AMERICAN UNIVERSITY OF BEIRUT
MATHEMATICS 230 FINAL EXAMINATION
Date: January 31, 2002
Time = 1 hour 30 minutes

Please, print your name and ID number on both the question sheet and the booklet.

You are allowed to use one formula sheet and a calculator.

1. An urn contains 10 red and 9 white balls. The balls are drawn from the urn at random, one at a time. Find the probability that the fourth white ball is the sixth ball drawn if the sampling is done

   (a) with replacement. [5 pts]

   (b) without replacement. [5 pts]

2. Bowl A contains two red chips, Bowl B contains two white chips, and Bowl C contains one red chip and one white chip. A bowl is selected at random (with equal probabilities), then one chip is drawn from that bowl.

   (a) Compute the probability of selecting a white chip. [5 pts]

   (b) If the chip selected is white, compute the conditional probability that the other chip in the bowl is red. [5 pts]

3. Let the *pdf* of the random variable X be $f(x) = k(2/3)^x$ if $x = 2, 3, \ldots$ and zero elsewhere.

   (a) Find the value of k such that $f(x)$ is a *pdf*? [4 pts]

   (b) Find the mean and variance of X. [6 pts]

4. Let X have the *pdf* $f(x) = e^{-x}/(1 + e^{-x})^2$ if $-\infty < x < \infty$. Find the *pdf* of the transformation $Y = |X|$. [10 pts]

5. Let X and Y be independent random variables with respective binomial distributions $b(n_1, p)$ and $b(n_2, p)$. Find the distribution of the random variable $Z = n_1 + n_2 - X - Y$. [10 pts]

over →

6. Let X equal the birth weight (in Kg) of babies in Singapore. Assume that the distribution of X is $N(3.1, 1)$. Let Y equal the number of babies that weigh less than 3.1 kg at birth among 20 of these babies selected independently. Find $P(Y \le 3)$. [10 pts]