AMERICAN UNIVERSITY OF BEIRUT

MATH 234 FINAL EXAM

Time = 1 hour 30 minutes

February 3, 1997

1. Suppose $X_1, X_2, ..., X_n, ...$ is a sequence of random variables
   (a) Define what is meant by the sequence $\{X_n\}$ converges in distribution to random variable X.
   (b) Define what is meant by the sequence $\{X_n\}$ converges in probability to random variable X.
2. State clearly the central limit theorem.
3. Give an example or a situation where the maximum likelihood estimator of parameter θ is unbiased and an example or situation of a biased maximum likelihood estimator but efficient.
4. Let X have a Gamma distribution with parameters $\alpha = 2$ and $\beta > 0$.
   (a) Find the Fisher information $I(\beta)$.
   (b) If $X_1, X_2, ..., X_n$ is a random sample from this distribution, show that the maximum likelihood estimator of $\beta$is an efficient estimator of β.
5. Let the random variable X have the p.d.f. $f(x;\theta) = (1/\theta)e^{-x/\theta}$, $0 < \theta < \infty$, zero elsewhere. Consider the simple hypothesis $H_0 : \theta = 2$ and the alternative hypothesis $H_1 : \theta = 4$. Let X_1 and X_2 be a random sample of size 2 from this distribution. Show that the best test of H_0 against H_1 may be carried out by use of the statistic $X_1 + X_2$.