AUB
Physics Department

Physics 303
Classical Electrodynamics
Final Exam

January 29, 1997
Time: 3 hours

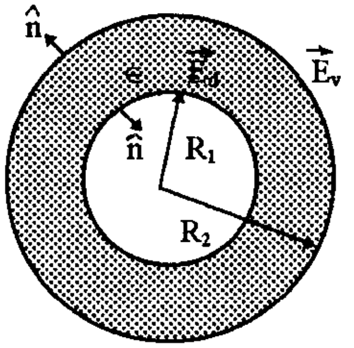
Choose two out of the three problems 1,2 and 3 and solve 4.

30 marks

1) Consider a conducting sphere of radius R_1, on which is placed a charge Q_S. In contact and concentric with this sphere is dielectric material having a dielectric constant $\epsilon = 1 + 4\pi\chi_e$, that extends out to a radius R_2. We wish to find the fields and charge densities generated everywhere.

1) Does $\vec{E}$ depend on θ and φ ?

2) Apply Gauss' Law to find $\vec{D}_d$ and $\vec{E}_d$ for $R_1 \le r \le R_2$.



3) Find $\vec{D}_v$ and $\vec{E}_v$ where $r > R_2$.

4) What are the fields $\vec{D}$ and $\vec{E}$ at $r < R_1$.

5) What is the polarization $\vec{P}$ at $r = R_1$.

6) Deduce the charge density σ_S at $r = R_1$.

7) Find σ_ρ at $r = R_2$.

8) Determine the potential of the conductor ϕ_c.

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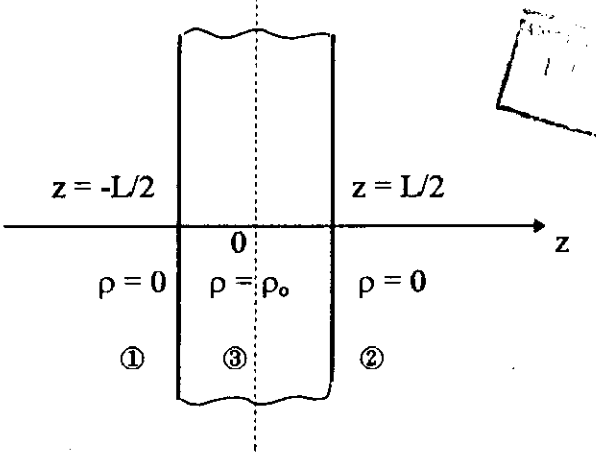
30 marks

2) We shall examine the potential function ϕ, associated with a charge distribution shown in the figure, and given by :

$$\rho = \rho_o \text{ for } -\frac{L}{2} < z < +\frac{L}{2}.$$

$$\rho = 0 \text{ for } |z| > \frac{L}{2}.$$



Use symmetry arguments,
and $\phi = 0$ at $z = 0$ (Indicate why !)
plus other boundary conditions to solve

Poisson's (or Laplace's) equation in the three regions 1, 2, and 3. (Assume that there is no surface charge density in this problem)

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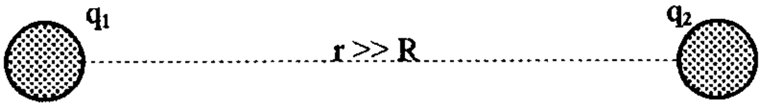
30 marks

3) Find the electrostatic energy of two identical small, charged conducting spheres at a large distance from each other.



Then calculate the change in this electrostatic energy when a thin wire is used to connect the spheres electrically.

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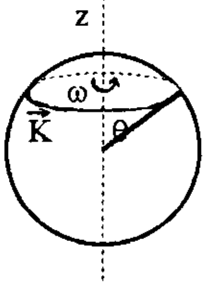
40 marks

4) Consider a sphere of radius R that has a charge uniformly distributed over its surface. Let σ be the surface charge density. The sphere is spinning with a constant angular velocity ω about an axis going through its center. The density of the surface current produced as a result of spinning is $\vec{K} = \sigma\ \vec{v}$ where $\vec{v} = \omega R \sin\theta \,.\, \vec{\varphi}$ is the velocity of the charge element at angle θ.



The vector potential inside and outside the sphere satisfy Laplace's equation. $\vec{A}$ is taken in the φ direction and independent of φ : $\vec{A} = A_\varphi(r, \theta)\ \vec{\varphi}$. Determine expressions for $\vec{A}$ when $r > R$ or $r < R$. The undetermined constants in $\vec{A}$ may be solved with the help of 2 conditions: the continuity of $\vec{A}$ and the condition relating the tangential components of $\vec{H}$ at $r = R$. Compare your result with the well known problem of the potential produced by a uniformly magnetized sphere.
