Phys 305

Final Exam

June 1995

Time: 3 hours

Name: ______________________

Id-no: ______________________

I. Short Questions (Please short answers)

Q1. If the energy eigenvalues of an observable form a continuous spectrum, how is it possible to form a physically acceptable state?. Explain the term "physically".

Q2. If $\varphi(x)$ is an energy eigenfunction of an observable and is a stationary state. In which way it becomes time-dependent?

Q3. A spin-$\frac{1}{2}$ particle is placed in a constant magnetic field along the z-axis. What is the period of rotation of the spin as time progress?. Can this rotation be described in quantum mechanics as precessing about the z-axis? If not why?

Q4. What is the uncertainty relation $(\Delta X)_n (\Delta P)_n$ for a harmonic oscillator in the state $|n\rangle$?. Give the value for the ground state $|n=0\rangle$. Explain the physical meaning of the fact that $(\Delta X)_0 (\Delta P)_0 > 0$.

<u>Hint</u>: You need a short calculation of $(\Delta X)_n^2$ and $(\Delta P)_n^2$ by using the operator a and a^+.

Q5. The energy eigenfunctions of the radial wave equation are $\langle r,\theta,\phi|E,l,m\rangle = R(r)\, Y_{lm}(\theta,\phi)$. What is the physical reason for this separation between the radial and angular part?

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II. Problems

There are 5 problems. The problems ①, ② and ③ are obligatory. You can choose between problem ④ or ⑤

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II 1. Hydrogen Atom

Given is the wave function:

$$\psi(r,\theta,\phi) = \alpha\, r\, e^{-r/2a_B}\, Y_{11}(\theta,\phi),$$

where $a_B = \dfrac{4\pi\varepsilon_0 \hbar^2}{m_e e^2}$ is the Bohr radius, and α is a constant.

(a) Solve the time-independent Schrödinger equation to show that $\psi(r,\theta,\phi)$ is an eigenfunction of the electron in the hydrogen atom, if its spin degree of freedom is not considered.

(b) What are the corresponding energy eigenvalues?

(c) Which quantum numbers (n, l, m) characterize the state of the electron?

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II 2. A spin-$\frac{1}{2}$ particle is at time $t=0$ in the $|+z\rangle$ state. It is placed in a strong magnetic field $\vec{B} = (0, 0, B_0)$ and in an oscillating field $\vec{B_1} = B_1 \cos\omega t\ \vec{i} - B_1 \sin\omega t\ \vec{j}$.

(a) Show that the transition probability to the $|-z\rangle$ state at a time t is given by

$$W_- = \frac{\mu_s^2 B_1^2}{4\Delta^2} \sin^2(\Delta \cdot t),$$

Where $\Delta = \frac{1}{2}\left[(\omega - B_0 \mu_s)^2 + B_1^2 \mu_s^2\right]^{1/2}$.

You assume only spin degree of freedom for the particle, that is the Hamiltonian is given by:

$$H = -\mu_s \vec{S} \cdot \vec{B}$$

(b) Discuss the result obtained for this so called "Paramagnetic resonance absorption".

Hint: After constructing the Hamiltonian, the basic issue is to follow the evolution of the state $|\psi(t)\rangle = \begin{pmatrix} a_+(t) \\ a_-(t) \end{pmatrix}$

This problem is relatively long!

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II 3. The spin Hamiltonian of a spin-$\frac{1}{2}$ particle is given as in problem (II.2). However, the particle is placed in an external magnetic field:

$$\vec{B} = (0, B_2, B_0),$$

where $B_0 \gg B_2$.

(a) First, determine the energy eigenvalues exactly.

(b) Then, calculate in the framework of perturbation theory the energy eigenvalues through second order in B_2/B_0, and compare the results with the exact values in part (a). Do you find agreement?

Hints:

In part (a), during your trip define $\omega_0 \equiv -\mu_s B_0$ and $\omega_2 = -\mu_s B_2$

In part (b): Your trip is not going to end before second-order energy shift.

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II 4. A molecule rotates rigidly in space about the origin of the coordinate system with two degree of freedom discribed by the polar angles Θ and ϕ. The Hamiltonian is given by

$$H = \frac{\vec{L}^2}{2I}, \quad I = \text{moment of inertia}$$

(a) Find the eigenvalues and eigenfunctions and their possible degeneracy

(b) At a given time, the rotator is in the state

$$\psi(\Theta, \phi) = a\,(\cos^2\Theta + \sin^2\Theta \cos 2\phi),$$

where a is a normalization constant (real!).

If L^2 is measured, what is the probability of finding the values:

$$6\hbar^2 \text{ and } 0 \text{ (zero) } ?$$

(c) If a simultaneous measurment of L^2 and L_z is done, What is the probability of finding the values $(6\hbar^2, -2\hbar)$?

Hint: Take a look at the spherical harmonics on page 266 in the book.

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II 5. Cold Emission from metals

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It is experimentally known that an applied strong electric field normal to the surface of a metal leads to emission current outside the metal. The explanation of this "cold emission effect" is possible by the tunnel effect. According to the Pauli Principle, the ~~el~~ conduction electrons occupy at zero temperature ($T=0$) the conduction band up to the Fermi-Energy $\mathcal{E}_f$ (see figure). The work function is defined as

$$W = V_0 - \mathcal{E}_f \,.$$

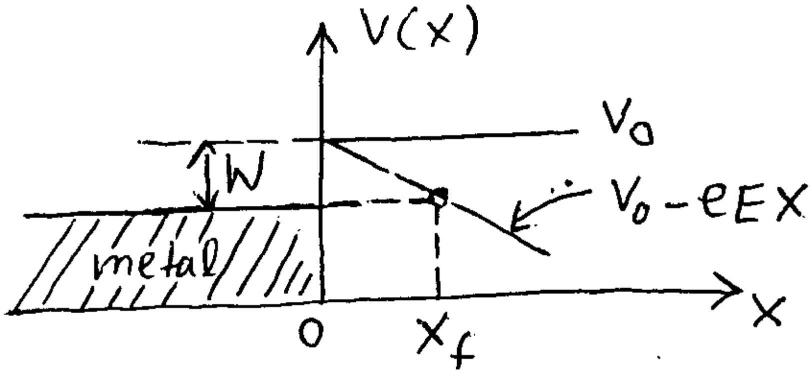
The effect of the external field is to change the potential V_0 to $V(x) = V_0 - eEX$ as shown in the figure.

It is reasonable to assume that the electrons near the Fermi-surface leave the metal, since the tunnel distance is the shortest.

Calculate the emission current:

$$j_T = j_0 \, T(E = \mathcal{E}_f)$$

as a function of W and E, and j_0.



Hint: see Equation (6.158) as a starting point