



AMERICAN UNIVERSITY OF BEIRUT

MATH 230, Final Examination

February 5, 2001

Time= 1 hour and 30 minutes

Instructions: You are allowed to bring with you one formula sheet and a calculator

1. Die A has orange on one face and blue on five faces, die B has orange on two faces and blue on four faces, and die C has orange on three faces and blue on three faces. These are fair balanced dice. If the three dice are rolled, find the probability that exactly two of the three dice come up the same color. (10 pts)
2. In a certain factory, machines I, II, and III are all producing springs of the same length. Of their production, machines I, II, and III produce 2%, 1%, and 3% defective springs, respectively. Of the total production of springs in the factory, machine I produces 35%, machine II produces 25%, and machine III produces 40%. If one spring is to be randomly selected from this factory,
 - (a) What is the probability of getting a non-defective spring? (5 pts)
 - (b) If the selected spring was defective, what is the probability that it was produced by machine III? (5 pts)
3. Let X_1 and X_2 be mutually independent random variables with Poisson distribution having means 1 and 2, respectively.
 - (a) Find the moment generating function of $Y = X_1 + X_2$. (5 pts)
 - (b) Compute $P(1 \leq Y \leq 3)$. (5 pts)
4. Let the probability density function of the random variable X be

$$f(x) = \frac{1}{\theta} e^{-(x-\alpha)/\theta}, \text{ for } \alpha < x < \infty, \text{ and } \theta > 0 \quad (1)$$

- (a) Find the distribution function of X . (5 pts)
- (b) Find the mean and variance of X . (5 pts)

5. Let X have the probability density function

$$f(x) = 3(1 - x)^2 \text{ for } 0 < x < 1 \quad (2)$$

Find the distribution of the transformation $Y = (1 - X)^3$. (10 pts)

6. Let X have the probability density function

$$f(x) = \frac{e^{-x}}{(1 + e^{-x})^2} \text{ for } -\infty < x < \infty \quad (3)$$

Find the distribution of the transformation $Y = e^{-X}/(1 + e^{-X})$. (10 pts)