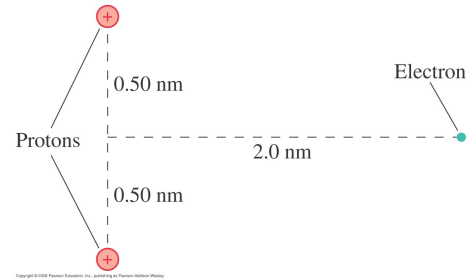


Physics 9 Fall 2009

Homework 4 - Solutions

1. Chapter 29 - Exercise 5.

What is the electric potential energy of the electron in the figure? The protons are fixed and cannot move.



Solution

The potential energy of the electron from a single proton is $PE = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$, and since there are two protons, each at the same distance, the total potential energy is just $PE = -\frac{1}{4\pi\epsilon_0} \frac{2e^2}{r}$. So, plugging in the numbers gives

$$PE = -\frac{1}{4\pi\epsilon_0} \frac{2e^2}{r} = -\frac{(9 \times 10^9) (2 \times 1.602 \times 10^{-19})}{\sqrt{0.5^2 + 2^2} \times 10^{-9}} = -2.24 \times 10^{-19} \text{ J.}$$

2. **Chapter 29 - Exercise 10.**

What is the speed of an electron that has been accelerated from rest through a potential difference of 1000 V?

Solution

The electron starts from rest, so its initial energy is all potential, $E_i = PE = eV$, where V is the voltage. After it has been accelerated, it has fallen through the full potential, so its final potential energy is zero, meaning its final energy is all kinetic, $E_f = KE$. Conservation of energy says that $E_i = E_f$, and so $\frac{1}{2}mv^2 = eV$, and thus

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 10^3}{9.11 \times 10^{-31}}} = 1.87 \times 10^7 \text{ m/s.}$$

3. Chapter 29 - Exercise 21.

Two 2.0 cm-diameter disks spaced 2.0 mm apart form a parallel-plate capacitor. The electric field between the disks is 5.0×10^5 V/m.

- (a) What is the voltage across the capacitor?
 - (b) How much charge is on each disk?
 - (c) An electron is launched from the negative plate. It strikes the positive plate at a speed of 2.0×10^7 m/s. What was the electron's speed as it left the negative plate?
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Solution

- (a) Since the electric field is constant inside the capacitor, the potential is just $V = Ed$, where d is the separation distance. So, $V = Ed = (5 \times 10^5)(.002) = 1000$ V.
- (b) The electric field between the capacitor is $E = \eta/\epsilon_0$, where $\eta = Q/A = Q/\pi R^2$ is the surface charge density. So, $Q = \pi R^2 \epsilon_0 E$. Thus,

$$Q = \pi (10^{-2})^2 (8.85 \times 10^{-12}) (5 \times 10^5) = 1.39 \times 10^{-9} \text{ C.}$$

- (c) The initial energy is both kinetic and potential, $E_i = KE_i + PE_i = \frac{1}{2}mv_i^2 + qV$, while the final energy is all kinetic, $E_f = \frac{1}{2}mv_f^2$. Setting the two energies equal and solving for the initial velocity gives

$$v_i = \sqrt{v_f^2 - \frac{2qV}{m}} = \sqrt{(2.0 \times 10^7)^2 - 2 \frac{1.602 \times 10^{-19} \times 1000}{9.11 \times 10^{-31}}} = 7.0 \times 10^6 \text{ m/s.}$$

4. Chapter 29 - Problem 39.

Bead A has a mass of 15 g and a charge of -5.0 nC. Bead B has a mass of 25 g and a charge of -10.0 nC. The beads are held 12 cm apart (measured between their centers) and released. What maximum speed is achieved by each bead? **Hint:** There are *two* conserved quantities. Make use of both.

