/(n + 1)$$

Thus the Weibull plotting position $i/(n + 1)$ is one logical choice. Other commonly used plotting positions are the *Hazen plotting position* $(i - {}^3/_8)/(n + {}^1/_4)$. The plotting position $(i - {}^3/_8)/(n + {}^1/_4)$ is a reasonable choice because its use provides a good approximation to the expected value of X_i. In particular for standard normal variables

$$E[X_i] \cong \Phi^{-1}[(i - {}^3/_8)/(n + {}^1/_4)]$$

where $\Phi(\,\cdot\,)$ is the cumulative distribution function of a standard normal variable. While much debate centers on the appropriate plotting position to use to estimate $p_i = F_X(X_i)$, often people fail to realize how imprecise all such estimates must be. Noting that

$$\mathrm{Var}(p_i) = \frac{i(n - i - 1)}{(n + 1)^2(n + 2)},$$

contrast the difference between the estimates $\hat{p}_i$ of p_i provided by these three plotting positions and the standard deviation of p_i. Provide a numerical example. What do you conclude?

7.17 The following data represent a sequence of annual flood flows, the maximum flow rate observed each year, for the Sebou River at the Azib Soltane gaging station in Morocco.

Date	Maximum Discharge (m^3/s)	Date	Maximum Discharge (m^3/s)
03/26/33	445	03/13/54	750
12/11/33	1410	02/27/55	603
11/17/34	475	04/08/56	880
03/13/36	978	01/03/57	485
12/18/36	461	12/15/58	812
12/15/37	362	12/23/59	1420
04/08/39	530	01/16/60	4090
02/04/40	350	01/26/61	376
02/21/41	1100	03/24/62	904
02/25/42	980	01/07/63	4120
12/20/42	575	12/21/63	1740
02/29/44	694	03/02/65	973
12/21/44	612	02/23/66	378
12/24/45	540	10/11/66	827
05/15/47	381	04/01/68	626
05/11/48	334	02/28/69	3170
05/11/49	670	01/13/70	2790
01/01/50	769	04/04/71	1130
12/30/50	1570	01/18/72	437
01/26/52	512	02/16/73	312
01/20/53	613		

(a) Construct a histogram of the Sebou flood flow data to see what the flow distribution looks like.

(b) Calculate the mean, variance, and sample skew. Based on Table 7.3, does the sample skew appear to be significantly different from zero?

(c) Fit a normal distribution to the data and use the Kolmogorov-Smirnov test to determine if the fit is adequate. Draw a quantile-quantile plot of the fitted quantiles $F^{-1}[(i - {}^3/_8)/(n + ¼)]$ versus the observed quantiles x_i and include on the graph the Kolmogorov-Smirnov bounds on each x_i, as shown in Figures 7.2a and 7.2b.

(d) Repeat part (c) using a two-parameter lognormal distribution.

(e) Repeat part (c) using a three-parameter lognormal distribution. The Kolmogorov-Smirnov test is now approximate if applied to $\log_e[X_i - \tau]$, where τ is calculated using Equation 7.81 or some other method of your choice.

(f) Repeat part (c) for two- and three- parameter versions of the gamma distribution. Again, the Kolmogorov-Smirnov test is approximate.

(g) A powerful test of normality is provided by the correlation test. As described by Filliben (1975), one should approximate $p_i = F_X(x_i)$ by

$$
\hat{p}_i = \begin{cases} 1 - (0.5)^{1/n} & i = 1 \\ (i - 0.3175)/(n + 0.365) & i = 2,\ldots, n - 1 \\ (0.5)^{1/n} & i = n \end{cases}
$$

Then one obtains a test for normality by calculation of the correlation r between the ordered observations X_i and m_i the median value of the ith largest observation in a sample of n standard normal random variables so that

$$
m_i = \Phi^{-1}(\hat{p}_i)
$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution. The value of r is then

$$
r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2 (m_i - \bar{m})^2}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(m_j - \bar{m})^2}}
$$

Some significance levels for the value of r are (Filliben 1975)

	SIGNIFICANCE LEVEL		
n	1%	5%	10%
10	0.876	0.917	0.934
20	0.925	0.950	0.960
30	0.947	0.964	0.970
40	0.958	0.972	0.977
50	0.965	0.977	0.981
60	0.970	0.980	0.983

The probability of observing a value of r less than the given value, were the observations actually drawn from a normal distribution, equals the specified probability. Use this test to

determine whether a normal or two-parameter lognormal distribution provides an adequate model for these flows.

7.18 A small community is considering the immediate expansion of its wastewater treatment facilities so that the expanded facility can meet the current deficit of 0.25 MGD and the anticipated growth in demand over the next 25 years. Future growth is expected to result in the need of an additional 0.75 MGD. The expected demand for capacity as a function of time is

$$\text{Demand} = 0.25 \text{ MGD} + G(1 - e^{-0.23t})$$

where t is the time in years and $G = 0.75$ MGD. The initial capital costs and maintenance and operating costs related to capital are $\$1.2 \times 10^6\ C^{0.70}$ where C is the plant capacity (MGD). Calculate the loss of economic efficiency LEE and the misrepresentation of minimal costs (MMC) that would result if a designer incorrectly assigned G a value of 0.563 or 0.938 ($\pm$ 25%) when determining the required capacity of the treatment plant. [Note: When evaluating the true cost of a non-optimal design which provides insufficient capacity to meet demand over a 25-year period, include the cost of building a second treatment plant; use an interest rate of 7% per year to calculate the present value of any subsequent expansions.] In this problem, how important is an error in G compared to an error in the elasticity of costs equal to 0.70? One MGD, a million gallons per day, is equivalent to 0.0438 m^3/s.

7.19   A municipal water utility is planning the expansion of their water acquisition system over the next 50 years. The demand for water is expected to grow and is given by

$$D = 10t(1 - 0.006t)$$

where t is the time in years. It is expected that two pipelines will be installed along an acquired right-of-way to bring water to the city from a distant reservoir. One pipe will be installed immediately and then a second pipe when the demand just equals the capacity C in year t is

$$PV = (\alpha + \beta C^{\gamma})e^{-rt}$$

where

$\alpha = 29.5$
$\beta = 5.2$
$\gamma = 0.5$
$r = 0.07/\text{year}$

Using a 50-year planning horizon, what is the capacity of the first pipe which minimizes the total present value of the construction of the two pipelines? When is the second pipe built? If a $\pm$ 25% error is made in estimating γ or r, what are the losses of economic efficiency (LEE) and the misrepresentation of minimal costs (MMC)? When finding the optimal decision with each set of parameters, find the time of the second expansion to the nearest year; a computer program that finds the total present value of costs as a function of the time of the second expansion t for $t = 1, \ldots, 50$ would be helpful. (A second pipe need not be built.)

7.20 A national planning agency for a small country must decide how to develop the water resources of a region. Three development plans have been proposed, which are denoted d_1, d_2, and d_3. Their respective costs are 200f, 100 f, and 100 f where f is a million farths, the national currency. The national benefits which are derived from the chosen development plan depend, in part, on the international market for the goods and agricultural commodities that would be produced. Consider three possible international market outcomes, m_1, m_2, and m_3. The national benefits if development plan 1 selected would be, receptively, 400, 290, 250. The

national benefits from selection of plan 2 would be 350, 160, 120, while the benefits from
selection of plan 3 would be 250, 200, 160.

(a) Is any plan inferior or dominated?
(b) If one felt that probabilities could not be assigned to m_1, m_2, and m_3 but wished to
avoid poor outcomes, what would be an appropriate decision criterion, and why?
Which decisions would be selected using this criterion?
(c) If $Pr[m_1] = 0.50$ and $Pr[m_2] = Pr[m_3] = 0.25$, how would each of the expected net
benefits and expected regret criteria rank the decisions?

7.21 Show that if one has a choice between two water management plans yielding benefits X
and Y, where X is stochastically smaller than Y, then for any reasonable utility function, plan Y
is preferred to X.

7.22 A reservoir system was simulated for 100 years and the average annual benefits and
their variance was found to be

$$\bar{B} = 4.93$$

$$s_B^2 = 3.23$$

The correlation of annual benefits was also calculated and is:

k	r_k
0	1.000
1	0.389
2	0.250
3	0.062
4	0.079
5	0.041

(a) Assume that $\rho(l) = 0$ for $l > k$, compute (using Equation 7.137) the standard error
of the calculated average benefits for $k = 0, 1, 2, 3, 4,$ and 5. Also calculate the
standard error of the calculated benefits, assuming that annual benefits may be
thought of as a stochastic process with a correlation structure $\rho_B(k) = [\rho_B(1)]^k$.
What is the effect of the correlation structure among the observed benefits on the
standard error of their average?

(b) At the 90% and 95% levels, which of the r_k are significantly different from zero,
assuming that $\rho_B(l) = 0$ for $l > k$?

7.23 Replicated reservoir simulations using two operating policies produced the following
results:

		BENEFITS	
Replicate		Policy 1	Policy 2
1		6.27	4.20
2		3.95	2.58
3		4.49	3.87
4		5.10	5.70
5		5.31	4.02
6		7.15	6.75
7		6.90	4.21
8		6.03	4.13
9		6.35	3.68
10		6.95	7.45
11		7.96	6.86
Mean, X_i		6.042	4.859
Standard deviation of values, s_{xi}		1.217	1.570

(a) Construct a 90% confidence limits for each of the two means $\overline{X}_i$.

(b) With what confidence interval can you state that Policy 1 produces higher benefits than Policy 2 using the sign test and using the t-test?

(c) If the corresponding replicate with each policy were independent, estimate with what confidence one could have concluded that Policy 1 produces higher benefits with the t-test.

7.24 Assume that annual streamflow at a gaging site have been grouped into three categories or states. State 1 is 5 to 15 m³/s, state 2 is 15 to 25 m³/s, and state 3 is 25 to 35 m³/s, and these grouping contain all the flows on records. The following transition probabilities have been computed from record:

P_{ij}	1	$\begin{array}{c} j = \\ 2 \end{array}$	3
1	0.5	0.3	0.2
$i = 2$	0.3	0.3	0.4
3	0.1	0.5	0.4

(a) If the flow for the current year is between 15 and 25 m³/s, what is the probability that the annual flow 2 years from now will be in the range 25 to 35 m³/s?

(b) What is the probability of a dry, an average, and a wet year many years from now?

7.25 A Markov chain model for the streamflows in two different seasons has the following transition probabilities

STREAMFLOW IN SEASON 1	STREAMFLOW next Season 2		
	0-3 m³/s	3-6 m³/s	≥ 6 m³/s
0-10 m³/s	0.25	0.50	0.25
≥ 10 m³/s	0.05	0.55	0.40

STREAMFLOW IN SEASON 2	STREAMFLOW next Season 1	
	0-10 m³/s	≥ 10 m³/s
0-3 m³/s	0.70	0.30
3-6 m³/s	0.50	0.50
≥ 6 m³/s	0.40	0.60

Calculate the steady-state probabilities of the flows in each interval in each season.

7.26 Can you modify the deterministic discrete DP reservoir-operating model to include the uncertainty, expressed as $P_{ij}{}^t$, of the inflows, as in Exercise 7.25?

(*Hints*: The operating policy would define the release (or final storage) in each season as a function of not only the initial storage but also the inflow. If the inflow changes, so might the release or final storage volume. Hence you need to discretize the inflows as well as the storage volumes. Both storage and inflow are state variables. Assume for this model you can predict with certainty the inflow in each period at the beginning of the period. So, each node of the network represents a known initial storage and inflow value. You cannot predict with certainty the following period's flows, only their probabilities. What does the network look like now?

