

## Final Exam

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Name:

Duration: 90 minutes

## Question 1 (60pts)



Let  $A$  be a sparse  $m \times m$  matrix and  $B$  a dense  $m \times n$  matrix with  $m \geq n$ . Consider the matrix-matrix multiplication  $C = AB$ .

- What is the time complexity of a straightforward sequential algorithm?
- Choose distributions for  $A$ ,  $B$ , and  $C$  and formulate a corresponding parallel algorithm. **Motivate your choices of any distribution you adopt.**
- Analyse the time complexity of the parallel algorithm.

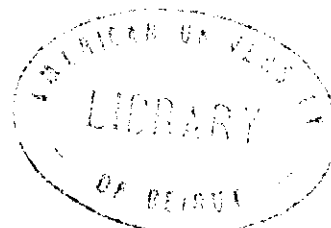
## Question 2 (40pts)

The Conjugate Gradient (CG) method is an iterative method for solving a symmetric positive definite linear system of equations  $Ax = b$ . (A matrix  $A$  is **positive definite** if  $x^T Ax > 0$  for all  $x \neq 0$ .)

The algorithm computes a sequence of approximations  $x^k$ ,  $k = 0, 1, 2, \dots$ , that converges towards the solution  $x$ . The algorithm is usually considered converged when  $\|r^k\| \leq \epsilon_{conv} \|b\|$ , where  $r^k = b - Ax^k$ . Recall that for a vector  $x = (x_1, x_2, \dots, x_n)$ , the (Euclidean) norm is given by  $(x_1^2 + x_2^2 + \dots + x_n^2)^{1/2}$ .

A sequential CG algorithm is given on the next page <sup>1</sup>.

- Design a parallel CG algorithm based on the sparse matrix-vector multiplication discussed in class. How do you distribute the various vectors and matrix  $A$ ? **Motivate your design choices.**
- Analyse the time complexity of your algorithm.



<sup>1</sup>All bold-face variables denote vectors, and non-bold ones denote scalars

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input:  $A$ : sparse  $n \times n$  matrix,
        $\mathbf{b}$ : dense vector of length  $n$ .
output:  $\mathbf{x}$ : dense vector of length  $n$ , such that  $A\mathbf{x} \approx \mathbf{b}$ .

 $\mathbf{x} := \mathbf{x}^0$ ; { initial guess }
 $k := 0$ ; { iteration number }
 $\mathbf{r} := \mathbf{b} - A\mathbf{x}$ ;
 $\rho := \|\mathbf{r}\|^2$ ;
while  $\sqrt{\rho} > \epsilon_{conv} \|\mathbf{b}\|$  &&  $k < k_{max}$  do
  if  $k = 0$  then
     $\mathbf{p} := \mathbf{r}$ ;
  else
     $\beta := \rho / \rho_{old}$ ;
     $\mathbf{p} := \mathbf{r} + \beta \mathbf{p}$ ;
   $\mathbf{w} := A\mathbf{p}$ ;
   $\gamma := \mathbf{p}^T \mathbf{w}$ ;
   $\alpha := \rho / \gamma$ ;
   $\mathbf{x} := \mathbf{x} + \alpha \mathbf{p}$ ;
   $\mathbf{r} := \mathbf{r} - \alpha \mathbf{w}$ ;
   $\rho_{old} := \rho$ ;
   $\rho := \|\mathbf{r}\|^2$ ;
   $k := k + 1$ ;

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