American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Fall 2012

quiz # 2

Exercise 1 (answer on page 1 of the booklet)

a) Find the domain and the range of the function $f(x,y) = \ln\left(\dfrac{1}{e - x^2 - y^2}\right)$; determine if the domain open or closed, also determine if the domain is bounded or unbounded; find the equation of the level curve of f that passes through the point $(1,0)$

b) Find $\frac{dy}{dx}$ at $P(0,1)$ if the equation $1 - x - y^2 - \sin(xy) = 0$ define y as differentiable function of x

Exercise 2 (answer on page 2 of the booklet)

a) Can the function $f(x,y) = \dfrac{y^2 \sin x}{2x^2 + 3y^2}$ be extended by continuity at $(0,0)$?

b) Use the two path test to show that $f(x,y) = \dfrac{xy - x}{x^2 + 2(y-1)^2}$ has no limit at $(0,1)$

Exercise 3 (answer on pages 3 and 4 of the booklet)

Suppose that the derivative of the function $f(x,y,z)$ at $P(1,1,1)$ is greatest in the direction of $\vec{u} = 6\vec{i} - 3\vec{j} + 3\vec{k}$, and that in this direction the value of the derivative is $\sqrt{6}$. Also suppose that

$$f(3,0,0) = 1 \;, \quad \vec{\nabla} f(3,0,0) = 3\vec{i} - \vec{j} + 5\vec{k} \;, \quad f(3,2,1) = 3 \;, \quad \textit{and} \quad \vec{\nabla} f(3,2,1) = 6\vec{i} - 2\vec{j} + \vec{k}$$

a) Find the derivative of f at the point $(3,2,1)$ in the direction of $\vec{v} = \vec{i} + \vec{j} + \sqrt{2}\,\vec{k}$

b) Find $\vec{\nabla} f(1,1,1)$

c) Give an approximate value of $f(2.99, 2.02, 1)$

d) Let $x = r^2 + s \;,\; y = rs \;,\; z = \frac{2r}{s} \;,\; \textit{and} \;\; w = f(x,y,z)$

Find $\frac{\partial w}{\partial s}$ at the point $(r,s) = (1,2)$

e) Let $w = w(r,s)$ be defined as in part d). Find a plane tangent to the surface

$$\frac{2}{1-t} + 3 = w(r,s)$$

in the rst-space

(hint: start by finding a point (r_0, s_0, t_0) on the surface)

Exercise 4 (answer on page 5 of the booklet)

Find the points on the surface $xy + yz + zx - x - z^2 = 0$ where the tangent plane is perpendicular to the xz-plane

Exercise 5 (answer on page 6 of the booklet, and last page if needed)

Find the extreme values of $f(x,y) = x^2 + y^2 - 3x - xy$ on the region R in plane defined by: $R = \{x^2 + y^2 \leq 9 \;,\; x \geq 0 \;,\; y \geq 0\}$