7.27 Assume that there exist two possible discrete flows Q_{it} into a small reservoir in each of two periods t each year having probabilities P_{it}. Find the steady-state operating policy (release as a function of initial reservoir volumes and current period's inflow) for the reservoir that minimizes the expected sum of squared deviations from storage and release targets. Limit the storage volumes to integer values that vary from 3 to 5. Assume a storage volume target of 4 and a release target of 2 in each period t. (Assume only integer values of all states and decision variables and that each period's inflow is known at the beginning of the period.) Find the annual expected sum of squared deviations from the storage and release targets.

	FLOWS, Q_{it}		PROBABILITIES, P_{it}	
Period, t	i = 1	i = 2	i = 1	i = 2
1	1	2	0.17	0.83
2	3	4	0.29	0.71

This is an application of Exercise 7.26 except the flow probabilities are independent of the previous flow.

7.28 Assume that the streamflow Q at a particular site has cumulative distribution function $F_Q(q) = q/(1 + q)$ for $q \geq 0$. The withdrawal x at that location must satisfy a chance constraint of the form $\Pr[x \geq Q] \leq 1 - \alpha$. Write the deterministic equivalent for each of the following chance constraints:

$$\Pr[x \leq Q] \geq 0.90 \qquad \Pr[x \geq Q] \leq 0.80$$
$$\Pr[x \leq Q] \leq 0.95 \qquad \Pr[x \leq Q] \leq 0.10$$
$$\Pr[x \geq Q] \geq 0.75$$

7.29 Assume that a potential water user can withdraw water from an unregulated stream, and that the probability distribution function $F_Q()$ of the available streamflow Q is known. Calculate the value of the withdrawal target T that will maximize the expected net benefits from the water's use given the two short-run benefit functions specified below.

(a) The benefits from streamflow Q when the target is T are

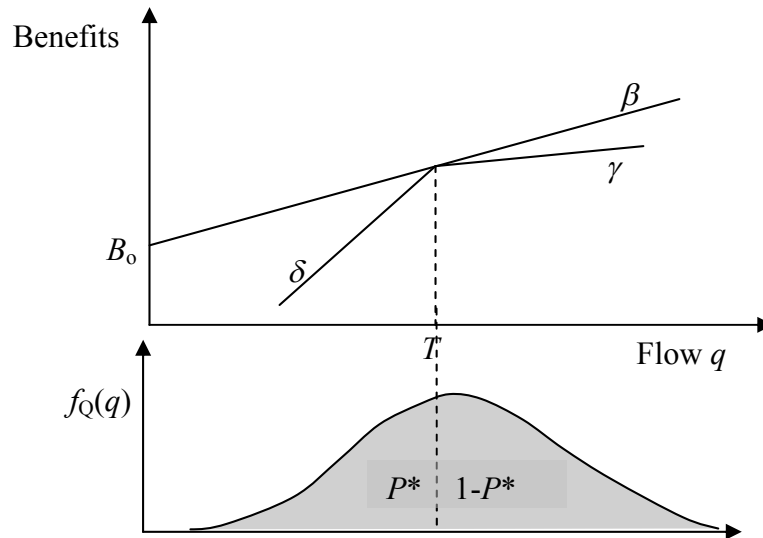$$B(Q \mid T) = \begin{cases} B_o + \beta T + \gamma(Q - T) & Q \geq T \\ B_o + \beta T + \delta(Q - T) & Q < T \end{cases}$$

where $\delta > \beta > \gamma$. In this case, the optimal target T^* can be expressed as a function of $P^* = F_Q(T) = \Pr\{Q \leq T\}$, the probability that the random streamflow Q will be less than or equal to T. Prove that

$$P^* = (\beta - \gamma)/(\delta - \gamma).$$

    (b) The benefits from streamflow Q when the target is T are

$$B(Q \mid T) = B_o + \beta T - \delta(Q - T)^2$$

7.30 If a random variable is discrete, what effect does this have on the specified confidence of a confidence interval for the median or any other quantile? Give an example.

7.31 (a) Use Wilcoxon test for unpaired samples to test the hypothesis that the distribution of the total shortage TS in Table 7.14 is stochastically less than the total shortage TS reported in Table 7.15. Use only the data from the second 10 simulations reported in the table. Use the fact that observations are paired (i.e., simulation j for $11 \leq j \leq 20$ in both tables were obtained with the same streamflow sequence) to perform the analysis with the sign test.

    (b) Use the sign test to demonstrate that the average deficit with Policy 1 (Table 7.14) is stochastically smaller than with Policy 2 (Table 7.15); use all simulations.

7.32 The accompanying table provides an example of the use of non-parametric statistics for examining the adequacy of synthetic streamflow generators. Here the maximum yield that can be supplied with a given size reservoir is considered. The following table gives the rank of the maximum yield obtainable with the historic flows among the set consisting of the historic yield and the maximum yield achievable with 1000 synthetic sequences of 25 different rivers in North America.

    (a) Plot the histogram of the ranks for reservoir sizes $S/\mu_Q = 0.85$, 1.35, 2.00. (*Hint*: Use the intervals 0-100, 101-200, 201-300, etc.) Do the ranks look uniformly distributed?

  Rank of the Maximum Historic Yield among 1000 Synthetic Yields

River Number	NORMALIZED ACTIVE STORAGE, S/μ_O			
	0.35	0.85	1.35	2.00
1	47	136	128	235
2	296	207	183	156
3	402	146	120	84
4	367	273	141	191
5	453	442	413	502
6	76	92	56	54
7	413	365	273	279
8	274	191	86	51
9	362	121	50	29
10	240	190	188	141
11	266	66	60	118
12	35	433	562	738
13	47	145	647	379
14	570	452	380	359
15	286	392	424	421
16	43	232	112	97
17	22	102	173	266
18	271	172	260	456
19	295	162	272	291
20	307	444	532	410
21	7	624	418	332
22	618	811	801	679
23	1	78	608	778
24	263	902	878	737
25	82	127	758	910

Source: A. I. McLeod and K. W. Hipel, Critical Drought Revisited,
Paper presented at the international Symposium on Risk and Reliability
in Water Resources, Waterloo, Ont., June 26-28, 1978

(b) Do you think this streamflow generation model produces streamflows which are consistent with the historic flows when one uses as a criterion the maximum possible yield? Construct a statistical test to support your conclusion and show that it does support your conclusion. (*Idea*: You might want to consider if it is equally likely that the rank of the historical yield is 500 and below 501 and above. You could then use the binomial distribution to determine the significance of the results.)

(c) Use the Kolmogrov-Smirnov test to check if the distribution of the yields obtainable with storage $S/\mu_Q = 1.35$ is significantly different from uniform $F_U(u) = u$ for $0 \leq u \leq 1$. How important do you feel this result is?

7.33 Section 7.3 dismisses the bias in v_x^2 for correlated X's as unimportant to its variance.

(a) Calculate the approximate bias in v_x^2 for the cases corresponding to Table 7.10 and determine if this assertion is justified.

(b) By numerically evaluating the bias and variance of v_x^2, when $n = 25$, determine if the same result holds if $\rho_x(k) = 0.5(0.9)^k$, which is the autocorrelation function of an ARMA (1, 1) process sometimes used to describe annual streamflow series.

7.34 Consider the crop irrigation problem in Exercise 4.31. For the given prices 30 and 25 for crop A and B, the demand for each crop varies over time. Records of demands show for crop A the demand ranges from 0 to 10 uniformly. There is an equal probability of that the demand will be any value between 0 and 10. For crop B the demand ranges from 5 units to 15 units, and the most likely demand is 10. At least 5 units and no more than 15 units of crop

B will be demanded. The demand for crop B can be defined by a triangular density function, beginning with 5, having a mean of 10 and an upper limit of 15. Develop and solve a model for finding the maximum expected net revenue from both crops, assuming the costs of additional resources are 2/unit of water, 4/unit of land, 8/unit of fertilizer, and 5/unit of labor. The cost of borrowed money, i.e., the borrowing interest rate, is 8 percent per growing season. How does the solution change if the resource costs are $1/10^{th}$ of those specified above?

7.35 In Section 9.2 generated synthetic streamflows sequences were used to simulate a reservoir's operation. In the example, a Thomas-Fiering model was used to generate $\ln Q_{1y}$ and $\ln Q_{2y}$, the logarithms of the flows in the two seasons of each year y, so as to preserve the season-to-season correlation of the untransformed flows. Noting that the annual flow is the sum of the untransformed seasonal flows Q_{1y} and Q_{2y}, calculate the correlation of annual flows produced by this model. The required data are given in Table 7.13. (*Hint*: You need to first calculate the covariance of $\ln Q_{1y}$ and $\ln Q_{1,y+1}$ and then of Q_{1y} and $Q_{2,y+1}$).

7.36 Part of New York City's municipal water supply is drawn from three parallel reservoirs in the upper Delaware River basin. The covariance matrix and lag-1 covariance matrix, as defined in Equations 7.166 and 7.168, were estimated based on the 50-year flow record to be (in m^3/sec):

$$S_0 = \begin{bmatrix} 20.002 & 21.436 & 6.618 \\ 21.436 & 25.141 & 6.978 \\ 6.618 & 6.978 & 2.505 \end{bmatrix} = [Cov(Q_y^i, Q_y^j)]$$

$$S_1 = \begin{bmatrix} 6.487 & 6.818 & 1.638 \\ 7.500 & 7.625 & 1.815 \\ 2.593 & 2.804 & 0.6753 \end{bmatrix} = [Cov(Q_{y+1}^i, Q_y^j)]$$

Other statistics of the annual flow are:

Site	Reservoir	Mean Flow	Standard Deviation	r_1
1	Pepacton	20.05	4.472	0.3243
2	Cannosville	23.19	5.014	0.3033
3	Neversink	7.12	1.583	0.2696

(a) Using these data, determine the values of the **A** and **B** matrices of the lag 1 model defined by Equation 7.165. Assume that the flows are adequately modeled by a normal distribution. A lower triangular **B** matrix that satisfies $M = \mathbf{BB}^T$ may be found by equating the elements of $\mathbf{BB}^T$ to those of M as follows:

$$M_{11} = b_{11}^2 \rightarrow b_{11} = \sqrt{M_{11}}$$

$$M_{21} = b_{11}b_{21} \rightarrow b_{21} = \frac{M_{21}}{b_{11}} = \frac{M_{21}}{\sqrt{M_{11}}}$$

$$M_{31} = b_{11}b_{31} \rightarrow b_{31} = \frac{M_{31}}{b_{11}} = \frac{M_{31}}{\sqrt{M_{11}}}$$

$$M_{22} = b_{21}^2 + b_{22}^2 \rightarrow b_{22} = \sqrt{M_{22} - b_{21}^2} = \sqrt{M_{22} - M_{21}^2 / M_{11}}$$

and so forth for M_{23} and M_{33}. Note that $b_{ij} = 0$ for $i < j$ and M must be symmetric because $\mathbf{BB}^T$ is necessarily symmetric.

(b) Determine **A** and **BB**^{T} for the Markov model which would preserve the variances and cross covariances of the flows at each site, but not necessarily the lag 1 cross covariances of the flows. Calculate the lag 1 cross covariances of flows generated with your calculated **A** matrix.

(c) Assume that some model has been built to generate the total annual flow into the three reservoirs. Construct and calculate the parameters of a disaggregation model that, given the total annual inflow to all three reservoirs, will generate annual inflows into each of the reservoirs preserving the variances and cross covariances of the flows. [*Hint*: The necessary statistics of the total flows can be calculated from those of the individual flows.]

7.37 Derive the variance of an ARMA (1, 1) process in terms of ϕ_1, θ_1, and σv^2. [*Hint*: Multiply both sides of the equation to obtain a second. Be careful to remember which V_t's are independent of which Z_t's.]

7.38 The accompanying table presents a 60-year flow record for the normalized flows of the Gota River near Sjotop-Vannersburg in Sweeden.