Solution

The initial energy is all potential, $E_i = PE = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$. The final energy is all kinetic, $E_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$. Energy conservation gives $E_i = E_f \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$. This is one equation with *two* unknowns! Fortunately, we can find another one - we know that the momentum is conserved! So, $p_i = p_f \Rightarrow m_1 v_1 = m_2 v_2$, since the initial momentum is zero. Solving for, say, $v_2 = \frac{m_1}{m_2} v_1$. Substituting this back into the energy expression gives

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \frac{m_1^2}{m_2} v_1^2 = \frac{1}{2} \left(1 + \frac{m_1}{m_2} \right) m_1 v_1^2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

So,

$$v_1 = \sqrt{\frac{2m_2}{m_1} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r (m_1 + m_2)}}.$$

From momentum conservation, $v_2 = \frac{m_1}{m_2} v_1$, which gives

$$v_2 = \sqrt{\frac{2m_1}{m_2} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r (m_1 + m_2)}}.$$

With the numbers, we find

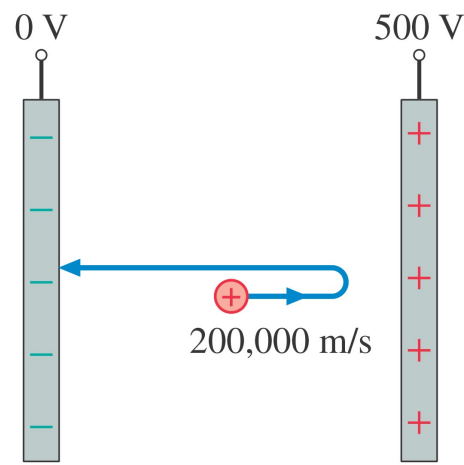
$$\begin{aligned} v_1 &= \sqrt{\frac{2m_2}{m_1} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r (m_1 + m_2)}} \\ &= \sqrt{2 \frac{25}{15} (9 \times 10^9) \frac{(-5 \times 10^{-9})(-10 \times 10^{-9})}{(0.12)(0.025 + 0.015)}} \\ &= 1.77 \text{ cm/s,} \end{aligned}$$

while $v_2 = \frac{m_1}{m_2} v_1 = \frac{15}{25} (1.77) = 1.06 \text{ cm/s}$.

5. Chapter 29 - Exercise 45.

In the figure to the right, a proton is fired with a speed of 200,000 m/s from the midpoint of the capacitor toward the positive plate.

- Show that this is insufficient to reach the positive plate.
- What is the proton's speed as it collides with the negative plate?



Solution

- Suppose that we define the potential energy to be zero at the center of the capacitor (remember that we're always able to do that). Then, the proton's initial energy is all kinetic, $E_i = KE = \frac{1}{2}mv^2$. If the proton just reached the positive plate, then all of its energy would be potential. The question is whether the proton has enough initial kinetic energy to overcome the repulsive force. If that is the case, then it should be able to overcome a voltage of 250 volts. So, let's check this. From energy conservation, $E_i = \frac{1}{2}mv^2 = eV = E_f$, and so $V = \frac{mv^2}{2e}$. Plugging in the numbers gives

$$V = \frac{mv^2}{2e} = \frac{1.67 \times 10^{-27} \times (2 \times 10^5)^2}{2 \times 1.602 \times 10^{-19}} \approx 209 \text{ V}.$$

So, since the proton could only overcome a potential difference of about 209 volts, it *won't* reach the positive plate.

- Now, because we've chosen the potential energy to be zero at the center, the proton starts with only kinetic energy, so $E_i = \frac{1}{2}mv_i^2$, where v_i is the initial speed. When the proton hits the negative plate, it has picked up speed, but also *lowered* its final potential energy. So, $E_f = \frac{1}{2}mv_f^2 - eV$. Solving for the final velocity gives $v_f = \sqrt{v_i^2 + \frac{2eV}{m}}$. Plugging in the numbers gives

$$v_f = \sqrt{v_i^2 + \frac{2eV}{m}} = \sqrt{(2 \times 10^5)^2 + 2 \frac{1.602 \times 10^{-19} \times 250}{1.67 \times 10^{-27}}} = 2.96 \times 10^5 \text{ m/s}.$$

