Physics 303 Final Exam

Please return before the date of the 2006 Final Exam

Problem 1

In a certain reference frame A the expressions for the uniform electric and magnetic fields are given by

$$\vec{E} = a\vec{i} + b\vec{j}$$
$$\vec{H} = c\vec{k} \tag{1}$$

where a, b, c are some constants. Show that, for certain values of c there exists a reference frame B where the magnetic field is zero. What are these values of c? Find the velocity of the frame B with respect to the frame A and the expresion for the electric field in the frame B.

Problem 2

Find the solutions of the Maxwells equations in the Loretz gauge:

$$\begin{aligned} \partial_k F^{ik} &= \frac{4\pi}{c} j^i \\ \partial_i A^i &= 0 \\ i, k &= 0, 1, 2 \\ A^i \equiv (\phi, A_x, A_y); j^i &\equiv (c\rho, j_x, j_y) \end{aligned} \tag{2}$$

for the two-dimensional electrodynamics (i.e. the electrodynamics in the $2 + 1$ Minkowski space), describing the analogue of the electromagnetic radiation in two dimensions. **Hint:** Follow the same logic we used to derive the retarding potentials in the usual (3-dimensional) case, i.e. find the spherically symmetric solutions in the vacuum first, then consider a point-like time-dependent infinitezimal charge in an infinitely small volume element and finally write the solution for the arbitrary charge distribution. It may be helpful to use the Fourier transformation for the time variable. Recall the asymptotic properties of the Bessel's functions and the form of the Green's functions in two dimensions.

Problem 3

It is known that the classical theory of electricity and magnetism leads to contradictions when one tries to apply it to describe physics of atoms and molecules. Attempts to resolve the contradictions eventually led to the discovery of quantum mechanics. One particular contradiction is the inability of classical electromagnetic theory to explain the stability of atoms. E.g. the simplest model of the hydrogen atom involves the electron moving around the proton along a circular orbit. Such an atom cannot be stable since the electron would radiate the electromagnetic waves due to its centripetal acceleration and would eventually lose all of its energy due to the radiation and fall on the proton. Using the dipole approximation for the radiation, estimate the lifetime of such a “classical” hydrogen atom, if its initial “classical” radius is approximately $r = 10^{-8}$ cm.