(a) Fit an autoregressive Markov model to the annual flow record.

(b) Using your model, generate a 50-year synthetic flow record. Demonstrate the mean, variance, and correlation of your generated flows deviate from the specified values no more than would be expected as a result of sampling error.

(c) Calculate the autocorrelations and partial autocovariances of the annual flows for a reasonable number of lags. Calculate the standard errors of the calculated values. Determine reasonable value of p and q for an ARMA (p, q) model of the flows. Determine the parameter values for the selected model.

Annual Flows, Gota River near Sjotop-Vannersburg, Sweden					
1898	1.158	1918	0.948	1938	0.892
1899	1.267	1919	0.907	1939	1.020
1900	1.013	1920	0.991	1940	0.869
1901	0.935	1921	0.994	1941	0.772
1902	0.662	1922	0.701	1942	0.606
1903	0.950	1923	0.692	1943	0.739
1904	1.120	1924	1.086	1944	0.813
1905	0.880	1925	1.306	1945	1.173
1906	0.802	1926	0.895	1946	0.916
1907	0.856	1927	1.149	1947	0.880
1908	1.080	1928	1.297	1948	0.601
1909	0.959	1929	1.168	1949	0.720
1910	1.345	1930	1.218	1950	0.955
1911	1.153	1931	1.209	1951	1.186
1912	0.929	1932	0.974	1952	1.140
1913	1.158	1933	0.834	1953	0.992
1914	0.957	1934	0.638	1954	1.048
1915	0.705	1935	0.991	1955	1.123
1916	0.905	1936	1.198	1956	0.774
1917	1.000	1937	1.091	1957	0.769

Source: V. M. Yevdjevich, Fluctuations of Wet and Dry Years, Part I, Hydrology Paper No. 1, Colorado State University, Fort Collins, Colo., 1963

(d) Using the estimated model in (c), generate a 50-year synthetic streamflow record and demonstrate that the mean, variance, and show that first autocorrelations of the synthetic flows deviate from the modeled values by no more than would be expected as a result of sampling error.

7.39   (a) Assume that one wanted to preserve the covariance matrices S_0 and S_1 of the flows at several site Z_y by using the multivariate or vector ARMA (0, 1) model

$$Z_{y+1} = AV_y - BV_{y-1}$$

where V_y contains n independent standard normal random variables. What is the relationship between the values of S_0 and S_1 and the matrices A and B?

(b) Derive estimates of the matrices A, B, and C of the multivariate AR(2) model

$$Z_{y+1} = AZ_y + BZ_{y-1} + CV_y$$

using the covariance matrices S_0, S_1 and S_2.

7.40 Formulate a model for the generation of monthly flows. The generated monthly flows should have the same marginal distributions as were fitted to the observed flows of record and should reproduce (i) the month-to-month correlation of the flows, (ii) the month-to-season correlation between each monthly flow and the total flow the previous season, and (iii) the month-to-year correlation between each monthly flow and the total 12-month flow in the previous year. Show how to estimate the model's parameters. How many parameters does your model have? How are the values of the seasonal model? How do you think this model could be improved?

Chapter 8 Modelling Uncertainty

8.1 Can you modify the deterministic discrete DP reservoir-operating model to include the uncertainty, expressed as $P_{ij}{}^t$, of the inflows, as in Exercise 7.25?

(*Hints*: The operating policy would define the release (or final storage) in each season as a function of not only the initial storage but also the inflow. If the inflow changes, so might the release or final storage volume. Hence you need to discretize the inflows as well as the storage volumes. Both storage and inflow are state variables. Assume for this model you can predict with certainty the inflow in each period at the beginning of the period. So, each node of the network represents a known initial storage and inflow value. You cannot predict with certainty the following period's flows, only their probabilities. What does the network look like now?

8.2 Assume that there exist two possible discrete flows Q_{it} into a small reservoir in each of two periods t each year having probabilities P_{it}. Find the steady-state operating policy (release as a function of initial reservoir volumes and current period's inflow) for the reservoir that minimizes the expected sum of squared deviations from storage and release targets. Limit the storage volumes to integer values that vary from 3 to 5. Assume a storage volume target of 4 and a release target of 2 in each period t. (Assume only integer values of all states and decision variables and that each period's inflow is known at the beginning of the period.) Find the annual expected sum of squared deviations from the storage and release targets.

	FLOWS, Q_{it}		PROBABILITIES, P_{it}	
Period, t	*i = 1*	*i = 2*	*i = 1*	*i = 2*
1	1	2	0.17	0.83
2	3	4	0.29	0.71

> This is an application of Exercise 7.26 except the flow probabilities are independent of the previous flow.

8.3 Develop a linear model for defining the optimal joint probabilities of predefined discrete initial storage volumes, discrete inflows, and discrete final storage volumes in a reservoir in each period t. Let values of the index k represent the different discrete initial storage volumes, S_{kt}. Similarly, let the index i represent the inflows, Q_{it}, and the index l represent the final storage volumes, $S_{l,t+1}$, in period t. Let the index j represent the discrete inflows, $Q_{j,t+1}$, and m represent the discrete final storage volumes, $S_{m,t+2}$, in period $t+1$. Let PR_{kilt} be the unknown joint probability of a discrete initial storage, S_{kt}, an inflow, Q_{it}, and a final storage volume, $S_{l,t+1}$, in period t. It is also the probability of a release associated with a particular combination of k, i, and l in period t. The objective is to maximize the expected net benefits, however measured. The net benefits associated with any combination represented by k, i, and l in period t is B_{kilt}. These net benefits and the conditional inflow probabilities. $P_{ij}{}^t =$ Pr$\{Q_{j,t+1}|Q_{it}\}$, are known. Show how the optimal operating policy can be determined once the values of the joint probabilities, PR_{kilt}, are known.

The same policy can be found by DP. Develop a DP model to find the optimal operating policy.

8.4 Referring to Exercise 8.3, instead of defining a final volume subscript l and m for computing joint probabilities PR_{kilt}, assume that subscripts d and e were used to denote different reservoir release volumes. How would the linear programming model developed be

altered to include d and e in place of l and m? How would the dynamic programming recursion equation be altered?

8.5 Given joint probabilities PR_{kilt} found from Exercise 8.3, how would one derive the probability distribution of reservoir releases and storage volumes in each period t?

Chapter 9 Model Sensitivity and Uncertainty Analysis

9.1   Distinguish between sensitivity analysis and uncertainty analysis.

9.2   Consider the allocation model you have been using in previous chapters involving three water consumers i. Allocations x_i of water can be made from a given total amount Q to the three consumers. The respective benefits are $(6x_1 - x_1^2)$, $(7x_2 - 1.5x_2^2)$ and $(8x_3 - 0.5x_3^2)$. Discuss possible sources of uncertainty in model structure and model output, and identify and display parameter sensitivity.

9.3 Discuss how model output uncertainty is impacted by both model input uncertainty as well as parameter sensitivity.

9.4 In many water resources studies considerable attention is given to the uncertainty of water supplies (precipitation, streamflows, evaporation, infiltration, etc.) and much less attention is given to the uncertainty of the management objectives, the costs and benefits of infrastructure, the political support associated with alternative possible decisions, and the like. Develop a simple water resources planning model involving the management of water quantity and quality and show how these management objective uncertainties may actually dominate the hydrologic uncertainties.

9.5 Perform a deterministic sensitivity analysis for the consumer 1 in Exercise 9.2. Consider the three parameters, Q, 6 and 1; the later two numbers are the parameters of the benefit function. Low values of these three parameters are 3, 3, and 0.5 respectively. Most likely values are 6, 6, and 1. High values are 12, 9, and 1.5. Display the results using a Pareto chart, a tornado diagram, and a spider plot.

9.6 Referring to water allocation problem defined in Exercise 9.2, assume the available amount of water Q is uncertain. Its cumulative probability distribution is defined by $q/(6+q)$ for values $q \geq 0$ of the random variable Q. The expected value of Q is 6. Perform an uncertainty analysis showing how to define, at least approximately:

- Estimating the mean and standard deviation of the outputs.
- Estimating the probability the performance measure will exceed a specific threshold.
- Assigning a reliability level on a function of the outputs, e.g., the range of function values that is likely to occur with some probability.
- Describing the likelihood of different potential outputs of the system.

Show the application of Monte Carlo sampling and analysis, Latin hypercube sampling, generalized likelihood uncertainty estimation and factorial design methods.

Chapter 10 Performance Criteria

10.1 Distinguish between multiple purposes and multiple objectives and give some examples of complementary and conflicting purposes and objectives of water resources projects.

10.2 Assume that farmers' demand for water q is a linear function $a - bp$ of the price p, where $a, b > 0$. Calculate the farmers' willingness to pay for a quantity of water q. If the cost of delivering a quantity of water q is cq, $c > 0$, how much water should a public agency supply to maximize willingness to pay minus total cost? If the local water district is owned and operated by a private firm whose objective is to maximize profit, how much water would they supply and how much would they earn? The farmers' consumer surplus is their willingness to pay minus what they must pay for the resource. Compare the farmers' consumer surplus in two cases. Do the farmers loose more than the private firm gains by moving from the social optimum to the point that maximize the firm's profit? Illustrate these relationships with a graph showing the demand curve and the unit cost c of water. Which areas on the graph represent the firm's profits and the farmers' consumer surplus?

10.3 Consider the water allocation problem used in the earlier chapters of this book. The returns, $B_i(X_i)$ from allocating X_i amount of water to each of three uses i are as follows, along with the optimal allocations from the point of view of each use.

$$B_1(X_1) = 6X_1 - X_1^2 \quad \rightarrow \quad X_1^{opt} = 3 \quad \text{and} \quad B_1^{max} = B_1(X_1^{opt}) = 9$$
$$B_2(X_2) = 7X_2 - 1.5X_2^2 \quad \rightarrow \quad X_2^{opt} = 7/3 \quad \text{and} \quad B_2^{max} = B_2(X_2^{opt}) = 147/18$$
$$B_3(X_3) = 8X_3 - 0.5X_3^2 \quad \rightarrow \quad X_3^{opt} = 8 \quad \text{and} \quad B_3^{max} = B_3(X_3^{opt}) = 32$$

Consider this a multi-objective problem. Instead of finding the best overall allocation that maximizes the total return assume the objectives are to maximize the returns from each user.

Show how the weighting, constraint, goal attainment, and goal programming methods can be used to identify the tradeoffs among each of the three objectives for any limiting total amount of water, for example 6.

