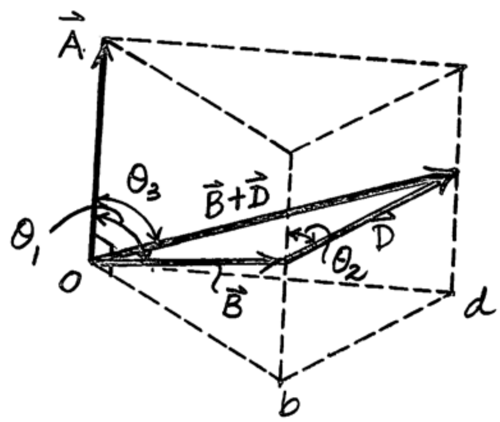
•4–1. If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.



Consider the three vectors; with **A** vertical.

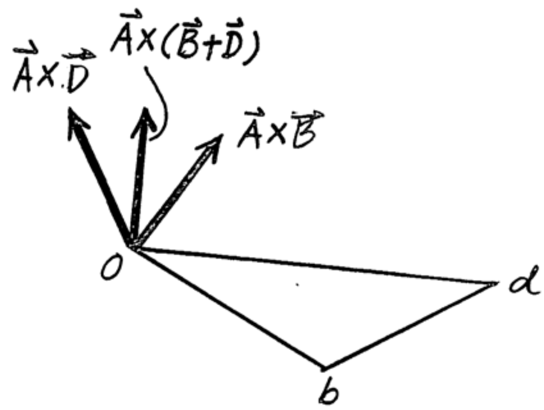
Note *obd* is perpendicular to **A**.

$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}|(|\mathbf{B} + \mathbf{D}|) \sin\theta_3$

$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta_1$

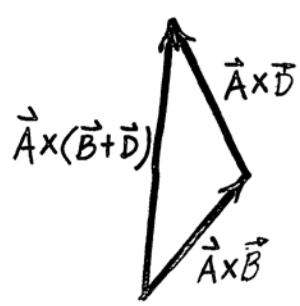
$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin\theta_2$

Also, these three cross products all lie in the plane *obd* since they are all perpendicular to **A**. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.



The three vector cross - products also form a closed triangle *o′b′d′* which is similar to triangle *obd*. Thus from the figure,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{D} \qquad \text{(QED)}$$



Note also,

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\mathbf{D} = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i}$$
$$-[A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j}$$
$$+ [A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k}$$
$$= [(A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}]$$
$$+ [(A_yD_z - A_zD_y)\mathbf{i} - (A_xD_z - A_zD_x)\mathbf{j} + (A_xD_y - A_yD_x)\mathbf{k}]$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \text{(QED)}$$

4–2. Prove the triple scalar product identity $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$.

As shown in the figure

$$\text{Area} = B(C\sin\theta) = |\mathbf{B} \times \mathbf{C}|$$

Thus,

Volume of parallelepiped is $|\mathbf{B} \times \mathbf{C}||h|$

But,

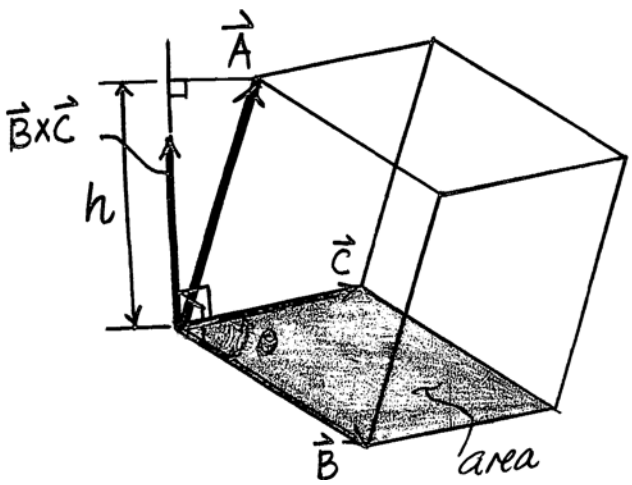
$$|h| = |\mathbf{A} \cdot \mathbf{u}_{(\mathbf{B}\times\mathbf{C})}| = \left|\mathbf{A} \cdot \left(\frac{\mathbf{B} \times \mathbf{C}}{|\mathbf{B} \times \mathbf{C}|}\right)\right|$$

