Physics Department

Phys 310
Astrophysics
Final Exam

June 29, 1996
Time: 3 hours

Name: ______________________

Id NO: ______________________

Remember these nice words:

Nature never deceives us, it is always
we who deceive ourselves.

J. Jaque Rousseau

Please, answer as many questions as you can within the given time.

(1) Virial Theorem

(a). Consider the Bohr model of the hydrogen atom. Show that the virial theorem is also satisfied in this case.

(b) Consider a star in hydrostatic equilibrium. Using the adiabatic equation of state

$$P = K \varrho^{\gamma} \quad (K, \gamma \text{ are constants})$$

show that the potential gravitational energy Ω of the star is

$$\Omega = -3(\gamma - 1)\, U,$$

where $U = \int_0^R \varepsilon \, 4\pi r^2 dr$ is the total internal energy. Note that ε is the energy per volume.

For $\gamma = 5/3$ your result should be in agreement with part (a). Explain why this agreement.

(2) Calculate the adiabatic temperature gradient ∇_{ad} for an ideal nondegenerate gas. Use the general expression derived in the lecture.

(3) If a star is in hydrostatic equilibrium, and its pressure is dominated by the radiation field ($P = \frac{1}{3} a T^4$). Show that the luminosity of the star is given by the so called "Eddington luminosity":

$$L_E = \frac{4\pi c G M}{\varkappa},$$

where M is the total mass of the star, $\varkappa$ is the opacity, c is the speed of light, and G is the gravitational constant.

Hint: You need to combine the equation of hydrostatic equilibrium with that of radiation transport.

(4) Assume a simple linear model for the density in a star, such as

$$\rho(r) = \rho_c (1 - r/R),$$

where R is the radius of the star, and ρ_c is the central density.

(a) Calculate the mass function $m(r)$ in terms of ρ_c and R.

(b) Use $m(r)$ to obtain the pressure P_c at the center of the star

Hint: assume the surface pressure $P_s = 0$.
Use the equation of hydrostatic equilibrium to show that $P_c = \frac{5}{4\pi} \frac{GM^2}{R^4}$.

(c) Determine the central pressure in the sun. Explain why the central pressure turns out to be very high compared with Earth's standards.

(5) Consider the reaction $^{12}C(p,\gamma)^{13}N$, and assume the following data given:

$X = 0.80$ (mass fraction of hydrogen)

$T_6 = 30$ $(T_6 = T/10^6 K)$

$S(0) = 1.40$ keV-barn (astrophysical factor)

$S_{eff} = S(0)\left(1 + \frac{5}{12\tau}\right)$;

$\rho = 15\ g/cm^3$

(a) With these data, determine the lifetime of ^{12}C against proton capture.

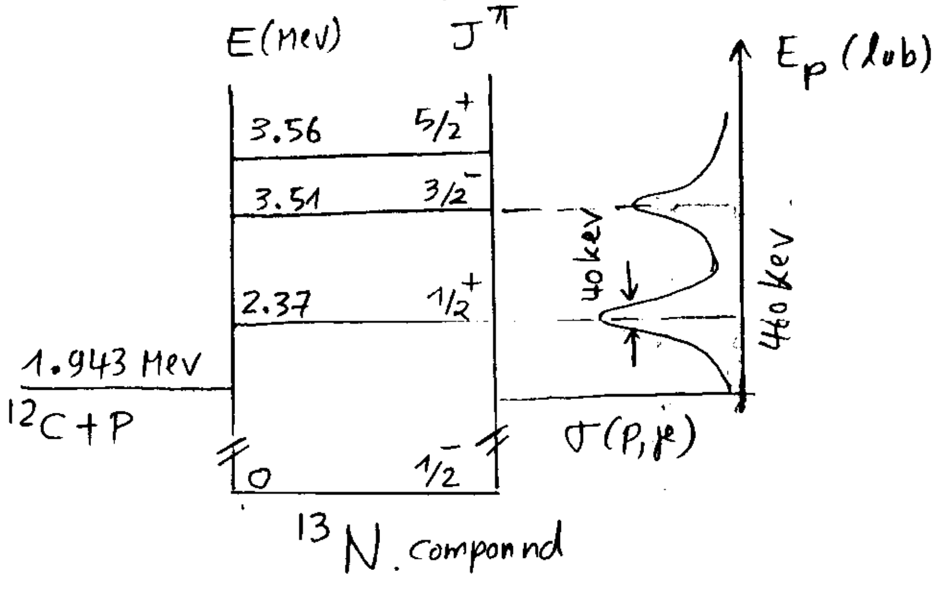
(b) Assume that the 460 keV (lab energy) resonance is artificially deplaced to 50 kev (see Figure). Knowing that:

$\Gamma_2 = \Gamma_\gamma = 1\ eV$

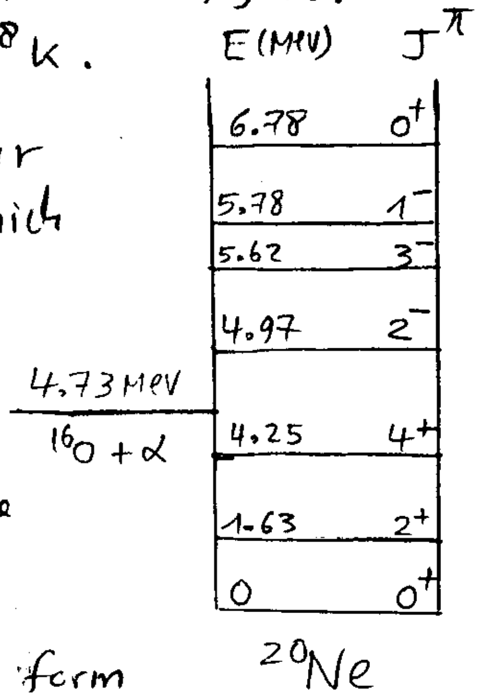
$\Gamma_1 = \Gamma_p = 40\ keV$ (at 460 keV)

$T_6 = 30$

Compare $\langle\sigma v\rangle_r$ with $\langle\sigma v\rangle$ as obtained under (a). What is the factor by which the lifetime of ^{12}C is reduced if such resonance could exist?



(6) Consider the reaction $^{16}O(\alpha,\gamma)^{20}Ne$. The level scheme of the ^{20}Ne system is shown in the figure. Assume the temperature to be $T = 10^8$ K.



(a) Calculate the most effective stellar energy (E_0) for this reaction. Which state falls in the center of the "Gamow-peak"?

(b) Is this state a resonance? Justify your answer. What is the significance of your result?

(c) What temperature is needed to form the excited state (6.78, 0^+)? Could this state serve as a resonance?

(7) In the approximate relation for the energy generation rate by the 3α-process:

$$\varepsilon_{3\alpha}(T) = \varepsilon_{3\alpha}(T_0)\left(\frac{T}{T_0}\right)^n,$$

determine the value of n near $T_8 = 1$.

8 In the reaction

$$^{12}C + P \rightarrow {}^{13}N + \gamma$$

determine the values of the orbital angular momentum l_{cp} that can form the following resonance states

(a) $\frac{1}{2}^-$ (b) $\frac{1}{2}^+$ (C) $\frac{5}{2}^-$.

Indicate which waves (s, p, d, ...) form these resonances

(b) Write down the configurations for the protons and neutrons in $^{14}_{7}N$. What is the spin of ^{14}N in the ground state. (2 values possible).

Hint: shell model

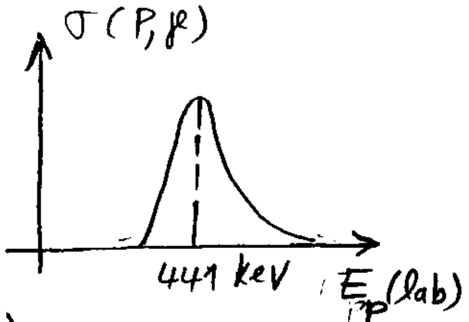
(C) Consider the reaction :

$$^{7}Li + P \rightarrow {}^{8}Be^{*} \rightarrow {}^{8}Be + \gamma.$$

It has a resonance at $(E_p)_{lab} = 441$ keV

Answer the following questions!