10.4   Under what circumstances will the weighting and constraint methods fail to identify efficient solutions?
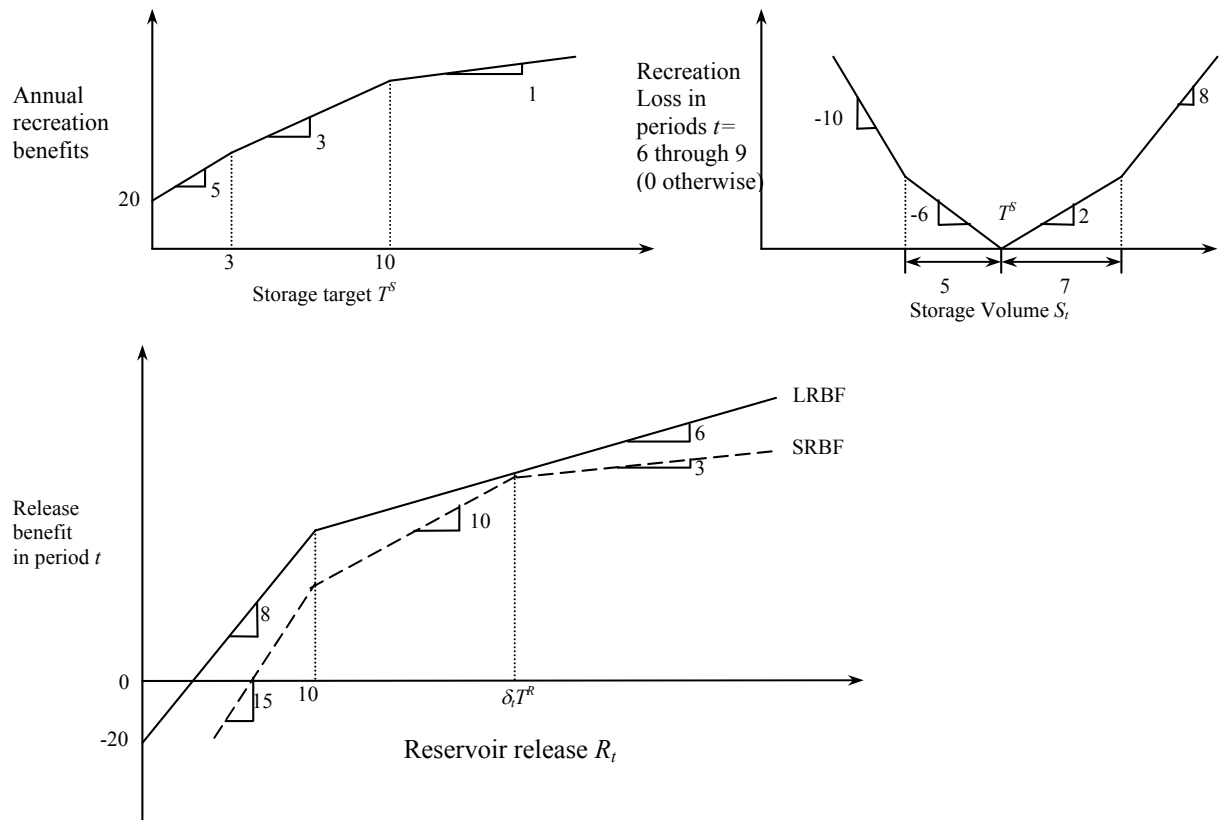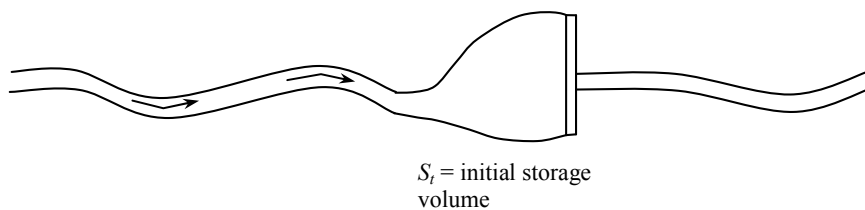
10.5 A reservoir is planned for irrigation and low flow augmentation for water quality control. A storage volume of 6×10^6 m³ will be available for those two conflicting uses each year. The maximum irrigation demand (capacity) is 4×10^6 m³. Let X_1 be the allocation of water to irrigation and X_2 the allocation for downstream flow augmentation. Assume that there are two objectives, expressed as

$$Z_1 = 4 X_1 - X_2$$
$$Z_2 = -2 X_1 + 6X_2$$

(a) Write the multi-objective planning model using a weighing approach and a constraint approach.
(b) Define the efficient frontier. This requires a plot of the feasible combinations of X_1 and X_2.
(c) Assume that various values are assigned to a weight W_1 for Z_1 whereas weight W_2 for Z_2 is constant and equal to 1, verify the following solutions to the weighing model.

W_1	X_1	X_2	Z_1	Z_2
>6	4	0	16	-8
6	4	0 to 12	16 to 14	-8 to 4
<6 to >1.6	4	2	14	4
1.6	4 to 0	2 to 6	14 to –6	4 to 36
<1.6	0	6	-6	36

10.6   Show that the following benefit, loss, and cost functions can be included in a linear optimization problem for finding the active storage volume target T^s, annual release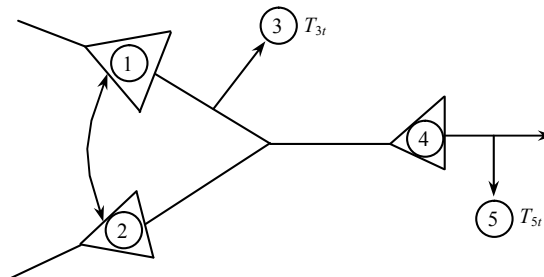 target T^R and the actual storage releases R_t in each within-year period t, and the reservoir capacity K. The objective is to maximize annual net benefits from the construction and operation of the reservoir. Assume that the inflows are known in each of 12 within-year periods t. Note that the loss function associated with reservoir recreation is independent of the value of T^s, unlike the loss function associated with reservoir releases. Structure the complete linear programming model. Define all variables used that are not defined below. Let $\delta_t T^R$ be the known release target in period t.

S_t = initial storage volume

Reservoir capacity K

10.7   For the river basin shown, potential reservoirs exist at sites $i = 1, 2,$ and 4 and a diversion can be constructed between sites 1 and 2. The cost $C_i(K_i)$ of each reservoir i is a function of its active storage capacity K_i. The cost of the diversion canal is $C_i(Q_i)$ where Q is the flow capacity of the canal. The cost of diverting a flow Q_{ijt} from site i to site j is $C_{ij}(Q_{ijt})$. The two users at sites 3 and 5 have known target allocations (demands) T_{it} in each period t. The return flow from use 3 is 40% of that allocated to use 3. Construct a model for finding the least cost of meeting various percentages of the target demands. Assume that the natural streamflows Q^i_t at each site i in each period t, are known.

10.8   Suppose that there exist two polluters, A and B, who can provide additional treatment, X_A and X_B, at a cost of $C_A(X_A)$ and $C_B(X_B)$, respectively. Let W_A and W_B be the waste produced at sites A and B, and $W_A(1 - X_A)$ and $W_B(1 - X_B)$ be the resulting waste discharges at site A and B. These discharges must be no greater than the effluent standards E_A^{max} and E_B^{max}. The resulting pollution concentration $a_{Aj}(W_A(1 - X_A)) + a_{Bj}(W_B(1 - X_B)) + q_j$ at various sites j must not exceed the stream standards S_j^{max}. Assume that total cost and cost inequity [i.e., $C_A(X_A) + C_B(X_B)$ and $| C_A(X_A) - C_B(X_B)|$] are management objectives to be determined
   (a) Discuss how you would model this multi-objective problem using the weighting and constraint (or target) approaches.
   (b) Discuss how you would use the model to identify efficient, non-inferior (Pareto-optimal) solutions.
   (c) Effluent standards at sites A and B and ambient stream standards at sites j could be replaced by other planning objectives (e.g., the minimization of waste discharged into the stream). What would these objectives be, and how could they be included in the multi-objective model?

10.9 (a) What conditions must apply if the goal attainment method is to produce only non-inferior alternatives for each assumed target T_k and weight w_k?
   (b) Convert the goal programming objective deviation components $w_i (z_i^* - z_i(\bar{x}))$ to a form suitable for solution by linear programming.

10.10   Water quality objectives are sometimes difficult to quantify. Various attempts have been made to include the many aspects of water quality in single water quality indices. One such index was proposed by Dinius (Social Accounting Systems for Evaluating Water Resources, Water Resources Research, Vol. 8, 1972. pp. 1159-1177). Water quality, Q, measured in percent is given by

$$Q = \frac{w_1 Q_1 + w_2 Q_2 + ... + w_n Q_n}{w_1 + w_2 + ... + w_n}$$

where Q_i is the ith quality constituent (dissolved oxygen, chlorides, etc.) and w_i is the weight or relative importance of the ith quality constituent. Write a critique on the use of such an index in multi-objective water resources planning.

10.11   Let objective $Z_1(\mathbf{X}) = 5X_1 - 2X_2$ and objective $Z_2(\mathbf{X}) = -X_1 + 4X_2$. Both are to be minimized. Assume that the constraints on variables X_1 and X_2 are:

1. $-X_1 + X_2 \leq 3$
2. $X_1 \leq 6$
3. $X_1 + X_2 \leq 8$
4. $X_2 \leq 4$
5. $X_1, X_2 \geq 0$

(a) Graph the Pareto-optimal or non-inferior solutions in decision space.
(b) Graph the efficient combination of Z_1 and Z_2 in objective space.
(c) Reformulate the problem to illustrate the weighting method for defining all efficient solutions of part (a) and illustrate this method in decision and objective space.
(d) Reformulate the problem to illustrate the constraint method of defining all efficient solutions of part (a) and illustrate this method in decision and objective space.
(e) Solve for the compromise set of solutions using compromise programming as defined by

$$\text{Minimize } [w_1(Z_1^* - Z_1)^\alpha + w_2(Z_2^* - Z_2)^\alpha]^{1/\alpha}$$

where Z_i^* represents the best value of objective i with all weights w equal to 1 and α equal to 1, 2, and ∞.

10.12   Illustrate the procedure for selecting among three plans, each having three objectives, using indifference analysis. Let Z_{ji} represent the value of objective i for plan j. The values of each objective for each plan are given below. Assume that each objective is to be maximized. Assume that an identical indifference function for all trade-offs between pairs of objectives, namely one that implies you are willing to give up twice as many units of your higher (larger) objective value to gain one unit of your lower (smaller) objective value. [For example, you would be indifferent to two plans having as their three objective values (30, 5, 10) and (20, 5, 15).] Rank these three plans in order of preference.

Chapter 11 River Basin Planning Models

11.1 Using the following information pertaining to the drainage area and discharge in the Han River in South Korea, develop an equation for predicting the natural unregulated flow at any site in the river, by plotting average flow as a function of catchment area. What does the slope of the function equal?

Gage Point	Catchment Area (km^2)	Average Flow $(10^6\,m^3/yr)$
First bridge of the Han River	25,047	17,860
Pal Dang dam	23,713	16,916
So Yang dam	2,703	1,856
Chung Ju dam	6,648	4,428
Yo Ju dam	10,319	7,300
Hong Chun dam	1,473	1,094
Dal Chun dam	1,348	1,058
Kan Yun dam	1,180	926
Im Jae dam	461	316

11.2 In watersheds characterized by significant elevation changes, one can often develop reasonable predictive equations for average annual runoff per hectare as a function of elevation. Describe how one would use such a function to estimate the natural average annual flow at any gage in a watershed which is marked by large elevation changes and little loss of water from stream channels due to evaporation or seepage.
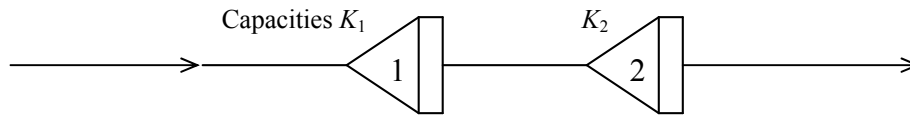
11.3 Compute the storage yield function for a single reservoir system by the mass diagram and modified sequent peak methods given the following sequences of annual flows: (7, 3, 5, 1, 2, 5, 6, 3, 4). Next assume that each year has two distinct hydrologic seasons, one wet and the other dry, and that 80% of the annual inflow occurs in season $t = 1$ and 80% of the yield is desired in season $t = 2$. Using the modified sequent peak method, show the increase in storage capacity required for the same annual yield resulting from within-year redistribution requirements.

Assume that each year has wet season = 1 and dry season = 2 and 80 % of inflow occurs in season 1, while 80 % of the yield is desired in season 2. Using the modified sequent peak method to show the increase in storage capacity required for the same annual yield resulting from within-year redistribution requirements.

11.4 Write two different linear programming models for estimating the maximum constant reservoir release or yield Y given a fixed reservoir capacity K, and for estimating the minimum reservoir capacity K required for a fixed yield Y. Assume that there are T time periods of historical flows available. How could these models be used to define a storage capacity-yield function indicating the yield Y available from a given capacity K?