6. Chapter 29 - Problem 57.

One form of nuclear radiation, *beta decay*, occurs when a neutron changes into a proton, an electron, and a neutral particle called an *antineutrino*: $n \rightarrow p^+ + e^- + \bar{\nu}_e$, where $\bar{\nu}_e$ is the symbol for an antineutrino. When this change happens to a neutron within the nucleus of an atom, the proton remains behind in the nucleus while the electron and neutrino are ejected from the nucleus. The ejected electron is called a *beta particle*. One nucleus that exhibits beta decay is the isotope of hydrogen ${}^3\text{H}$, called *tritium*, whose nucleus consists of one proton (making it hydrogen) and two neutrons (giving tritium an atomic mass $m = 3u$). Tritium is radioactive, and it decays to helium: ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$.

- Is charge conserved in the beta decay process? Explain.
- Why is the final product a helium atom? Explain.
- The nuclei of both ${}^3\text{H}$ and ${}^3\text{He}$ have radii of 1.5×10^{-15} m. With what minimum speed must the electron be ejected if it is to escape from the nucleus and not fall back?

Solution

- The initial particle is a neutron, which is neutral, and so contains no charge. The final particles are a proton, an electron, and an antineutrino (note that the textbook has it *wrong*). The proton has a positive charge, $+e$, the electron has a negative charge, $-e$, while the antineutrino is neutral, carrying no charge. The products have a net zero charge, and so this decay conserves charge.
- When the neutron decays, the proton stays in the nucleus, raising its atomic number by one. Since elements are defined by their number of protons, beta decay changes hydrogen to an isotope of helium.
- The electron is launched from the nucleus with some kinetic energy. The nucleus tries to pull the electron back, so it needs enough kinetic energy to overcome the attraction, which is all potential energy. This is just an escape velocity problem. So, we need, $KE = PE \Rightarrow \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$, where q_1 is the charge of the electron, q_2 is the charge of the helium nucleus, $(+2e)$, and r is the initial distance from the nucleus to the electron. Thus, the minimum velocity is $v = \sqrt{\frac{1}{\pi\epsilon_0} \frac{e^2}{mr}}$. Plugging in the numbers we get $v = 8.21 \times 10^8$ m/s, which is faster than light! So, does this mean that the electron never escapes? No - it just means that we should have used the theory of relativity to get the correct answer. The correct analysis gives $v \approx 0.98c$, which *doesn't* violate relativity!

7. Chapter 29 - Problem 58.

The sun is powered by *fusion*, with four protons fusing together to form a helium nucleus (two of the protons turn into neutrons) and, in the process, releasing a large amount of thermal energy. The process happens in several steps, not all at once. In one step, two protons fuse together, with one proton then becoming a neutron, to form the “heavy hydrogen” isotope *deuterium* (${}^2\text{H}$). A proton is essentially a 2.4 fm-diameter sphere of charge, and fusion occurs only if two protons come into contact with each other. This requires extraordinarily high temperatures due to the strong repulsion between protons. Recall that the average kinetic energy of a gas particle is $\frac{3}{2}k_B T$.

- (a) Suppose two protons, each with exactly the average kinetic energy, have a head-on collision. What is the minimum temperature for fusion to occur?
- (b) Your answer to part a is much hotter than the 15 million K in the core of the sun. If the temperature were as high as you calculated, every proton in the sun would fuse almost instantly, and the sun would explode. For the sun to last for billions of years, fusion can only occur in collisions between two protons with kinetic energies much higher than average. Only a very tiny fraction of the protons have enough kinetic energy to fuse when they collide, but that fraction is enough to keep the sun going. Suppose two protons with the same energy collide head-on and just barely manage to fuse. By what factor does each proton’s energy exceed the average kinetic energy at 15 million K?