Thus,

$$\text{Volume} = |\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}|$$

Since $|\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}|$ represents this same volume then

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C} \qquad \text{(QED)}$$



Also,

$$LHS = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$$

$$= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - B_zC_x) + A_z(B_xC_y - B_yC_x)$$
$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

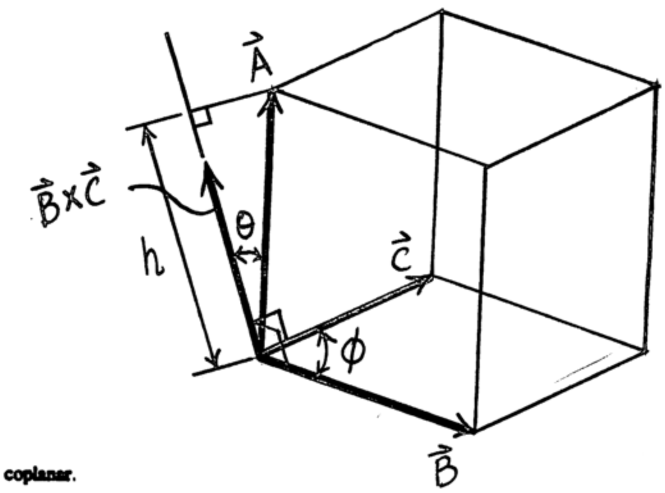
$$RHS = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \cdot (C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$
$$= C_x(A_yB_z - A_zB_y) - C_y(A_xB_z - A_zB_x) + C_z(A_xB_y - A_yB_x)$$
$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

Thus, $\quad LHS = RHS$

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C} \qquad \text{(QED)}$$

4–3. Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, the three vectors *must* lie in the same plane.



Consider,

$$|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| = |\mathbf{A}||\mathbf{B} \times \mathbf{C}| \cos\theta$$

$$= (|\mathbf{A}|\cos\theta)|\mathbf{B} \times \mathbf{C}|$$

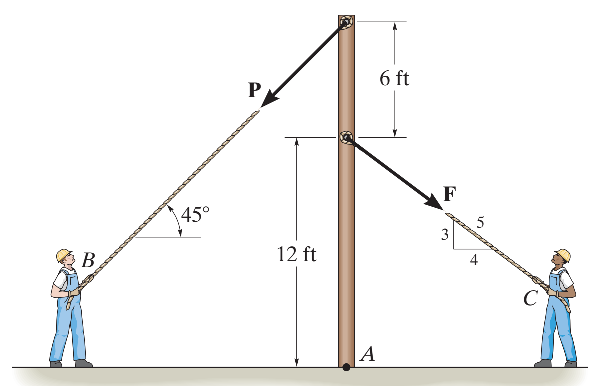
$$= |h||\mathbf{B} \times \mathbf{C}|$$

$$= BC|h| \sin\phi$$

= volume of parallelepiped.

If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, then the volume equals zero, so that **A**, **B**, and **C** are coplanar.

***4–4.** Two men exert forces of $F = 80$ lb and $P = 50$ lb on the ropes. Determine the moment of each force about A. Which way will the pole rotate, clockwise or counterclockwise?



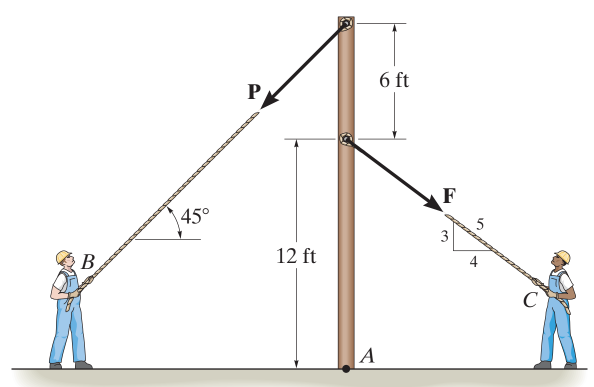
$\curvearrowright +$ $(M_A)_C = 80\left(\frac{4}{5}\right)(12) = 768$ lb·ft $\curvearrowright$ **Ans**

$\curvearrowleft +$ $(M_A)_B = 50(\cos 45°)(18) = 636$ lb·ft $\curvearrowleft$ **Ans**

Since $(M_A)_C > (M_A)_B$

Clockwise **Ans**

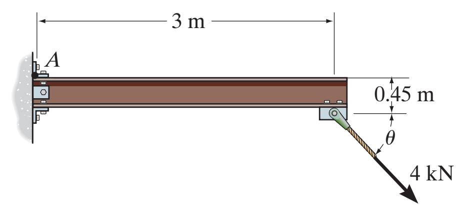
•4–5. If the man at B exerts a force of $P = 30$ lb on his rope, determine the magnitude of the force **F** the man at C must exert to prevent the pole from rotating, i.e., so the resultant moment about A of both forces is zero.



