Fall 2008-2009	Final Exam	Date: January 30
Prof: M. Egeileh	Math 201 - Sections 17 to 20	Duration: 2 hours

---

Exercise 1 *(5 pts)* Is the series $\sum_{n\geqslant 0} \frac{(-1)^n}{n!}$ convergent ? absolutely convergent ? Justify your answers.

Exercise 2 *(5 pts)* Does $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2-y^2}$ exist ? Justify your answer.

Exercise 3 *(17 pts)* We consider the function defined on $\mathbb{R}^2$ by : $f(x,y) = x^2 + 2y^2 - \frac{y^3}{3}$.

1. Find all the critical points of f. *(4 pts)*
2. Give the nature (local minimum, local maximum or saddle point) of each of the critical points you found in question 1. *(8 pts)*
3. Give an equation for the tangent plane to the graph of f at the point $(0,4,f(0,4))$. *(5 pts)*

Exercise 4 *(35 pts)* Let R be the region in the plane bounded by the triangle of vertices $O(0,0)$, $A(2,0)$ and $B(1,1)$. We denote by C_1 the line segment joining O to A, C_2 the line segment joining A to B, and C_3 the line segment joining B to O.

1. (a) Compute $\iint_R (y-x^2)\,dx\,dy$, using the order $dx\,dy$. *(6 pts)*
   (b) Write the above double integral as iterated integrals with the order $dy\,dx$ (you do **not** have to re-calculate its value). *(4 pts)*

2. Let $\vec{F}$ be the vector field defined on $\mathbb{R}^2$ by : $\vec{F}(x,y) = (x+x^2y)\vec{i} + (xy+y)\vec{j}$.
   (a) Find, by direct computation, the value of $\int_{C_3} \vec{F}\cdot\vec{dl}$ (work of $\vec{F}$ along C_3). *(6 pts)*
   (b) Compute curl $\vec{F}(x,y)$ (where curl $\vec{F}$ is the $\vec{k}$-component of the vector field $\overrightarrow{\text{curl}}\ \vec{F}$). *(2 pts)*
   (c) Apply Green's theorem to find the value of $\int_{C_1} \vec{F}\cdot\vec{dl} + \int_{C_2} \vec{F}\cdot\vec{dl}$. *(5 pts)*

3. Let $\vec{G}$ be the vector field defined on $\mathbb{R}^2$ by : $\vec{G}(x,y) = (2x)\vec{i} + (4y-y^2)\vec{j}$.
   (a) Show that $\vec{G}$ is conservative. *(4 pts)*
   (b) Give a potential function for $\vec{G}$. *(4 pts)*
   (c) What is the value of $\int_{C_1} \vec{G}\cdot\vec{dl} + \int_{C_2} \vec{G}\cdot\vec{dl}$? *(4 pts)*

Exercise 5 *(15 pts)* Compute the volume of the region D of the space lying in the first octant, bounded from below by the xy-plane, from the sides by the planes $x=2$ and $y=2$, and from above by the plane $x+y+2z=6$. The region D is sketched in page 2.

Exercise 6 *(12 pts)*

1. For each the following two power series, give the radius of convergence. It is sufficient to provide **justification for only one** of them. *(2 pts)*

   (a) $\sum_{n\geqslant 0} \frac{x^{2n}}{(2n)!}$

   (b) $\sum_{n\geqslant 0} \frac{x^{2n+1}}{(2n+1)!}$

2. For every $x \in \mathbb{R}$, we set $\text{ch}(x) = \frac{1}{2}(e^x + e^{-x})$ and $\text{sh}(x) = \frac{1}{2}(e^x - e^{-x})$. Find the Maclaurin series generated by each of the functions ch and sh. *(6 pts)*

3. Show that for any $t \in [0, 1]$, $\text{ch}(t) \leqslant \frac{5}{3}$. (*Hint : you may use the fact that ch is an increasing function on* $[0, +\infty[$, *and that* $1 \leqslant ln(3)$). *(2 pts)*

4. Deduce from the preceding question an upper bound for the error committed when approximating $\frac{1}{2}(e+\frac{1}{e})$ by 1.5. You may leave the final answer as a fraction $\frac{a}{b}$. (*Hint : start by writing that* $ch(x) = 1 + \frac{x^2}{2!} + 0\frac{x^3}{3!} + R_3(x)$, *then use Taylor remainder's estimation theorem...*). *(2 pts)*

Exercise 7 *(11 pts)* Let R be the region inside the reversed empty ice-cream cone $z = -\sqrt{x^2 + y^2}$ and sandwiched between the planes $z = -1$ and $z = -2$.

1. Find the volume of R using a triple integral in spherical coordinates, with the order $d\rho\ d\phi\ d\theta$. *(9 pts)*

2. Setup the triple integral whose evaluation would give you the volume of R, in spherical coordinates, with the order $d\phi\ d\rho\ d\theta$ (do **not** re-calculate the volume of R). *(2 pts)*