(i) What is the excitation energy in ^{8}Be if the proton has the above energy



Hint: You need the atomic masses in atomic mass unit (u)

$M(^{7}Li) = 7.016005$ u

$M(P) = 1.007825$ u, $M(^{8}Be) = 8.005305$ u

$1u = 935.1$ Mev/c^2

9 Synthesis of heavy elements

(a) Why do the nuclei $^{88}_{39}Sr$, $^{90}_{40}Zr$, $^{140}_{58}Ce$, $^{142}_{60}Nd$ have smallest neutron capture cross sections ?

(b) Why is the number of pure s-nuclei, or pure r-nuclei so small ? Explain the term "pure s-nuclei", or "pure r-nuclei".

(c) Given the lifetime of a nucleus against neutron capture $\tau_n = 10^4$ years, and a typical average cross section $\langle \sigma \rangle = 100$ millibarns, and a thermal velocity of $v_T = 3 \times 10^8$ cm/s.

What is the corresponding neutron density n_n (in cm^{-3})

(d) Repeat part (c) assuming $\tau_n = 1\ \mu s$ (10^{-6} second).

(e) If the neutron capture cross section of thermalized neutrons varies as $\sigma(v) \propto \frac{1}{v}$, what is the average cross section $\langle \sigma \rangle$?
Hint: use the expressions presented in the lecture. $\int_0^\infty x^2 e^{-a^2x^2} = \sqrt{\pi}/a^3$.

(f) What is the main advantage of the so called σN_s curve ?

(g) Explain briefly the "waiting points" encountered during the r-process nucleosynthesis.

(h) Why is not possible to form Uranium or Thorium by the s-process nucleosynthesis ?

Review Questions

Q1 What is the main physical reason that the sun can survive about 10^{10} years on the main sequence?

Q2 After leaving the main sequence, the evolution of the sun (and other low mass stars) along the red giant branch (RGB) is relatively long. How can this fact be related to the internal structure at this stage?

Q3 What makes the more massive stars evolve faster than the sun on the RGB?

Q4 Consider the solar neutrino spectrum:

(a) Explain qualitatively why the PP-neutrinos and ^8B-neutrinos have continuous spectra.

(b) How do you explain the two lines associated with the ^7Be-neutrinos?

(c) Which neutrino flux is most sensitive to temperature?. Which neutrino flux has the weakest dependence on temperature?

Q5 Why there is an outer convective zone in the sun?, that is which physical quantity is responsible for the formation of the convective zone. Which observation indicates the existence of this zone?

Q6 Why do massive stars (like a $5 M_\odot$ star) develop a convective core on the main sequence?

[Q7] The liberated energy from hydrogen burning (effectively: $4p \rightarrow {}^4He$) is about 26 MeV, while the liberated energy from the 3α-process is 7.37 MeV.

(a) How much energy (erg/g) is gained from hydrogen burning? How much energy (erg/g) is gained from the 3α-process?

(b) Using the results under (a), estimate the life-time of a star during helium burning (τ_{He}) to the life-time during hydrogen burning (τ_{He}).

Hint: You need to express 1 MeV in erg.
Mass of a proton $= 1.67 \times 10^{-24}$ g.

[Q8] When a massive star of 15 $M_\odot$ forms a core consisting of iron-group nuclei, it ultimately undergoes gravitational collapse. Why is not possible to stabilize the star at this stage?

[Q9] What are the physical conditions that make the ^{12}C production possible in stars by the 3α-process? List at least three conditions.

[Q10] The driving mechansim for the "helium flash", and for "Thermal Pulsations" is the 3α-process.

(a) Describe why the helium flash is so violent, but not the thermal pulsation.

(b) Where are the stars which suffer the helium flash in the Hertzsprung-Russel diagram? Where are the stars which exhibit