The American University of Beirut

STAT 235 FINAL EXAM

Time = 1 hour and 30 minutes

June 9, 2004

1. A regression analysis using an intercept and one independent variable gave the following:

$$\hat{Y} = 1.841246 + 0.10934X, \text{ and}$$

$$S^2(\hat{\beta}) = \begin{bmatrix} 0.1240363 & -0.002627 \\ -0.002627 & 0.0000909 \end{bmatrix}$$

$MSE = 1.6360$

   (a) Construct a 95% confidence interval estimate of β_1.

   (b) compute $\hat{Y}$ when $X = 4$ and its variance

2. (Simple linear regression through the origin) Let $Y_i = \beta_1 X_i + \epsilon_i$ with the usual assumptions of a linear regression model.

   (a) Show that $\hat{\beta}_1 = \sum X_i Y_i / \sum X_i^2$

   (b) Show that $s^2 = [\sum Y^2 - \hat{\beta}_1 \sum X_i Y_i]/(n-1)$ is an unbiased estimator for σ^2.

3. For a simple linear regression model with intercept, $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$ with the usual assumptions. One may parametrize the model by considering the following model: $Y_i = \alpha + \beta(X_i - \bar{X}) + \epsilon_i$

   (a) Show that the least squares of $\hat{\beta} = \hat{\beta}_1$ and $\hat{\alpha} = \bar{Y}$.

   (b) Show that $Cov(\hat{\alpha}, \hat{\beta}) = 0$.

4. Consider the following two linear models:

$$\underline{Y} = X\underline{\beta} + \underline{\epsilon}$$
$$\text{and } \underline{Y} = \overline{W}\underline{\gamma} + \underline{\epsilon}$$

where X and W are $n \times p$ matrices with linearly independent columns. Suppose further that $W = X.B$, where B is a $p \times p$ non-singular matrix.

   (a) Show that $\hat{\beta} = B.\hat{\gamma}$.

   (b) Show that both models give the same predicted vector $\hat{\underline{Y}}$ and the same residual vector $\hat{\underline{\epsilon}}$.