$\curvearrowleft +$ $30(\cos 45°)(18) - F\left(\frac{4}{5}\right)(12) = 0$

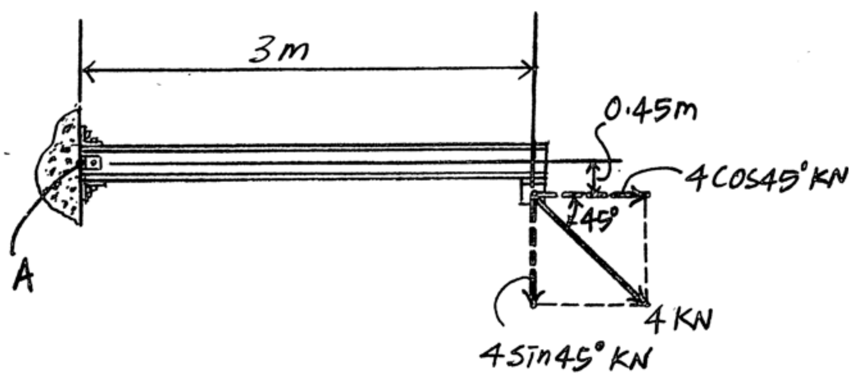
$F = 39.8$ lb **Ans**

4–6. If $\theta = 45°$, determine the moment produced by the 4-kN force about point A.



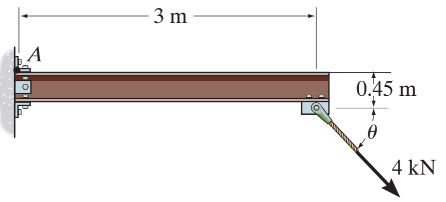
Resolving the 4 - kN force into its horizontal and vertical components, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = 4\cos 45°(0.45) - 4\sin 45°(3)$$
$$= -7.21\ \text{kN}\cdot\text{m} = 7.21\ \text{kN}\cdot\text{m}\quad \text{(clockwise)} \qquad \textbf{Ans.}$$



(a)

4–7. If the moment produced by the 4-kN force about point A is 10 kN · m clockwise, determine the angle θ, where $0° \leq \theta \leq 90°$.



Resolving the 4 - kN force into its horizontal and vertical components, Fig. a, and applying the principle of moments,

$$\circlearrowleft +M_A = -10 = 4\cos\theta(0.45) - 4\sin\theta(3)$$

$$12\sin\theta - 1.8\cos\theta = 10 \qquad (1)$$

Referring to the geometry of Fig. a,

$$\cos\phi = \frac{12}{\sqrt{147.24}} \qquad \sin\phi = \frac{1.8}{\sqrt{147.24}} \qquad (2)$$

Dividing Eq. (1) by $\sqrt{147.24}$ yields

$$\frac{12}{\sqrt{147.24}}\sin\theta - \frac{1.8}{\sqrt{147.24}}\cos\theta = \frac{10}{\sqrt{147.24}} \qquad (3)$$