Solution

- (a) Initially, the protons only have kinetic energy. We’re told that they have the same kinetic energy, $\frac{3}{2}k_B T$. So, the initial kinetic energy of the system is $3k_B T$. The final energy is all potential, $E_f = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$, and so, since $E_f = E_i$,

$$T = \frac{1}{4\pi\epsilon_0} \frac{e^2}{3k_B r}.$$

With numbers, $T = \frac{1}{3 \times 1.38 \times 10^{-23}} (9 \times 10^9) \frac{(1.602 \times 10^{-19})^2}{2.5 \times 10^{-15}} = 2.3 \times 10^9$ K. So, $T = 2.3$ billion Kelvins!

- (b) At a temperature of 15 million Kelvins, the average kinetic energy is $\frac{3}{2}k_B$ (15 millions), and so the ratio of kinetic energy in part (a) to that for this temperature is

$$\frac{\frac{3}{2}k_B (2.3 \times 10^9)}{\frac{3}{2}k_B (1.5 \times 10^6)} = 153.$$

So, the kinetic energy of a proton undergoing fusion is about 153 times bigger than that of the average proton.

It turns out that at any temperature the particles have a distribution of energies. So, some particles have energies a little higher, while some have energies a little lower than average. There are so many particles in the sun, that there are enough with high enough energy to fuse, and so fusion continues in the sun!

8. **Chapter 29 - Problem 64.**

A thin spherical shell of radius R has total charge Q . What is the electric potential at the center of the shell?

Solution

We know that the electric field inside the shell is zero, which means that the potential is constant. Outside the shell, the potential is $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$. At the surface of the sphere $r = R$, and so the potential at the surface is

$$V(R) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}.$$

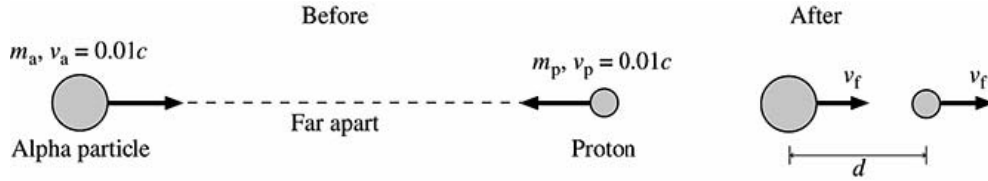
This has to be the potential *inside* the sphere, too! Why? Because, if the potential was a different value inside the sphere, then the potential wouldn't be constant inside - it would change going from the center to the edge. This would lead to an electric field inside the sphere, which we know has to be zero, since there is no charge inside the shell. So, everywhere inside the sphere, $V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$, for $r \leq R$.

9. Chapter 29 - Problem 77.

A proton and an alpha particle ($q = +2e$, $m = 4u$) are fired directly toward each other from far away, each with an initial speed of $0.010c$. What is their distance of closest approach, as measured between their centers?

Solution

Let's draw a picture of the situation, seen below.



The particles are *not* at rest in the final state! Why? They start off with the same velocity, but the alpha particle is *heavier*. So, it pushes the proton away, just slowing it down a certain amount. How can we figure out d ?

The initial energy is all kinetic, $E_i = KE_{i\alpha} + KE_{ip}$. The final energy is *both* kinetic and potential, $E_f = KE_{f\alpha} + KE_{fp} + PE$. So, since energy is conserved, and since the two particles have the same initial speed,

$$\frac{1}{2}m_\alpha v_{i\alpha}^2 + \frac{1}{2}m_p v_{ip}^2 = \frac{1}{2}(m_\alpha + m_p) v_i^2 = \frac{1}{2}m_\alpha v_{f\alpha}^2 + \frac{1}{2}m_p v_{fp}^2 + \frac{1}{4\pi\epsilon_0} \frac{qQ}{d}.$$

Now, we know that momentum has to be conserved, $p_i = p_f$. The initial momentum is $p_i = (m_\alpha - m_p) v_i$, since the initial speeds are the same for both particles. Now, the final momentum is the sum of the momenta of both particles, $p_f = m_\alpha v_{f\alpha} + m_p v_{fp}$, since both particles are moving in the same direction.

