



Z.Khachadourian

Math. 230—Final Exam.

Aug 13 , 2003

NAME:.....

ID.:.....

I . Derive the moment generating function of the binomial variable ,X , and use it to verify that :

$$\mu = np \text{ and } \sigma^2 = npq .$$

II. Find the probability that a person tossing three coins will get either all heads or all tails for the second time on the fifth toss.

III. Let A and B be two events such that  $P(A | B) = P(A | B^c)$  . Are the events independent? Why?

IV. Let the pdf of X be:

$$f(x) = \begin{cases} x ; & 0 < x < 1 \\ k ; & 2 < x < 4 \end{cases}$$

1. Show that  $k = 1/4$  .
2. Find  $P( 1/2 < x < 4 )$ .
3. Determine the distribution function  $F(x) = P(X \leq x)$ .
4. Find the values of  $q_1$  and  $q_3$  (the first and the third quartiles resp.) and deduce the I.Q.R.
5. Calculate  $E(X^2)$

V. Approximate :  $\sum_{x=3}^5 \binom{200}{x} (0.02)^x (0.98)^{200-x}$

VI. Find the values of the constant c , so that each of the following will be a pdf.

1.  $f(x)=c(x+2)^2 ; x = 0 , 1 , 2$  .
2.  $f(x)=cx^2 ; 0 < x < 2$  .

VII. The moment generating function of X is  $M(t)=\exp(4t+12.5t^2)$ .

1. Find  $P(| X- 4 | < 7.65 )$ .
2. Find the value of C ,  $P(X < C) = 0.025$  .



VIII. Let  $X$  have the exponential distribution with pdf:  $f(x) = 2e^{-2x}$ ;  $0 < x < \infty$ .

1. Find  $P(0 < X < 1/2)$ .
2. Let  $X_1, X_2, X_3, X_4$ , and  $X_5$  be a random sample of size  $n=5$  values from the given distribution. Find the probability that exactly two of the values are between 0 and  $1/2$ .
3. Find  $P(X_1 < X_2 < X_3)$ .

IX. Given 20 identical envelopes and 5 distinct boxes(box 1, box 2, ..., box 5).

1. In how many ways can you place the envelopes into the boxes?
2. In how many ways can you put the envelopes so that there will be at least one envelope in each of the first three boxes and at least two in each of the two other boxes?
3. In how many ways can you put the envelopes so that box  $i$  will have at least  $i$  envelopes;  $i=1,2,3,4,5$ ?
4. In how many ways can you put the envelopes so that box 1 will have at most 12 envelopes?

X. The 11 letters of the word MISSISSIPPI.

1. How many words can you form using all 11 letters.
2. How many of these words will have no two I's next to each other?  
*HINT: (For each arrangement of the letters MSSSSPP, find out where the I's should be put.)*
3. What is the probability that a random arrangement of the 11 letters will produce a word that has no two I's next to each other?

XI.(BONUS QUESTION)

A and B are equals in tennis. They play a match where the player who wins 3 sets first is the winner of the match.

1. If A wins the first set, what is the probability that B wins the match.
2. If B wins the match, what is the probability that A won the first set?