American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Fall 2010-2011

Final Exam

Exercise 1 Determine if the following **series** converges or diverges. Justify your answers.

$$a) \sum_{n=0}^{\infty} e^{-n^2} \qquad b) \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n \qquad c) \sum_{n=2}^{\infty} \frac{5}{\ln^2 n}$$

Exercise 2 Give the parametric equations of the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at the point $P(1,1,1)$

Exercise 3 a) Find the absolute maximum and minimum values of the function

$$f(x,y) = 2x^2 - xy + y^2 - 7x$$

on the region $R = \{(x,y)\ ;\ x \geq 0\ ,\ y \leq 3\ ,\ 2x - y \leq 3\}$
(group your points and their values in a table)

b) Use Lagrange multipliers to find the point on the plane $2x + 3y + 4z = 12$ at which the function $f(x,y,z) = 4x^2 + y^2 + 5z^2$ has its least value.

Exercise 4 Change the order of integration, then evaluate the integral $\displaystyle\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$

Exercise 5 Let V be the the volume of the region D enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.

a) Express, *but do not evaluate*, V as a triple integral in cartesian coordinates in the order $dzdydx$.

b) Express V as a triple integral in cylindrical coordinates, then evaluate the resulting integral.

Exercise 6 Convert to spherical coordinates the integral:

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} dz\, dy\, dx$$

then evaluate the resulting integral.

Exercise 7 Evaluate

$$\int_0^1 \int_0^{1-x} \sqrt{x+y}\ (y-2x)^2\, dy\, dx$$

by applying the transformation $u = x + y$ and $v = y - 2x$

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Exercise 8 a) Find the work done by the force $\mathbf{F} = xy\mathbf{i} + (y - x)\mathbf{j}$ over the straight line from $(1, 1)$ to $(2, 3)$.

b) Find the counterclockwise circulation of the field $\mathbf{F} = xy\mathbf{i} + (x - y)\mathbf{j}$ around the curve C in the first quadrant, bounded by the curve $y = x^2$ and $y = x$.

i) by direct calculation

ii) by Green's theorem