Now, how do $v_{f\alpha}$ and v_{fp} compare? They aren't zero, but right at the distance of closest approach *they are traveling at the same speed!* (If they were traveling at a different speed, then they would be getting either closer or further to each other, and so it wouldn't be the closest approach!) This gives the minimum distance.

So, $p_i = p_f = (m_\alpha + m_p) v_f$, at the distance of closest approach. Momentum conservation gives $v_f = \frac{m_\alpha - m_p}{m_\alpha + m_p} v_i$. Plugging this result back into the energy equation gives, after some rearranging

$$\begin{aligned} \frac{1}{4\pi\epsilon_0} \frac{qQ}{d} &= \frac{1}{2}(m_\alpha + m_p) v_i^2 - \frac{1}{2}(m_\alpha + \frac{1}{2}m_p) v_f^2 \\ &= \frac{1}{2}(m_\alpha + m_p) \left[1 - \left(\frac{m_\alpha - m_p}{m_\alpha + m_p} \right)^2 \right] v_i^2 \\ &= \frac{1}{2}(m_\alpha + m_p) \left[\frac{4m_\alpha m_p}{(m_\alpha + m_p)^2} \right] v_i^2 \\ &= 2 \frac{m_\alpha m_p}{m_\alpha + m_p} v_i^2 \end{aligned}$$

Solving for the distance gives

$$d = \frac{1}{8\pi\epsilon_0} \frac{qQ}{v_i^2} \left(\frac{m_\alpha + m_p}{m_\alpha m_f} \right).$$

So, with the numbers, we find

$$\begin{aligned} d &= \frac{1}{8\pi\epsilon_0} \frac{qQ}{v_i^2} \left(\frac{m_\alpha + m_p}{m_\alpha m_f} \right) \\ &= \frac{1}{2} (9 \times 10^9) \frac{(2)(1.60 \times 10^{-19})}{(0.01 \times 3 \times 10^8)^2} \frac{(1+4)u}{(1 \times 4)u^2} \frac{1u}{1.67 \times 10^{-27}} \\ &= 1.92 \times 10^{-14} \text{ m.} \end{aligned}$$

So, the particles come within about 19 fm, which isn't quite close enough to touch.

10. **Chapter 29 - Problem 82.**

A sphere of radius R has charge q .

- (a) What is the infinitesimal increase in electric potential energy dU if an infinitesimal amount of charge dq is brought to infinity to the surface of the sphere?
 - (b) An uncharged sphere can acquire a total charge Q by the transfer of charge dq over and over and over. Use your answer to part a to find an expression for the potential energy of a sphere of radius R with total charge Q .
 - (c) Your answer to part b is the amount of energy needed to assemble a charged sphere. It is often called the *self-energy* of the sphere. What is the self-energy of a proton, assuming it to be a charged sphere with a diameter of 1.0×10^{-15} m?
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Solution

- (a) A uniformly charged sphere of radius R has potential $V_0(R) = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$ at the surface. The increase in potential energy, dU is just $dU = dqV$, and so

$$dU = \frac{1}{4\pi\epsilon_0} \frac{q dq}{R}.$$

- (b) Here we just integrate dU to get the total energy,

$$U = \int dU = \frac{1}{4\pi\epsilon_0} \frac{1}{R} \int_0^Q q dq = \frac{Q^2}{8\pi\epsilon_0 R}$$

- (c) Plugging in the numbers for a proton gives

$$U = \frac{e^2}{8\pi\epsilon_0 R_p} = \frac{1}{2} \frac{9 \times 10^9 \times (1.602 \times 10^{-19})^2}{\frac{1}{2} \times 10^{-15}} = 2.30 \times 10^{-13} \text{ J}.$$