

American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Fall 2012

quiz # 1

Exercise 1 (10 points) Find the limit of the following **sequences**:

a) $\frac{n^2}{2n+1} \sin(3/n)$ b) $\frac{n^n + 1}{2^n + n!}$ c) $\left(1 + \frac{1}{3n}\right)^{2n}$

Exercise 2 (35 points) Determine if the following **series** converges or diverges **Justify your answers**

a) $\sum_{n=1}^{+\infty} \frac{10^n}{(\ln n)^n}$

b) $\sum_{n=1}^{+\infty} \frac{1}{n2^n - 1}$

c) $\sum_{n=1}^{+\infty} \frac{2 \cos(n!) - 1}{n(n+1)}$

d) $\sum_{n=2}^{+\infty} \frac{\ln(1 + e^{3n^2})}{n\sqrt{n}}$

e) $\sum_{n=1}^{+\infty} (e^{2/n} - 1)$

Exercise 3 (20 points) a) Find the interval of convergence of the power series

$$\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n4^n} (3x - 1)^{2n}$$

(do not forget to check at the end points)

b) For what value(s) of x the series converges absolutely ? conditionally ?

Exercise 4 (15 points) Let $f(x) = \frac{x-1}{3+2x}$. Find the Taylor series of f about $x = 1$, then find $f^{(101)}(1)$

Exercise 5 (10 points) Find the following limit: $\lim_{x \rightarrow 0} \frac{\cos(\sqrt{x}) - 1 + \frac{x}{2}}{3x^2}$

Exercise 6 (10 points) By using the Maclaurin series of $\ln(1+x)$, give an estimate of $\ln(1.1)$ with an error of magnitude less than 10^{-3}