

# Calculus III

## Exam #1

1) (60 points) Compute the following integrals.

a)  $\int \sin^{-1} 2x \, dx$

b)  $\int \frac{(1-x^2)^{3/2}}{x^6} \, dx = -\frac{1}{5} \left( \frac{\sqrt{1-x^2}}{x} \right)^2 + C$

c)  $\int \frac{4x^3 - x^2 + 16x}{x^2 + 4} \, dx = \int \left( 4x - 1 + \frac{4}{x^2 + 4} \right) dx$

d)  $\int p^4 e^{-p} \, dp$

e)  $\int \frac{2\sqrt{x} \, dx}{x + 4x^2} = 2 \int \frac{\sqrt{x} \, dx}{x(1+4x)} = 2 \int \frac{u \, du}{u^2(1+4u)} = 2 \int \frac{u \, du}{1+4u^2}$

2) (40 points) Determine whether the following integrals converge or diverge.

a)  $\int_{-1}^5 \frac{dx}{(x-1)^3} = \int_{-1}^1 \frac{dx}{(x-1)^3} + \int_1^5 \frac{dx}{(x-1)^3}$  div  $p=3 > 1$

b)  $\int_0^{\infty} \frac{dx}{\sqrt{x}(x+1)} = \int_0^1 \frac{dx}{\sqrt{x}(x+1)} + \int_1^{\infty} \frac{dx}{\sqrt{x}(x+1)} \sim \int_0^1 \frac{dx}{\sqrt{x}} + \int_1^{\infty} \frac{dx}{x^{3/2}} < \infty$   
(  $\lim_{x \rightarrow 0} \frac{1/\sqrt{x}(x+1)}{1/\sqrt{x}} = 1$  ) or  $\left( \sqrt{x} \leq \sqrt{x} + \sqrt{x}x \leq \sqrt{x}(x+1) \right)$   
 $\frac{1}{\sqrt{x}(x+1)} \leq \frac{1}{\sqrt{x}}$

c)  $\int_2^{\infty} \frac{dx}{x^2-3} \sim \int_2^{\infty} \frac{dx}{x^2} < \infty$