11.5 (a) Construct an optimization model for estimating the least-cost combination of active storage capacities, K_1 and K_2, of two reservoirs located on a single stream, used to produce a reliable constant annual flow or yield (or greater) downstream of the two reservoirs. Assume that the cost functions $C_s(K_s)$ at each reservoir site s are known and there is no dead storage and no evaporation. (Do not linearize the cost functions; leave them in their functional form.)   Assume that 10 years of monthly unregulated flows are available at each site s.

(b) Describe the two-reservoir operating policy that you would incorporate into a simulation model to check the solution obtained from the optimization model.
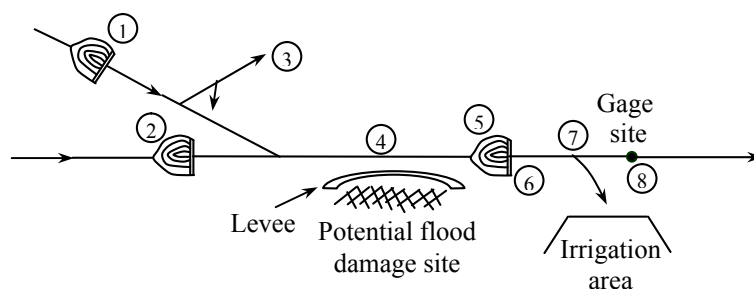

11.6   Given the information in the accompanying tables, compute the reservoir capacity that maximizes the net expected flood damage reduction benefits less the annual cost of reservoir capacity.

Reservoir Capacity	FLOOD STAGE FOR FLOOD OF RETURN PERIOD T					Annual Capacity Cost
	$T = 1$	$T = 2$	$T = 5$	$T = 10$	$T = 100$	
0	30	105	150	165	180	10^a
5	30	80	110	120	130	25
10	30	55	70	75	80	30
15	30	40	45	48	50	40
20	30	35	38	39	40	70

[a]10 is fixed cost if capacity > 0; otherwise, it is 0.

Flood Stage	Cost of Flood Damage
30	0
50	10
70	20
90	30
110	40
130	50
150	90
180	150

11.7 Develop a deterministic, static, within-year model for evaluating the development alternatives in the river basin shown in the accompanying figures. Assume that there are $t = 1$, 2, 3,…,n within-year periods and that the objective is to maximize the total annual net benefits in the basin. The solution of the model should define the reservoir capacities (active + flood storage capacity), the annual allocation targets, the levee capacity required to protect site 4 from a T-year flood, and the within-year period allocations of water to the uses at sites 3 and 7. Clearly define all variables and functions used, and indicate how the model would be solved to obtain the maximum-net-benefit solution.

Site	Fraction of Gage Flow	
1	0.	Potential reservoir for water supply,
2	0.3	Potential reservoir for water supply, flood control
3	0.15	Diversion to a use, 60% of allocation returned to river
4	-	Existing development, possible flood protection from levee
5	0.6	Potential reservoir for water supply, recreation
6	-	Hydropower; plant factor = 0.30
7	0.9	Potential diversion to an irrigation district
8	1.0	Gage site

For simplicity assume no evaporation losses or dead storage requirements. Omitting the appropriate subscripts t for time periods and s for site, let $T, K, D, E,$ and P be the target, reservoir capacity, deficit, excess, and power plant capacity variables, respectively. Let Q_t and R_t be the natural streamflows and reservoir releases, and S_t be the initial reservoir storage volumes in period t. K_f will denote the flood storage capacity at site 2 that will contain a peak flow of QS and QR is the downstream channel flood flow capacity. The relationship between QS and K_f is defined by the function $\kappa(QS)$. The unregulated design flood peak flow for which protection is required is QN. KWH will be the kilowatt hours of energy, H will be net storage head, h_t the hours in a period t. The variable q will be the water supply allocation. Benefit functions will be $B(\ )$, $L(\ )$ will denote loss functions and $C(\ )$ will denote the cost functions.

11.8 List the potential difficulties involved when attempting to structure models for defining:
     (a) Water allocation policies for irrigation during the growing season.
     (b) Energy production and capacity of hydroelectric plants.
     (c) Dead storage volume requirements in reservoirs.
     (d) Active storage volume requirements in reservoir.
     (e) Flood storage capacities in reservoirs.
     (f) Channel improvements for flood damage reduction.
     (g) Evaporation and seepage losses from reservoirs.
     (h) Water flow or storage targets using long-run benefits and short- run loss functions.

11.9 Assume that demand for water supply capacity is expected to grow as $t(60 - t)$, for t in years. Determine the minimum present value of construction cost of some subset of water supply options described below so as to always have sufficient capacity to meet demand over the next 30 years. Assume that the water supply network currently has no excess capacity so that some project must be built immediately. In this problem, assume project capacities are independent and thus can be summed. Use a discount factor equal to exp (- 0.07 t). Before you start, what is your best guess at the optimal solution?

Project Number	Construction Cost	Capacity
1	100	200
2	115	250
3	190	450
4	270	700

11.10 (a) Construct a flow diagram for a simulation model designed to define a storage-yield function for a single reservoir given known inflows in each month t for n years. Indicate how you would obtain a steady-state solution not influenced by an arbitrary initial storage volume in the reservoir at the beginning of the first period. Assume that evaporation rates (mm per month) and the storage volume/surface area functions are known.

(b) Write a flow diagram for a simulation model to be used to estimate the probability that any specific reservoir capacity, K, will satisfy a series of known release demands, r_t, downstream given unknown future inflows, i_t. You need not discuss how to generate possible future sequences of streamflows, only how to use them to solve this problem.

11.11   (a) Develop an optimization model for finding the cost-effective combination of flood storage capacity at an upstream reservoir and channel improvements at a downstream potential damage site that will protect the downstream site from a pre-specified design flood of return period T. Define all variables and functions used in the model.

(b) How could this model be modified to consider a number of design floods T and the benefits from protecting the potential damage site from those design floods? Let BF_T be the annual expected flood protection benefits at the damage site for a flood having return periods of T.

(c) How could this model be further modified to include water supply requirements of A_t to be withdrawn from the reservoir in each month t? Assume known natural flows Q_t^s at each site s in the basin in each month t.

(d) How could the model be enlarged to include recreation benefits or losses at the reservoir site?   Let T^s be the unknown storage volume target and D_t^s be the difference between the storage volume S_t^s and the target T^s if $S_t^s - T^s > 0$, and E_t^s be the difference if $T^s - S_t^s > 0$. Assume that the annual recreation benefits $B(T^s)$ are a function of the target storage volume T^s and the losses $L^D(D_t^s)$ and $L^E(E_t^s)$ are associated with the deficit D_t^s and excess E_t^s storage volumes.

11.12 Given the hydrologic and economic data listed below, develop and solve a linear programming model for estimating the reservoir capacity K, the flood storage capacity K_f, and the recreation storage volume target T that maximize the annual expected flood control benefits, $B_f(K_f)$, plus the annual recreation benefits, $B(T)$, less all losses $L^D(D_t)$ and $L^E(E_t)$ associated with deficits D_t or excesses E_t in the periods of the recreation season, minus the annual cost $C(K)$ of storage reservoir capacity K. Assume that the reservoir must also provide a constant release or yield of $Y = 30$ in each period t. The flood season begins at the beginning of period 3 and lasts through period 6. The recreation season begins at the beginning of period 4 and lasts though period 7.

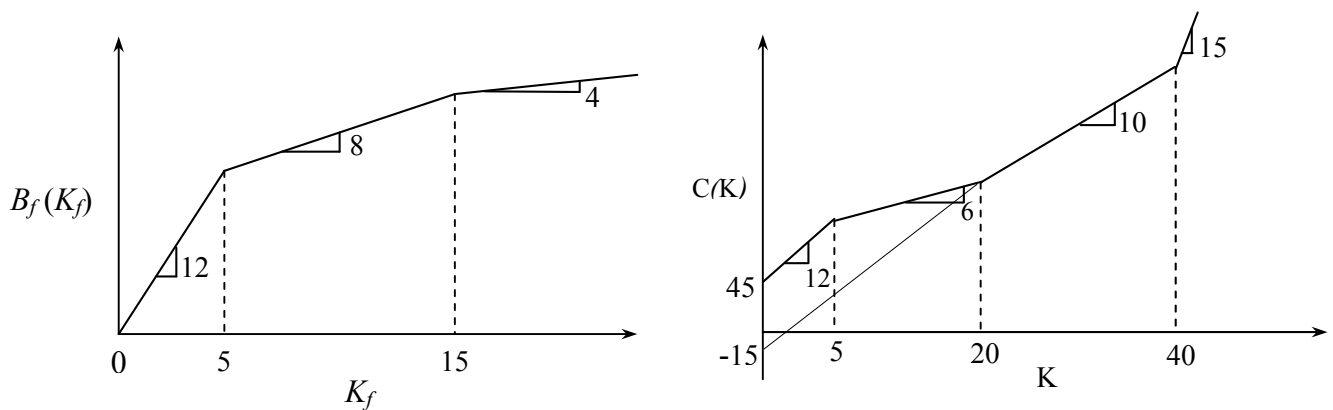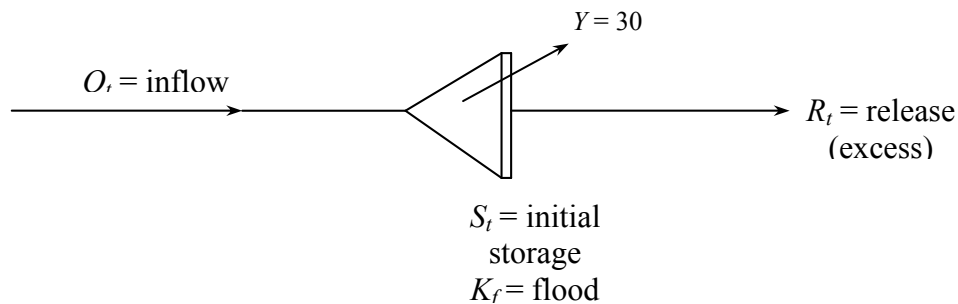| Period t	1	2	3	4	5	6	7	8	9
Inflows to reservoir	50	30	20	80	60	20	40	10	70

$$B_f(K_f) = \begin{cases} 12\,K_f & \text{if } K_f \le 5 \\ 60 + 8(K_f - 5) & \text{if } 5 \le K_f \le 15 \\ 140 + 4(K_f - 15) & \text{if } K_f \ge 15 \end{cases}$$

$$C(K) = \begin{cases} 45 + 10\,K & \text{if } 0 \le K_f \le 5 \\ 95 + 6(K - 5) & \text{if } 5 \le K_f \le 20 \\ 185 + 10(K - 20) & \text{if } 20 \le K_f \le 40 \\ 385 + 15(K - 40) & \text{if } K_f \ge 40 \end{cases}$$

$B(T) = 9T$ where T is a particular unknown value of - reservoir storage

$L^D(D_t) = 4D_t$ where D_t is $T - S_t$ if $T \ge S_t$

$L^E(E_t) = 2\,E_t$ where E_t is $S_t - T$ if $S_t \ge T$

O_t = inflow

$Y = 30$

R_t = release (excess)

S_t = initial storage

K_f = flood

11.13 The optimal operation of multiple reservoir systems for hydropower production presents a very nonlinear and often difficult problem.

