Practice Final Exam

Exercise 1 Discuss the convergence of the following series:

$$a)\ \sum_{n=0}^{+\infty} \frac{1}{\sqrt{n!}} \qquad b)\ \sum_{n=0}^{+\infty} \frac{n+10}{n\ln^3 n} \qquad c)\ \sum_{n=0}^{+\infty} (-1)^{n^2} \frac{1}{n^2+\sqrt{n}}$$

$$d)\ \sum_{n=1}^{+\infty} \frac{1-\cos n}{n^{\ln n}} \qquad e)\ \sum_{n=1}^{+\infty} n(e^{1/n}-1)$$

Exercise 2 a) Estimate the value of the integral $\int_0^1 e^{-3x^2}\,dx$ with an error of magnitude no less than 10^{-3}

(hint: Maclaurin then alternating series and ...)

b) Find $f^{(2001)}(0)$ if $f(x) = x^6 \sin(x^5)$

Exercise 3 If $w = f(x, y)$ is differentiable and $x = r + s, y = r - s$, show that

$$\frac{\partial w}{\partial r} \times \frac{\partial w}{\partial s} = \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2$$

Exercise 4 Find the absolute extremum of $f(x, y) = 5 + 4x - 2x^2 + 3y - y^2$ on the region R bounded by the lines $y = 2, y = x$, and $y = -x$.

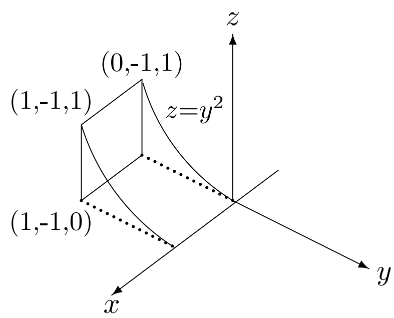
Exercise 5 Use Lagrange multipliers to find the maximum and minimum of $f(x, y) = 4xy$ subject to $x^2 + y^2 = 8$.

Exercise 6 Convert to polar coordinates, then evaluate the following integral

$$\int_0^2 \int_{-\sqrt{1-(y-1)^2}}^0 xy^2 dxdy$$

Exercise 7 Here is the region of integration of the integral

$$\int_0^1 \int_{-1}^0 \int_0^{y^2} dz\ dy\ dx$$



Rewrite the integral as an equivalent iterated integral in the other 5 orders, then evaluate one of them

Exercise 8 Let V be the volume of the region D enclosed by the paraboloid $z = x^2 + y^2$, and the plane $z = 2y$

a) Express V as an iterated triple integral in cartesian coordinates in the order $dz\ dx\ dy$ *(do not evaluate the integral)*

b) Express V as an iterated triple integral in cylindrical coordinates, then evaluate the resulting integral

(you may use the result: $\int \sin^4 x\ dx = -\frac{\sin^3 x \cos x}{4} - \frac{3\cos x \sin x}{8} + \frac{3x}{8}$*)*

Exercise 9 Let V be the volume of the region D that is bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$

a) Express V as an iterated triple integral in spherical coordinates in the order $d\rho\, d\phi\, d\theta$ *(do not evaluate the integral)*

b) Express V as an iterated triple integral in spherical coordinates in the order $d\phi\, d\rho\, d\theta$ *(do not evaluate the integral)*

Exercise 10 Let V be the volume of the region that is bounded form below by the sphere $x^2 + y^2 + (z-1)^2 = 1$ and from above by the cone $z = \sqrt{x^2 + y^2}$. Express V as an iterated triple integral in spherical coordinates, then evaluate the resulting integral *(sketch the region of integration)*

Exercise 11 Set up an integral in rectangular coordinates equivalent to the integral

$$\int_0^{\pi/2} \int_1^{\sqrt{3}} \int_1^{\sqrt{4-r^2}} r^3 (\sin\theta \cos\theta) z^2 dz dr d\theta$$

(do not evaluate the integral)

Exercise 12 a) Find the work done by the force $\overrightarrow{F} = x\,\overrightarrow{i} + y^2\,\overrightarrow{j} + (y-z)\,\overrightarrow{k}$ along the straight line from $(0,0,0)$ to $(1,1,1)$

b) Evaluate

$$\int_{(0,0,1)}^{(1,\pi/2,e)} (\ln z + e^x \sin y) dx + e^x \cos y dy + (x/z - z) dz$$

c) Find the *outward flux* of the field $\overrightarrow{F} = (y-2x)\,\overrightarrow{i} + (x+y)\,\overrightarrow{j}$ across the curve C in the first quadrant, bounded by the lines $y = 0$, $y = x$ and $x + y = 1$.

i) by direct calculation

ii) by Green's theorem

Exercise 13 a) The Curl vector of a vector field in space $F = M(x,y,z)\,\overrightarrow{i} + N(x,y,z)\,\overrightarrow{j} + P(x,y,z)\,\overrightarrow{k}$ is defined by:

$$Curl(F) = \nabla \times F = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\overrightarrow{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x}\right)\overrightarrow{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\overrightarrow{k}$$

Show that $div(Curl(F)) = 0$

b) Show that $Curl(grad\, f) = 0$ if $f(x,y,z)$ is a differentiable function

Exercise 14 Use the transformation $u = y - 2x$ and $v = x + 2y$ to find the value of the integral

$$\int\int_R \frac{x+2y}{y-2x}\, dA$$

where R is the region in the second quarter of the plane, and enclosed by the lines $y - 2x = 2$, and $y - 2x = 4$