Ex 14) Investigate $\sum \ln\left(\frac{1}{n} + \frac{3}{n\ln(n)}\right)$:

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Quiz 1
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- we know $\sum\left(\frac{1}{n} + \frac{3}{n\ln(n)}\right)$ diverges by nth term integral divergence test.

- So this is an opportunity to apply L.C.T.:

$$\lim_{n\to\infty} \frac{\ln\left(\frac{1}{n} + \frac{3}{n\ln(n)}\right)}{\frac{1}{n} + \frac{3}{n\ln(n)}} = \frac{\ln(0^+)}{0} = \frac{-\infty}{0}$$

$= -\infty$ $\therefore \sum \ln\left(\frac{1}{n} + \frac{3}{n\ln(n)}\right)$ diverges by L.C.T.

- Note: you could have also taken $\sum b_n = \frac{1}{n}$

Ex15) Investigate $\sum \ln\left(1+\frac{3}{n\ln(n)}\right)$:

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- here $\sum\left(1+\frac{3}{n\ln(n)}\right)$ diverges by n^{th} term test, however here if we apply L.C.T.:

we would get $\frac{\ln(1)+0}{1+0} = \frac{0}{1} = 0$

- Not the result we want
- With L.C.T. failing, we quickly switch to D.C.T.:
- Obviously:

$$\ln\left(1+\frac{3}{n\ln(n)}\right) > \ln\left(\frac{3}{n\ln(n)}\right)$$

$\ln\left(\frac{a}{b}\right) = \ln(a)-\ln(b)$

- but $\ln\left(\frac{3}{n\ln(n)}\right) = \ln(3) - \ln(n\ln(n))$

$\ln(a\cdot b) = \ln(a)+\ln(b)$

- but $\ln(n\cdot\ln(n)) = \ln(n)+\ln(\ln(n))$

$$\ln\left(\frac{a}{c\cdot b}\right) = \ln(a)-\ln(c)-\ln(b)$$

take -1 common so that we get $+\infty$ by n^{th} term test.

thus:

$$\ln\left(\frac{3}{n\ln(n)}\right) = \ln(3) - \ln(n) - \ln(\ln(n))$$

$$-\left(\ln\left(\frac{3}{n\ln(n)}\right)\right) = \ln(\ln(n)) + \ln(n) - \ln(3)$$

$$\hookrightarrow -\lim_{n\to\infty} \text{"}\text{"} = \infty + \infty - 3 = +\infty$$

then $-\sum \ln\left(\frac{3}{n\ln(n)}\right)$ diverges by n^{th} term test

- from 10.2 multiplying any non zero c to a divergent series, gives a divergent series
- multiply $-\sum \ln\left(\frac{3}{n\ln(n)}\right)$ again with -1

so that we get:

$\sum \ln\left(\frac{3}{n\ln(n)}\right)$ diverges by n^{th} term test and that property from 10.2

Serie $\sum \ln\left(\frac{3}{n\ln(n)}\right)$ diverges thus

$\sum \ln\left(1+\frac{3}{n\ln(n)}\right)$ diverges by P.C.T.