Use dynamic programming to determine the operating policy that maximizes the total annual hydropower production of a two-reservoir system, one downstream of the other. The releases R_{1t} from the upstream reservoir plus the unregulated incremental flow $(Q_{2t} - Q_{1t})$ constitute the inflow to the downstream dam. The flows Q_{1t} into the upstream dam in each of the four seasons along with the incremental flows $(Q_{2t} - Q_{1t})$ and constraints on reservoir releases are given in the accompanying two tables:

Upstream Dam (flow in 10^6 m^3/period)

Season t	Inflow Q_{It}	Minimum Release	Maximum Release Through Turbines
1	60	20	90
2	40	30	90
3	80	20	90
4	120	20	90

Downstream Dam (flow in 10^6 m^3/period)

Season t	Incremental Flow, $(Q_{2t} - Q_{1t})$	Minimum Release	Maximum Release Through Turbines
1	50	30	140
2	30	40	140
3	60	30	140
4	90	30	140

Note that there is a limit on the quantity of water that can be released through the turbines for energy generation in any season due to the limited capacity of the power plant and the desire to produce hydropower during periods of peak demand.

Additional information that affects the operation of the two reservoirs are the limitations on the fluctuations in the pool levels (head) and the storage-head relationships:

Data	Upstream Dam	**Downstream Dam**
Maximum head, H^{max}	70 m	90 m
Minimum head, H^{min}	30 m	60 m
Maximum storage Volume, S^{max}	150×10^6 m^3	400×10^6 m^3
Storage-net head relationship	$H = H^{max}(S/S^{max})^{0.64}$	$H = H^{max}(S/S^{max})^{0.62}$

In solving the problem, discretize the storage levels in units of 10×10^6 m^3. Do a preliminary analysis to determine how large a variation in storage might occur at each reservoir. Assume that the conversion of potential energy equal to the product R_iH_i to electric energy is 70% efficient independent of R_i and H_i. In calculating the energy produced in any season t at reservoir i, use the average head during the season

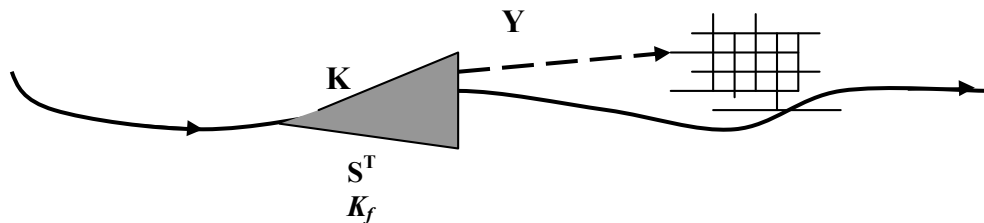$$\overline{H}_i = \frac{1}{2}[H_i(t) + H_i(t+1)]$$

Report your operating policy and the amount of energy generated per year. Find another feasible policy and show that it generates less energy than the optimal policy.

Show how you could use linear programming to solve for the optimal operating policy by approximating the product term $R_i\overline{H}_i$ by a linear expression.

11.14   You are responsible for planning a project that may involve the building of a reservoir to provide water supply benefits to a municipality, recreation benefits associated with the water level in the lake behind the dam, and flood damage reduction benefits. First you need to determine some design variable values, and after doing that you need to determine the reservoir operating policy.

      The design variables you need to determine include:
- the total reservoir storage capacity (K),
- the flood storage capacity (K_f) in the first season that is the flood season,
- the particular storage level where recreation facilities will be built, called the storage target (S^T) that will apply in seasons 3, 4 and 5 – the recreation seasons, and finally
- the dependable water supply or yield (Y) for the municipality.

      Assume you can determine these design variable values based on average flows at the reservoir site in six seasons of a year. These average flows are 35, 42, 15, 3, 15, and 22 in the seasons 1 to 6 respectively.

      The objective is to design the system to maximize the total annual net benefits derived from
- flood control in season 1,
- recreation in seasons 3 through 5, and
- water supply in all seasons,

less the annual cost of the
- reservoir and
- any losses resulting from not meeting the recreation storage targets in the recreation seasons.

The flood benefits are estimated to be 2 $K_f^{0.7}$.
The recreation benefits for the entire recreation season are estimated to be 8 ST.
The water supply benefits for the entire year are estimated to be 20 Y.
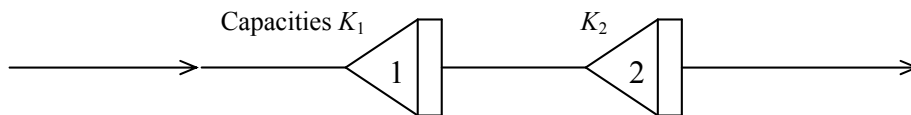The annual reservoir cost is estimated to be 3 $K^{1.2}$.
The recreation loss in each recreation season depends on whether the actual storage volume is lower or higher than the storage target. If it is lower the losses are 12 per unit average deficit in the season, and if they are higher the losses are 4 per unit

average excess in the season. It is possible that a season could begin with a deficit and end with an excess, or vice versa.

Develop and solve a non-linear optimization model for finding the values of each of the design variables: K, K_f, ST, and Y and the maximum annual net benefits. (There will be other variables as well. Just define what you need and put it all together in a model.)

Does the solution give you sufficient information that would allow you to simulate the system using a sequence of inflows to the reservoir that are different than the ones used to get the design variable values? If not how would you define a reservoir operating policy? After determining the system's design variable values using optimization, and then determining the reservoir operating policy, you would then simulate this system over many years to get a better idea of how it might perform.

11.15 Suppose you have 19 years of monthly flow data at a site where a reservoir could be located. How could you construct a model to estimate what the required over-year and within year storage needed to produce a specified annual yield Y that is allocated to each month t by some known fraction δ_t. What would be the maximum reliability of those yields? If you wanted to add to that an additional secondary yield having only 80% reliability, how would the model change? Make up 19 annual flows and assume the average monthly flows are specified fractions of those annual flows. Just using these annual flows and the average monthly fractions, solve your model.

11.16 a) Develop an optimisation model for estimating the least-cost combination of active storage K^1 and K^2 capacities at two reservoir sites on a single stream that are used to produce a reliable flow or yield downstream of the downstream reservoir. Assume 10 years of monthly flow data at each reservoir site. Identify what other data are needed.

b) Describe the two-reservoir operating policy that could be incorporated into a simulation model to check the solution obtained from the optimization model

Define

$C^s(K^s)$	= cost of active storage capacity at site s; where s = 1, 2
K_d^s	= dead storage capacity of reservoir at site s; $K_d^s = 0$
S_t^s	= storage volume at beginning of period t at site s.
L^s	= loss of water due to evaporation at site s; $L^s = 0$
R_t^{12}	= release from reservoir at site 1 to site 2 in period t
Y_t	= yield to downstream in period t
Q_t^s	= 10 years of monthly natural flows available at each site s
$a_o{}^s$	= area associated with dead storage volume at site s
a^s	= area per unit storage volume at site s
$e_t{}^s$	= evaporation depth in period t at site s

11.16   Given inflows to an effluent storage lagoon that can be described by a simple first-order Markov chain in each of T periods t, and an operating policy that defines the lagoon discharge as a function of the initial volume and inflow, indicate how you would estimate the probability distribution of lagoon storage volumes.

11.17   (a) Using the inflow data in the table below, develop and solve a yield model for estimating the storage capacity of a single reservoir required to produce a yield of 1.5 that is 90% reliable in both of the two within-year periods t, and an additional yield of 1.0 that is 70% reliable in period $t = 2$.
(b) Construct a reservoir-operating rule that defines reservoir release zones for these yields.
(c) Using the operating rule, simulate the 18 periods of inflow data to evaluate the adequacy of the reservoir capacity and storage zones for delivering the required yields and their reliabilities. (Note that in this simulation of the historical record the 90% reliable yield should be satisfied in all the 18 periods, and the incremental 70% reliable yield should fail only two times in the 9 years.)
(d) Compare the estimated reservoir capacity with that which is needed using the sequent peak procedure.

Year	Period	Inflow
1	1	1.0
	2	3.0
2	1	0.5
	2	2.5
3	1	1.0
	2	2.0
4	1	0.5
	2	1.5
5	1	0.5
	2	0.5
6	1	0.5
	2	2.5
7	1	1.0
	2	5.0
8	1	2.5
	2	5.5
9	1	1.5
	2	4.5

11.19 One possible modification of the yield model of would permit the solution algorithm to determine the appropriate failure years associated with any desired reliability instead of having to choose these years prior to model solution. This modification can provide an estimate of the extent of yield failure in each failure year and include the economic consequences of failures in the objective function. It can also serve as a means of estimating the optimal reliability with respect to economic benefits and losses. Letting F_y be the unknown yield reduction in a possible failure year y, then in place of $\alpha_{py}Y_p$ in the over-year continuity constraint, the term $(Y_p - F_y)$ can be used. What additional constraints are needed to ensure (1) that the average shortage does not exceed $(1 - \alpha_{py})Y_p$ or (2) that at most there are f failure years and none of the shortages exceed $(1 - \alpha_{py})Y_p$.

11.20 In Indonesia there exists a wet season followed by a dry season each year. In one are of Indonesia all farmers within an irrigation district plant and grow rice during the wet season. This crop brings the farmer the largest income per hectare; thus they would all prefer to continue growing rice during the dry season. However, there is insufficient water during the

dry season for irrigating all 5000 hectares of available irrigable land for rice production. Assume an available irrigation water supply of 32×10^6 m^3 at the beginning of each dry season, and a minimum requirement of 7000 m^3/ha for rice and 1800 m^3/ha for the second crop.

      (a) What proportion of the 5000 hectares should the irrigation district manager allocate for rice during the dry season each year, provided that all available hectares must be given sufficient water for rice or the second crop?

      (b) Suppose that crop production functions are available for the two crops, indicating the increase in yield per hectare per m^3 of additional water, upto 10, 000 m^3/ha for the second crop. Develop a model in which the water allocation per hectare, as well as the hectares allocated to each crop, is to be determined, assuming a specified price or return per unit of yield of each crop. Under what conditions would the solution of this model be the same as in part (a)?

11.21 Along the Nile River in Egypt, irrigation farming is practiced for the production of cotton, maize, rice, sorghum, full and short berseem for animal production, wheat, barley, horsebeans, and winter and summer tomatoes. Cattle and buffalo are also produced, and together with the crops that require labor, water. Fertilizer, and land area (feddans). Farm types or management practices are fairly uniform, and hence in any analysis of irrigation policies in this region this distinction need not be made. Given the accompanying data develop a model for determining the tons of crops and numbers of animals to be grown that will maximize (a) net economic benefits based on Egyptian prices, and (b) net economic benefits based on international prices. Identify all variables used in the model.

Known parameters:

C_i	= miscellaneous cost of land preparation per feddan
P_i^E	= Egyptian price per 1000 tons of crop i
P_i^I	= international price per 1000 tons of crop i
v	= value of meat and dairy production per animal
g	= annual labor cost per worker
f^P	= cost of P fertilizer per ton
f^N	= cost of N fertilizer per ton
Y_i	= yield of crop i, tons/feddan
α	= feddans serviced per animal
β	= tons straw equivalent per ton of berseem carryover from winter to summer
r^w	= berseem requirements per animal in winter
s^{wh}	= straw yield from wheat, tons per feddan
s^{ba}	= straw yield from barley, tons per feddan
r^s	= straw requirements per animal in summer
μ_i^N	= N fertilizer required per feddan of crop i
μ_i^P	= P fertilizer required per feddan of crop i
l_{im}	= labor requirements per feddan in month m, man-days
w_{im}	= water requirements per feddan in month m, 1000 m^3
h_{im}	= land requirements per month, fraction (1 = full month)

Required Constraints. (assume known resource limitations for labor, water, and land):

  (a) Summer and winter fodder (berseem) requirements for the animals.
  (b) Monthly labor limitations.
  (c) Monthly water limitations.
  (d) Land availability each month.
  (e) Minimum number of animals required for cultivation.

