AMERICAN UNIVERSITY of BEIRUT

MATH/STAT 230. FINAL EXAM

Time = 1 hour 30 minutes

Aug 26, 2002

You are allowed to use a calculator and one formula sheet of discrete and continuous distributions!

1. A bridge hand is defined as 13 cards selected at random without replacement from a deck of 52 playing cards.

   (a) What is the probability of being dealt at random a bridge hand that does not contain a spade? (5 pts)

   (b) What is the probability that a bridge hand contains exactly 5 hearts? (5 pts)

   (c) Find the probability that a bridge hand contains at most one ace. (5 pts)

2. Consider independent tosses of a fair coin. Find the probability that the fifth head is observed on the tenth toss. (5 pts)

3. Find the moment generating function, mean, and variance if the pdf of X is $f(x) = (1/2)(2/3)^x$ if $x = 1, 2, 3, \ldots$. (10 pts)

4. If $E(X^k) = 2^k$, for $k = 1, 2, 3, \ldots$, find $M(t)$, the moment generating function of X. Can you determine the pdf of X? (10 pts)

5. Find the point at which a chi-square with r degrees of freedom pdf obtains its maximum when $r > 2$ (10 pts)

6. Let X have a uniform distribution $U(-1, 3)$. Find the pdf of $Y = X^2$. (10 pts)

7. The pdf of X is $f(x) = \frac{1}{2\sqrt{x}}$ if $0 < x < 1$. Let $Y = -ln(x)$. Find the distribution of Y. [10 pts]