# MAT 213
# Calculus III
# Exam 2

**(15 points)** 1. Does the sequence with the following $n^{th}$ term converge or diverge? Find the limit of each convergent sequence.

a) $a_n = \left(-\frac{1}{2}\right)^n$ b) $a_n = \sqrt{\frac{2n}{n+1}}$.

**(10 points)** 2. Find the sum of $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$

**(20 points)** 3. Does the following series converge or diverge?

a) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\left(\sqrt{n}+1\right)}$ c) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$

d) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

**(15 points)** 4. Does the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+1}$ converge conditionally?

**(15 points)** 5. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+3}}$.

**(10 points)** 6. The estimate $\sqrt{1+x} = 1+\frac{x}{2}$ is used when $x$ is small. Estimate the error when $|x| < 0.001$.

**(15 points)** 7. Find series solution for the initial value problem $y' + y = 1$, $y(0) = 2$.