STAT 238

Final Exam

Time = 1 hour

1. Consider the following transition matrix:

$$\begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Which are the transient states?

   (b) Which are the recurrent states?

   (c) Is there any absorbing state? If yes, find the absorption probabilities starting from the transient states.

2. Consider the following two stocks:

   Stock 1 always sells for \$10 or \$20. if it sells for \$10 today, there is a 0.80 chance it will sell for \$10 tomorrow. If it sells for \$20 today, there is a 0.90 chance it will sell for \$20 tomorrow.

   Stock 2 always sells for \$10 or \$25. if it sells for \$10 today, there is a 0.90 chance it will sell for \$10 tomorrow. If it sells for \$25 today, there is a 0.85 chance it will sell for \$25 tomorrow.

   On the average, which stock will sell for a higher price?

3. For the following $M/M/1$ queuong system. Show that the folowing results hold:

   (a) $W = (L+1)W_s$

   (b) $W_q = LW_s$

4. For an $M/M/s$ queuing system. let $\rho = \lambda/s\mu$. Assume further that $\rho < 1$

   (a) Show that $P(j \geq s)$, the probability of all servers are busy, is $(s\rho)^s\pi_0/(s!(1-\rho))$.

   (b) Use the result of part (a) to show that $L_q = P(j \geq s)\rho/(1-\rho)$.