Substituting Eq. (2) into (3) yields

$$\sin\theta\cos\phi - \cos\theta\sin\phi = \frac{10}{\sqrt{147.24}}$$

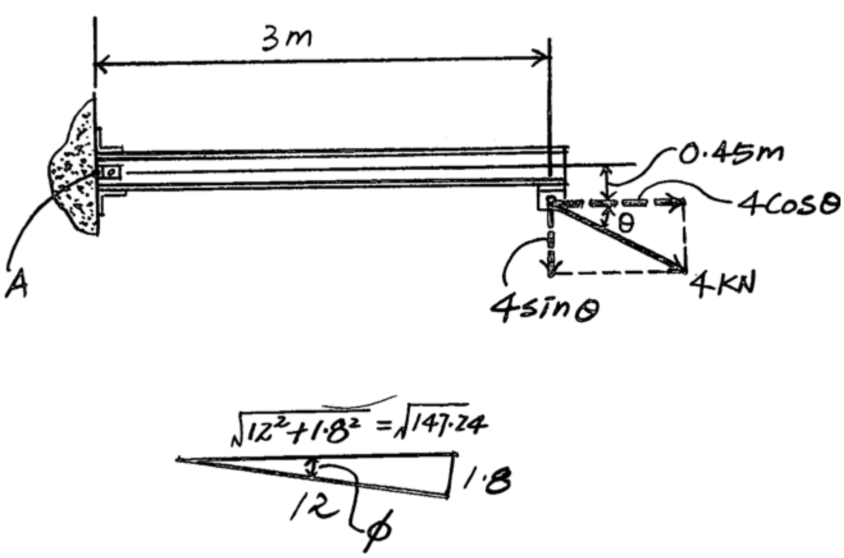
$$\sin(\theta - \phi) = \frac{10}{\sqrt{147.24}}$$

However, $\phi = \tan^{-1}\left(\frac{1.8}{12}\right) = 8.531°$. Thus,

$$\sin(\theta - 8.531°) = \frac{10}{\sqrt{147.24}}$$

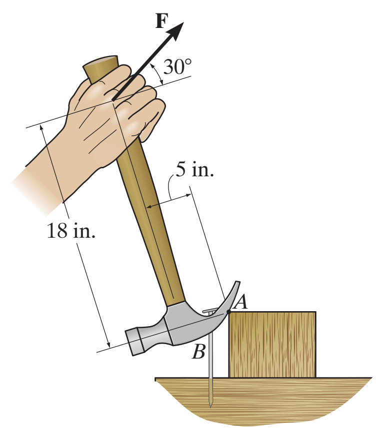
$$\theta - 8.531° = 55.50°$$

$$\theta = 64.0° \qquad \textbf{Ans.}$$



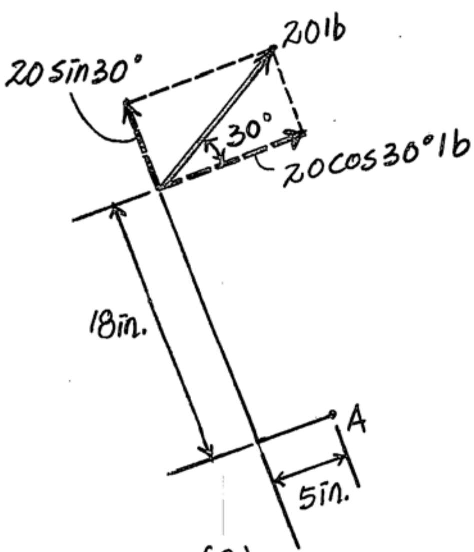
(a)

***4–8.** The handle of the hammer is subjected to the force of $F = 20$ lb. Determine the moment of this force about the point A.



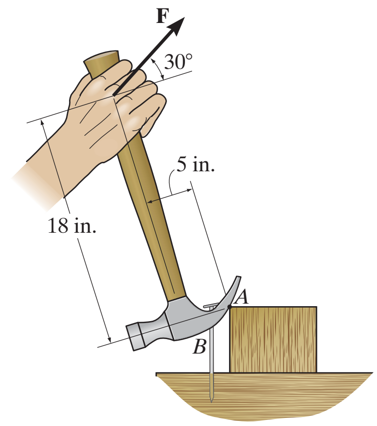
Resolving the 20 - lb force into components parallel and perpendicular to the hammer, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = -20\cos 30°(18) - 20\sin 30°(5)$$
$$= -361.77 \text{ lb}\cdot\text{in} = 362 \text{ lb}\cdot\text{in} \quad \text{(clockwise)} \qquad \textbf{Ans.}$$



(a)

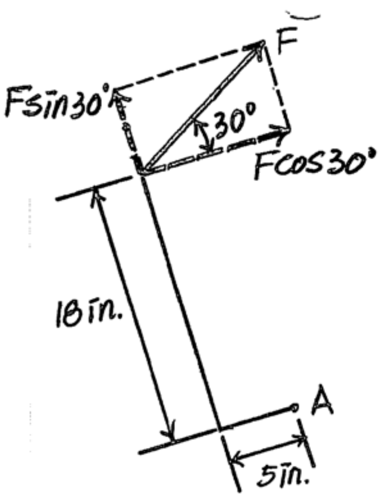
•**4–9.** In order to pull out the nail at B, the force **F** exerted on the handle of the hammer must produce a clockwise moment of 500 lb · in. about point A. Determine the required magnitude of force **F**.



