AMERICAN UNIVERSITY OF BEIRUT
STATISTICS 235, Final Exam

June 19, 2002

Time = 1 Hour and 30 Minutes

You are allowed to use a calculator.

1. (Any simple linear regression problem can be converted to a simple linear regression problem with no intercept.) To see this, consider the usual simple linear regression problem:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \tag{1}$$

where $i = 1, ..., n$, β_0 and β_1 are unknown parameters, ϵ_i are iid $N(0, \sigma^2)$, and σ^2 is an unknown constant. Show that, if we let $Y_i^* = Y_i - \bar{Y}$ and $X_i^* = X_i - \bar{X}$, then model (1) becomes $Y_i^* = \beta_1 X_i^* + \epsilon_i$. (10 pts)

2. A simple linear regression was run on a set of data using an intercept and one independent variable. You are given the following information from the S-PLUS output:

   (a) i. $\hat{Y}_i = 11.5 - 1.5X_i$.

   ii. The t-test for $H_0 : \beta_1 = 0$ was not significant at $\alpha = 0.05$ level. A computed t-value of -4.087 was compared to $t(\alpha = 0.05, df = 2) = 4.183$ which was read from the t-table.

   iii. The estimate of σ^2 was $s^2 = 1.75$

   (b) Complete the ANOVA table using the above results. (10 pts)

   (c) Compute and interpret the coefficient of determination R^2. (5 pts)

Over →

3. You are given the following matrices computed to a multiple regression problem:

$$X'X = \begin{bmatrix} 9 & 136 & 269 & 260 \\ 136 & 2114 & 4176 & 3583 \\ 269 & 4176 & 8257 & 7104 \\ 260 & 3583 & 7104 & 12276 \end{bmatrix}.$$

$$(X'X)^{-1} = \begin{bmatrix} 9.6109 & .0086 & -.2792 & -.0445 \\ .0086 & .5100 & -.2589 & .0008 \\ -.2792 & -.2589 & .1395 & .0007 \\ -.0445 & .0008 & .0007 & .0004 \end{bmatrix},$$

$X'Y$=[45, 648, 1283, 1821]', $\hat{\underline{\beta}} = [-1.1635, 0.1353, 0.0199, 0.1220]'$, and $Y'Y = 285$.

(a) Using the above results, compute the ANOVA table. [Hint: try to generalize for this problem the following fact of simple linear regression that states: SSR=$\hat{\beta}_1^2 \sum(X_i - \bar{X})^2$]. (10pts)

(b) Give the computed regression equation and the standard errors of the regression coefficients. (8 pts)

(c) show how you would test the following composite hypothesis: $\beta_0 = 0$, $\beta_1 = \beta_3$, and $\beta_2 = 0$ (7 pts)

Good luck!