      (f) Upper bounds on summer and winter tomatoes (assume these are known).
      (g) Lower bounds on cotton areas (assume this is known).

      Other possible constraints:
      (a) Crop balances.
      (b) Fertilizer balances.
      (c) Labor balance.
      (d) Land balance.

11.22 In Algeria there are two distinct cropping intensities, depending upon the availability of water. Consider a single crop that can be grown under intensive rotation or extensive rotation on a total of A hectares. Assume that the annual water requirements for the intensive rotation policy are 16000 m^3 per hectare, and for the extensive rotation policy they 4000 m^3 per hectare. The annual net production returns are 4000 and 2000 dinars, respectively. If the total water available is 320,000 m^3, show that as the available land area A increases, the rotation policy that maximizes total net income changes from one that is totally intensive to one that is increasingly extensive.

Would the same conclusion hold if instead of fixed net incomes of 4000 and 2000 dinars per hectares of intensive and extensive rotation, the net income depended on the quantity of crop produced? Assuming that intensive rotation produces twice as much produced by extensive rotation, and that the net income per unit of crop Y is defined by the simple linear function 5 – 0.05Y, develop and solve a linear programming model to determine the optimal rotation policies if A equals 20, 50, and 80. Need this net income or price function be linear to be included in a linear programming model?

Chapter 12 Water Quality Modelling and Prediction

12.1 The common version of the Streeter-Phelps equations for predicting biochemical oxygen demand *BOD* and dissolved oxygen deficit *D* concentrations are based on the following two differential equations:

(a) $\dfrac{d(BOD)}{d\tau} = -K_d(BOD)$

(b) $\dfrac{dD}{d\tau} = K_d(BOD) - K_a D$

where K_d is the deoxygenating rate constant (T^{-1}), K_a is the reaeration-rate constant (T^{-1}), and τ is the time of flow along a uniform reach of stream in which dispersion is not significant. Show the integrated forms of (a) and (b).

12.2 Based on the integrated differential equations in Exercise 12.1:
(a) Derive the equation for the distance X_c downstream from a single point source of *BOD* that for a given streamflow will have the lowest dissolved oxygen concentration.
(b) Determine the relative sensitivity of the deoxygenation-rate constant K_d and the reaeration-rate constant K_a on the critical distance X_c and on the critical deficit D_c. For initial conditions, assume that the reach has a velocity of 2 m/s (172.8 km/day), a K_d of 0.30 per day, and a K_a of 0.4 per day. Assume that the DO saturation concentration is 8 mg/l, the initial deficit is 1.0 mg/l, and the BOD concentration at the beginning of the reach (including that discharged into the reach at that point) is 15 mg/l.

12.3 To account for settling of *BOD*, in proportion to the *BOD* concentration, and for a constant rate of *BOD* addition *R* due to runoff and scour, and oxygen production ($A > 0$) or reduction ($A < 0$) due to plants and benthal deposits, the following differential equations have been proposed:

$$\frac{d(BOD)}{d\tau} = -(K_d + K_s)BOD + R \qquad (1)$$

$$\frac{dD}{d\tau} = K_d BOD - K_a D - A \qquad (2)$$

where K_s is the settling rate constant (T^{-1}) and τ is the time of flow. Integrating these two equations results in the following deficit equation:

$$D_\tau = \frac{K_d}{K_a - (K_d + K_s)}\left[(BOD_o - \frac{R}{K_d + K_s})\{\exp[-(K_d + K_s)\tau] - \exp(-K_a\tau)\} \right]$$

$$+ \frac{K_d}{K_a}\left\{ (\frac{R}{K_d + K_s} - \frac{A}{K_d})[1 - \exp(-K_a\tau)] \right\} + D_o \exp(-K_a\tau) \qquad (3)$$

where BOD_o and D_o are the BOD and DO deficit concentrations at $\tau = 0$
(a) Compare this equation with that found in Exercise 12.1 if K_s, R, and A are 0 Integrate equation (1) to predict the BOD at any flow time τ.

12.4 Develop finite difference equations for predicting the steady-state nitrogen component and DO deficit concentrations *D* in a multi-section one-dimensional estuary. Define every parameter or variable used.

12.5 Using Michaelis-Menten kinetics develop equations for

(a) Predicting the time rate of change of a nutrient concentration N (dN/dt) as a function of the concentration of bacterial biomass B;

(b) Predicting the time rate of change in the bacterial biomass B(dB/dt) as a function of its maximum growth rate μ_B^{max}, temperature T, B, N, and the specific-loss rate of bacteria ρ_B; and

(c) Predicting the time rate of change in dissolved oxygen deficit (dD/dt) also as a function of N, B, ρ_B, and the reaeration-rate constant K_a (T^{-1}).

How would these three equations be altered by the inclusion of protozoa P that feed on bacteria, and in turn require oxygen? Also write the differential equations for the time rate of change in the concentration of protozoa P(dP/dt).

12.6 Most equations for predicting stream temperature are expressed in Eulerian coordinates. The actual behavior of the stream temperature is more easily demonstrated if Lagrangian coordinates (i.e., time of flow t rather than distance X) are used. Assuming insignificant dispersion, the "time-of-flow" rate of temperature change of a water parcel as it moves down-stream is

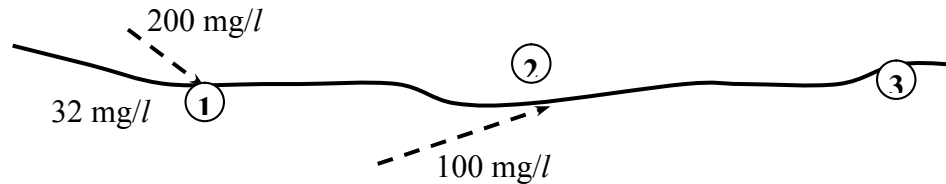$$\frac{dT}{d\tau} = \frac{\lambda}{\rho c D}(T_E - T)$$

(a) Assuming that λ, D, and T_E are constant over interval of time of flow $t_2 - t_1$, integrate the equation above to derive the temperature T_1 at locations X_1.

(b) Develop a model for predicting the temperature at a point in a nondispersive stream downstream from multiple point sources (discharges) of heat.

12.7 Consider three well-mixed bodies of water that have the following constant volumes and freshwater inflows:

Water Body	Volume (m^3)	Flow (m^3/s)	Displacement Time
1	3×10^{12}	3×10^3	3.17 years
2	3×10^8	3×10^2	11.6 days
3	3×10^4	3	2.8 hours

The first body is representative of the Great Lakes in North America, the second is characteristic in size to the upper New York harbor with the summer flow of the Hudson River, and the third is typical of a small bay or cove. Compute the time required to achieve 99% of the equilibrium concentration, and that concentration, of a substance having an initial concentration, and that concentration, of a substance having an initial concentration of 0 (at time = 0) and an input of N (MT^{-1}) for each of the three water bodies. Assume that the decay-rate constant K is 0, 0.01, 0.05, 0.25, 1.0, and 5.0 days^{-1} and compare the results.

12.8 Consider the water pollution problem as shown in the Figure below. There are two sources of nitrogen, 200 mg/l at site 1 and 100 mg/l at site 2, going into the river, whereas the nitrogen concentration in the river just upstream of site 1 is 32 mg/l. The unknown variables are the fraction of nitrogen removal at each of those sites that would achieve concentrations no greater than 20 mg/l and 25 mg/l just upstream of site 2 and at site 3 respectively, at a total minimum cost. Let those nitrogen removal fractions be X_1 and X_2.

200 mg/*l*

②

③

32 mg/*l* ①

100 mg/*l*

Assuming unit costs of removal as $30 and $20 at site 1 and site 2 respectively, the model can be written as:

Minimize $30 X_1 + 20 X_2$
Subject to: $200(1 - X_1)0.25 + 8 \le 20$
                 $200(1 - X_1)0.15 + 100(1 - X_2)0.60 + 5 \le 25$
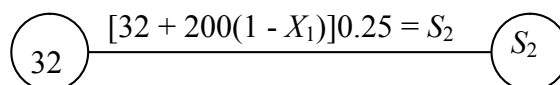 $X_1 \le 0.9, X_2 \le 0.9$

Another way to write the two quality constraints of this model is to define variables S_i (i=1,2,3) as the concentration of nitrogen just upstream of site i. Beginning with a concentration of 32 mg/*l* just upstream of site 1, the concentration of nitrogen just upstream of site 2 will be

$[32 + 200(1 - X_1)]0.25 = S_2$ and $S_2 \le 20$.

The concentration of nitrogen at site 3 will be

$[S_2 + 100(1 - X_2)]0.60 = S_3$ and $S_3 \le 25$.

This makes the problem easier to solve using discrete dynamic programming. The nodes or states of the network can be discrete values of S_i, the concentration of nitrogen in the river at sites i (just upstream of sites 1 and 2 and at site 3). The links represent the decision variable values, X_i that will result in the next discrete concentration, S_{i+1} given S_i. The stages i are the different source sites or river reaches. A section of the network in stage 1 (reach from site 1 to site 2) will look like:

$[32 + 200(1 - X_1)]0.25 = S_2$

(32) ———————————————————— (S_2)

So if S_2 is 20, X_1 will be 0.76; if S_2 is 15, X_1 will be 0.86. For S_2 values of 10 or less X_1 must exceed 0.90 and these values are infeasible. The cost associated with the link or decision will be $30 X_1$.

Setup the dynamic programming network. It begins with a single node representing the state (concentration) of 32 mg/*l* just upstream of site 1. It will end with a single node representing the state (concentration) 25 mg/*l*. The maximum possible state (concentration value just upstream of site 2 must be no greater than 20 mg/*l*. You can use discrete concentration values in increments of 5 mg/*l*. This will be a very simple network. Find the least-cost solution using both forward and backward moving dynamic programming procedures. Please show your work.

12.9 Identify a three alternative sets (feasible solutions) of storage lagoon volume capacities V and corresponding land application areas A and irrigation volumes Q_{2t} in each month t with in a year that satisfy a 10 mg/*l* maximum NO_3-N content in the drainage water of a land disposal system. In addition to the data listed below, assume that the influent nitrogen n_{1t} is 50 mg/*l* each month, with 10% ($\alpha = 0.1$) of the nitrogen in organic form. Also assume that the soil is a well-drained silt loam containing 4500 kg/ha of organic nitrogen in the soil above the drains. The soil has a monthly drainage capacity d of 60 cm and has a field capacity moisture

content M of 10 cm. Maximum plant nitrogen uptake values N_t^{max} are 35 kg/ha during April through October, and 70 kg/ha during May through September. Finally, assume that because of cold temperatures, no wastewater irrigation is permitted during November through March. December, January, and February's precipitation is in the form of snow and will melt and be added to the soil moisture inventory in March.

12.10 Consider the problem of estimating the minimum total cost of waste treatment in order to satisfy quality standards within a stream. Let the stream contain seven homogenous reaches r, reach $r = 1$ being at the upstream end and reach $r = 7$ at the downstream end. Reaches $r = 2$ and 4 are tributaries entering the mainstream at the beginning of 1, 3, 5, 6, and 7. Point sources of BOD enter the stream at the beginning of reaches 1, 2, 3, 4, 6, and 7. Assuming that at least 60% BOD removal is required at each discharge site, solve for the least-cost solution given the data in the accompanying table. Can you identify more than one type of model to solve this problem? How would this model be expanded to specifically include both carbonaceous BOD and nitrogenous BOD and nonpoint waste discharges?

