



1. Let X_1, X_2, \dots, X_n be a r.s. from a distribution with the following p.d.f.;

$$f(x; \theta) = \begin{cases} \frac{\theta^x e^{-\theta}}{x!} & x = 0, 1, 2, \dots \\ 0 & \text{elsewhere.} \end{cases}$$

Consider estimation of $e^{-2\theta}$.

- What is the m.l.e. of $e^{-2\theta}$?
- Show that the m.l.e. of $e^{-2\theta}$ is an asymptotically unbiased estimator.
- Find an unbiased estimator of $e^{-2\theta}$.
- Does there exist a unique unbiased minimum variance estimator of $e^{-2\theta}$ and if so, what is it?

2. Let X_1, X_2, \dots, X_n be a r.s. from a distribution with the following p.d.f.;

$$f(x; \alpha) = \begin{cases} \frac{1}{\alpha} x^{-(1+1/\alpha)} & x \geq 1, \alpha > 0 \\ 0 & \text{elsewhere.} \end{cases}$$



- What is the complete sufficient statistic for α ?
- What is the distribution of the complete sufficient statistic for α ?
- What is the unique unbiased minimum variance estimator of α ?

3. Let X_1, X_2, \dots, X_n be a r.s. from a distribution with the following p.d.f.;

$$f(x; \theta_1, \theta_2) = \begin{cases} \frac{1}{\theta_2 - \theta_1} & \theta_1 < x < \theta_2, -\infty < \theta_1 < \theta_2 < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

- What are the sufficient statistics for θ_1 and θ_2 ?
- Show that the sufficient statistics are complete
- What are the unique unbiased minimum variance estimators of θ_1 and θ_2 ?
- Is $Y_1 = \min(X_1, X_2, \dots, X_n)$ a consistent estimator of θ_1 ?