Resolving force **F** into components parallel and perpendicular to the hammer, Fig. *a*, and applying the principle of moments,

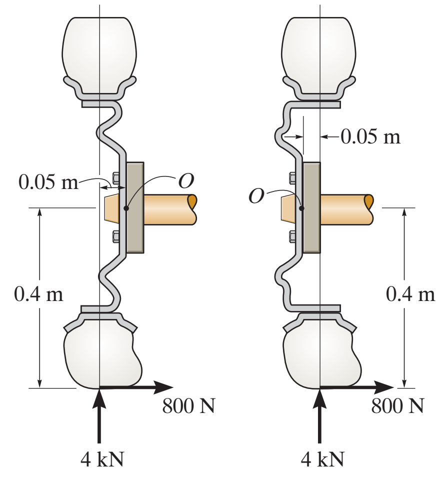
$$\circlearrowleft +M_A = -500 = -F\cos 30°(18) - F\sin 30°(5)$$

$$F = 27.6 \text{ lb} \qquad \textbf{Ans.}$$



(a)

4–10. The hub of the wheel can be attached to the axle either with negative offset (left) or with positive offset (right). If the tire is subjected to both a normal and radial load as shown, determine the resultant moment of these loads about point O on the axle for both cases.



Case 1　　Case 2

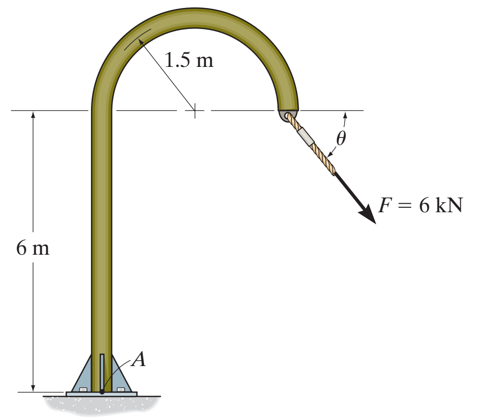
For case 1 with negative offset, we have

$$\circlearrowleft + \; M_O = 800(0.4) - 4000(0.05)$$
$$= 120\ \text{N}\cdot\text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans}$$

For case 2 with positive offset, we have

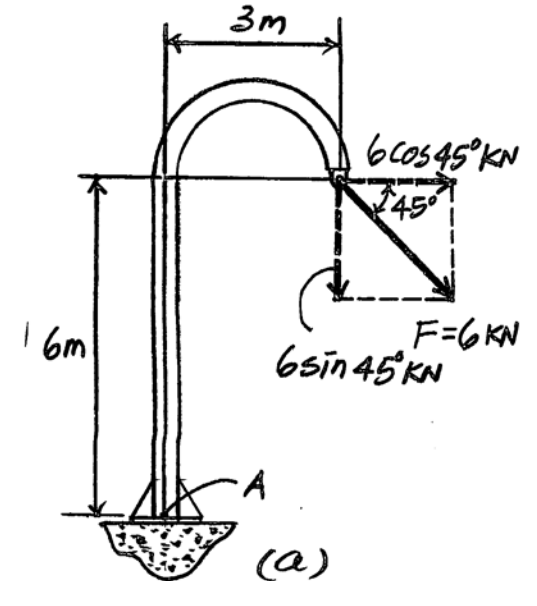
$$\circlearrowleft + \; M_O = 800(0.4) + 4000(0.05)$$
$$= 520\ \text{N}\cdot\text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans}$$

4–11. The member is subjected to a force of $F = 6$ kN. If $\theta = 45°$, determine the moment produced by **F** about point A.



