



AMERICAN UNIVERSITY of BEIRUT
Department of Mathematics
Statistics 237, Final Examination
Time = 1 hour and 30 minutes

You are allowed to use a standard normal table and a D_{α}^n table

1. to test the randomness of a pocket calculator random generator, the following 10 values were generated:

i	1	2	3	4	5	6	7	8	9	10
u_i	.010	.072	.855	.881	.698	.814	.816	.871	.255	.711

Use the Kolmogorov-Sminrov D_n statistic to check the conformity of the above data with the theoretical distribution.

2. Refer to data in previous question (1). Consider the following hypothesis:

$$H_0 : \xi_{0.5} = 1/2 \text{ vs } H_a : \xi_{0.5} > 1/2$$

Using the sign test, find an approximate p-value for the above hypothesis.

3. Refer to data in previous question (1) and the hypothesis in previous question (2). Test the hypothesis using the Wilcoxon rank sum test with a significance level $\alpha = 0.05$.
4. Consider two random samples $(X_1, \dots, X_m) \sim F(\cdot)$ and $(Y_1, \dots, Y_n) \sim G(\cdot)$. To test the equality of the two distribution functions F and G , one may use the Mann-Whitney-Wilcoxon U statistic. That is: $U = \sum_{j=1}^n \sum_{i=1}^m Z_{ij}$, where $Z_{ij} = I(X_i < Y_j)$. Let $h(u; m, n)$ be the density function of the random variable U . Compute $h(u; 2, 2)$
5. Refer to previous question (4) and let $m = n = 3$. Furthermore consider the following hypothesis,
 $H_0 : F = G$ vs $F > G$.

If we combine the two samples $(X_1, X_2, X_3, Y_1, Y_2, Y_3)$ and then order the combined sample $(W_1 < W_2 < W_3 < W_4 < W_5 < W_6)$. Let $R(Y_j)$ be the rank of the observation Y_j in the combined sample. Denote by $T = \sum_{j=1}^3 R(Y_j)$. Show that $U = T - 6$.