

AMERICAN UNIVERSITY OF BEIRUT
MATHEMATICS 238 FINAL EXAMINATION

Date: Feb 2, 2002

Time = 1 hour

Please, print your name and ID number on both the question sheet and the booklet.

1. Give an example of a sequence of random variables, X_n , if $n = 0, 1, 2, \dots$, which forms a martingale with respect to a filtration. [5 pts]
2. Give an example of a finite space and discrete time Markov chain. [5 pts]
3. You are given the following one step transition matrix, P :

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & \dots \\ 2/3 & 0 & 1/3 & 0 & \dots \\ 3/4 & 0 & 0 & 1/4 & \dots \\ \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix}$$

Determine the limiting probabilities π_j for $j = 0, 1, 2, \dots$. [10 pts]

4. State the laws of motion for a Birth and Death process. Then find the infinitesimal matrix Q for an M/M/3/GD/infinity/infinity. [10 pts]
5. The Decision Science Department is trying to determine whether to rent a slow or a fast copier. The department believes that an employee's time is worth \$15 per hour. The slow copier rents for \$4 per hour and it takes an employee an average of 10 minutes to complete copying (exponentially distributed). The fast copier rents for \$15 per hour and it takes an employee an average of 6 minutes to complete copying. An average of 4 employees per hour need to use the copying machine (interarrival times are exponential). Which machine should the department rent. [10 pts]