Reach No.	Design BOD Load	Present % Removal Load	ANNUAL COSTS OF VARIOUS DESIGN BOD REMOVALS			
			60%	75%	85%	90%
1	248	67	0	22,100	77,500	120,600
2	408	30	630,000	780,000	987,000	1,170,000
3	240	30	210,000	277,500	323,000	378,000
4	1440	30	413,000	523,000	626,000	698,000
6	2180	30	500,000	638,000	790,000	900,000
7	279	30	840,000	1,072,000	1,232,500	1,350,000

Reach No.	Time of Flow (days)	Waste Water Discharge ($10^3 m^3/day$)	Entering Reach Flow ($10^3 m^3/day$)	Total Reach Flow ($10^3 m^3/day$)	DO saturation conc. (mg/l)	Maximum allowable DO Deficit (mg/l)	DO Deficit of waste-water (mg/l)	DO Conc at begi-nning of Reach (mg/l)	BOD Conc at begi-nning of Reach (mg/l)	Av. Deoxgn rate constant for Reach (days⁻¹)	Reaeration-Rate constant (days⁻¹)
1	0.235	19	5,129	5,148	10.20	3.20	1.0	9.50	1.66	0.31	1.02
2	1.330	140	4,883	5,023	9.95	2.45	1.0	8.00	0.68	0.41	0.60
3	1.087	30	10,171	10,201	9.00	2.00	1.0	?	?	0.36	0.63
4	2.067	53	1,120	1,173	9.70	3.75	1.0	9.54	1.0	0.35	0.09
5	0.306	0	11,374	11,374	9.00	2.50	--	?	?	0.34	0.72
6	1.050	98	11,374	11,472	8.35	2.35	1.0	--	--	0.35	0.14
7	6.130	155	11,472	11,627	8.17	4.17	1.0	--	--	0.30	0.02

12.11 Discuss what would be required to analyze flow augmentation alternatives in Exercise 12.8 How would the costs of flow augmentation be defined and how would you modify the model(s) developed in Exercise 12.8 to include flow augmentation alternatives?

12.12 Develop a dynamic programming model to estimate the least-cost number, capacity, and location of artificial aerators to ensure meeting minimum allowable dissolved oxygen standards where they would otherwise be violated during an extreme low-flow design condition in a nonbranching section of a stream. Show how wastewater treatment alternatives, and their costs, could also be included in the dynamic programming model.

12.13 Using the data provided, find the steady-state concentrations C_t of a constituent in a well-mixed lake of constant volume 30×10^6 m³. The production N_{ti} of the constituent occurs at three sites i, and is constant in each of four seasons in the year. The required fractions of constituent removal P_i at each site i are to be set so that they are equal at all sites i and the maximum concentration in the lake in each period t must not exceed 20 mg/l.

Period, t	Days in Period	Flow, Q_t ($10^3\ m^3/day$)	Constituent Decay Rate, constant, K_t ($days^{-1}$)
1	100	90	0.02
2	80	150	0.03
3	90	200	0.05
4	95	120	0.04

Constituent Discharge Site, i	Constituent Production (kg/day)
1	38000
2	25000
3	47000

12.14 Suppose that the solution of a model such as that used in Exercise 12.13, or measured data, indicated that for a well-mixed portion of a saltwater lake, the concentrations of nitrogen ($i = 1$), phosphorus ($i = 2$), and silicon ($i = 3$) in a particular period t were 1.1, 0.1, and 0.8 mg/l, respectively. Assume that all other nutrients required for algal growth are in abundance. The algal species of concern are three in number and are denoted by $j = 1, 2, 3$. The data required to estimate the probable maximum algal bloom biomass concentration are given in the accompanying table. Compute this bloom potential for all k_i and k equal to 0, 0.8, and 1.0.

Parameter (Algae Species Index j)	PARAMETER VALUE		
	1	2	3
a_{1j} = mg N/mg dry wt of algae j	0.04	0.01	0.20
a_{2j} = mg P/mg dry wt of algae j	0.06	0.02	0.10
a_{3j} = mg Si/mg dry wt of algae j	0.08	0.01	0.03
D_j = morality and grazing rate constant (days^{-1})	0.6	0.4	0.20
d_j = morality rate constant, (days^{-1})	0.3	0.1	0.10
v = extinction reduction rate constant for dead algae, (days^{-1})	0.07	0.07	0.07
η_j^{max} = max. extinction coef. (m^{-1})	0.07	0.07	0.10
η_j^{min} = min. extinction coef. (m^{-1})	0.01	0.03	0.03
η_j = increase in extinction coef. per unit increase in mg/l (g/m^3) of dry wt of species j (m^2/g)	0.05	0.164	0.04
Nutrient Index i:	1	2	3
Nutrient:	N	P	Si
μi = mineralization rate constant , (days^{-1})	0.02	0.69	0.62
Extinction Subinterval s:	1	2	3
Extinction coefficient range:	0.01-0.03	0.03-0.07	0.07-0.10
Algae species $j\ E\ S_s$:	1	1, 2, 3	3
Extinction coefficient without algae = η_0 = 0.01 m^{-1}			

Note: Since there are three extinction subintervals, there are three models to solve for each value of $k_i = k$

Chapter 13 Urban Water Systems

13.1   Define the components of the infrastructure needed to bring water into your home and then collect the wastewater and treat it prior to discharging it back into a receiving water body. Draw a schematic of such a system and show how it can be modeled to determine the best design variable values. Define the data needed to model such a system and then make up values of the needed parameters and solve the model of the system.

13.2   Compare the curve number approach to the use of Manning's equation to estimate urban runoff quantities. Then define how you would predict quality and sediment runoff as well.

13.3   Develop a simple model for predicting the runoff of water, sediment and several chemicals from a 10-ha urban watershed in the northeastern United States during August 1976. Recorded precipitation was as follows:

Day	1	6	7	8	9	10	13	14	15	26	29
R_t (cm)	1.8	0.7	2.6	2.9	0.1	0.3	2.9	0.1	1.4	3.7	0.8

Solids (sediment) buildup on the watershed at the rate of 50 kg/ha-day, and chemical concentrations in the solids are 100 mg/kg. Assume that each runoff event washes the watershed surface clean. Assume also that there is no initial sediment buildup on August. The watershed is 30% impervious. For each storm use your model to compute:
(a) Runoff in cm and m^3.
(b) Sediment loss (kg).
(c) Chemical loss (g), in dissolved and solid-phase form for chemicals with three different adsorption coefficients, $k = 5, 100, 1000$.

13.4   There exists a modest-sized urban subdivision of 100 ha containing 2000 people. Land uses are 60% single-family residential, 10% commercial, and 30% undeveloped. An evaluation of the effects of street cleaning practices on nutrient losses in runoff is required for this catchment.

This evaluation is to be based on the 7-month precipitation record given below. Present the results of the simulations as 7-month PO_4 and N losses as functions of street-cleaning interval and efficiency (i.e., show these losses for ranges of intervals and efficiencies). Assume a runoff threshold for washoff of $Q_o = 0.5$ cm.

	PRECIPITATION (cm)						
Day	A	M	J	J	A	S	O
1			0.6		1.4	0.7	1.9
2	1.1	0.4				0.5	
3						0.1	0.1
4	0.1	1.5					
5	0.1	0.9			1.9	0.3	
6	0.1		1.4	1.4	1.0		0.5
7			0.1	1.1	0.7		
8	0.1	0.1		0.7			0.4
9		0.6	1.6	0.1			1.5
10					0.1		
11							
12				0.2	0.2		0.2
13	0.1			0.2		1.5	
14			0.2		0.5	3.5	0.8
15							1.0
16						4.3	2.8
17			0.7	0.8	0.5	0.8	1.9
18		0.5	0.4	0.1		0.8	0.1
19			0.4			0.4	0.9
20			0.7	0.3		2.3	
21						0.3	
22	0.1			0.1	0.4		
23	2.0						
24	3.2	0.1			0.2	4.7	
25	0.1		0.6			2.8	
26				3.0		1.6	
27							
28			0.3			0.1	
29		0.2	1.1				
30		0.1			0.6		
31					0.2		

Chapter 14 A Synopsis

14.1   Identify, research, describe and critique a water resources planning and management case study. Describe the system being studied, how it was studied or modeled, the institutional setting, the objectives to be accomplished, and your evaluation of how the study was carried out. Was a systems approach applied to the particular water resources management problem. Evaluate the effectiveness of any modeling, the extent of stakeholder participation, and how, if applicable, with hindsight the study could have been improved.

14.2   The following factors are among those that are impacting our global as well as local water systems. Briefly identify their causes and resulting global impacts.

  a) Climate change
  b) Basin scale water balance changes
  c) River flow regulation
  d) Sediment fluxes
  e) Chemical pollution
  f) Microbial pollution
  g) Biodiversity changes

14.3   Presented below are some conclusions of a recent conference on water and sustainability (Schiffries and Brewster, editors, 2004, Water for a Sustainable and Secure Future, National Council for Science and the Environment, Washington, DC). Write a brief critical discussion of each of these statements.

  a) Water is an essential part of human welfare — maintaining our health and survival, protecting sensitive ecosystems, producing an ample food supply, promoting overall economic prosperity, enhancing recreation and aesthetics, and providing for the long-term security of individuals and nations.

  b) Providing enough water for human needs is challenging water policymakers, especially in the water scarce regions, largely because water has been viewed as a free commodity. For this reason, it is typically delivered at vastly below cost and used inefficiently.

  c) The United States had the worst water efficiency of 147 countries ranked by the World Water Council, a status that is linked to low water prices. The (2004) price of water in the United States aver-ages $0.54 per cubic meter, compared to $1.23 for the United Kingdom and $1.78 for Germany.

  d) Perhaps the most important management issue regarding water and sanitation, the one that could have the most benefit for the poor is progressive pricing — "charging more per unit the more water is used" — to ensure that people can afford enough water to live healthfully and still provide incentives for efficient use.

  e) The world is in "a water crisis" that is getting worse. Population is growing most rapidly where water is least available, and water will be among the first resources affected by rising global temperatures and the resulting climate change. This water crisis can be alleviated by pursuing solutions that involve community-scale water systems, open and decentralized decisionmaking, and greater efficiency.

  f) Profound misunderstanding of water science has been institutionalized in many states, where groundwater and surface water are legally two unrelated entities. This gap has

led to practices of unsustainable groundwater withdrawal in some areas and ineffective water management policies that do not take a holistic approach. Ground water and surface water are inextricably linked through the hydrologic cycle, and we need to reform the governance of surface and ground water to reflect actual hydrologic linkages.

g) The challenge of 21st century river management is to better balance human water needs with the water needs of rivers themselves. Meeting this challenge may require a fundamentally new approach to valuing and managing rivers. each component of a river's flow pattern — the highs, the lows, and the levels in between — is important to the health of the river system and the life within it. He is optimistic that new policies will be based on a growing scientific consensus that restoring some degree of a river's natural flow pattern is the best way to protect and restore river health and functioning.

h) Hydrological and ecological linkages, rather than political boundaries, should form the basis for water management. Governance structures should be designed to facilitate a watershed, basin or ecosystem approach to water management. For example, researchers are increasingly attributing coastal pollution problems, such as nutrient over-enrichment, dead zones, and toxic contamination, to diffuse sources far inland from coastal environments. Therefore, effective solutions to these issues must be holistic, entering at the watershed level and connecting coastal pollution with inland sources.

14.4 How would you prioritize and implement the following water sustainability recommendations contained in the report cited in exercise 14.3?

1. Develop a Robust Set of Indicators for Sustainable Water Management.
2. Improve Data and Monitoring Systems for Sustainable Water Management.
3. Advance Interdisciplinary Scientific Research on Sustainable Water Management.
4. Integrate Social and Natural Science Research on Sustainable Water Management.
5. Close the Gap Between Water Science and Water Policy.
6. Develop a Spectrum of Technologies to Advance Sustainable Water Management.
7. Improve Education and Outreach on Sustainable Water Management.
8. Promote International Capacity Building on Sustainable Water Management.
9. Establish National Commissions on Water Sustainability.