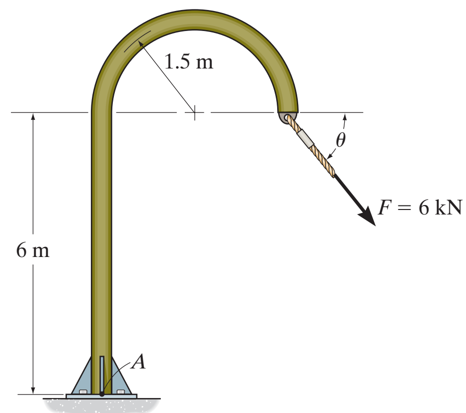
Resolving force **F** into horizontal and vertical components, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft +M_A = -6\cos 45°(6) - 6\sin 45°(3)$$
$$= -38.18 \text{ kN}\cdot\text{m} = 38.2 \text{ kN}\cdot\text{m} \text{ (clockwise)} \qquad \textbf{Ans.}$$



(a)

***4–12.** Determine the angle θ ($0° \le \theta \le 180°$) of the force **F** so that it produces a maximum moment and a minimum moment about point A. Also, what are the magnitudes of these maximum and minimum moments?



In order to produce the maximum moment about point A, force **F** must act perpendicular to line AB, Fig. a. From the geometry of this diagram,

$$\phi = \tan^{-1}\left(\frac{6}{3}\right) = 63.43°$$

$$\theta = 90° - \phi = 90° - 63.43° = 26.6° \qquad \textbf{Ans.}$$

Also,

$$d = \sqrt{6^2 + 3^2} = \sqrt{45}\ \text{m}$$

The maximum moment of **F** about point A is given by

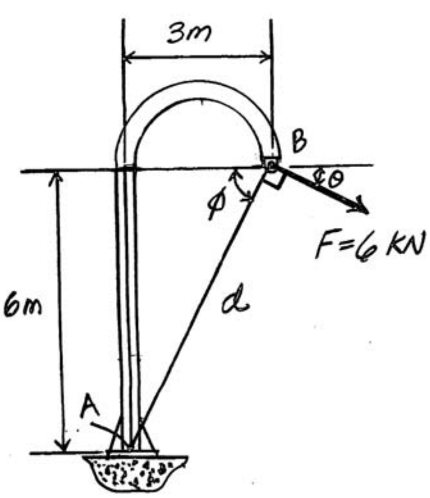
$$(M_A)_{max} = Fd = 6\left(\sqrt{45}\right) = 40.2\ \text{kN}\cdot\text{m}$$

The minimum moment of **F** about point A occurs when the line of action of **F** passes through point A. Referring to Fig. b,

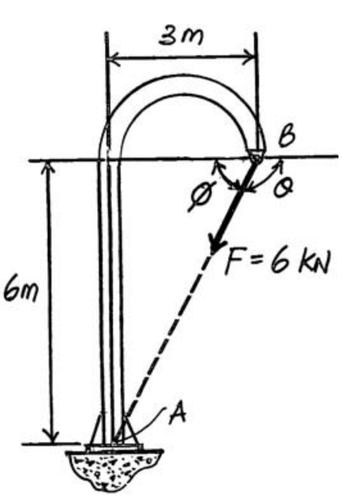
$$\theta = 180° - \phi = 180° - 63.43° = 117° \qquad \textbf{Ans.}$$

and

$$(M_A)_{min} = Fd = 6(0) = 0 \qquad \textbf{Ans.}$$

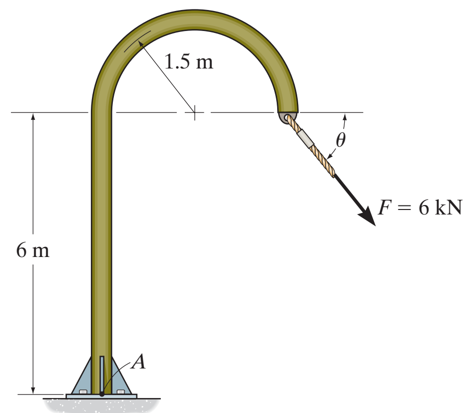


(a)



(b)

•**4–13.** Determine the moment produced by the force **F** about point A in terms of the angle θ. Plot the graph of M_A versus θ, where $0° \leq \theta \leq 180°$.



Moment Function: Resolving force **F** into horizontal and vertical components, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = -6\cos\theta(6) - 6\sin\theta(3)$$
$$= -(36\cos\theta + 18\sin\theta)\ \text{kN}\cdot\text{m}$$
$$= (36\cos\theta + 18\sin\theta)\ \text{kN}\cdot\text{m} \quad \text{(clockwise)}$$

The maximum moment occurs when $\dfrac{dM_A}{d\theta} = 0$.

$$\frac{dM_A}{d\theta} = -36\sin\theta + 18\cos\theta = 0$$
$$\theta = 26.6°$$

The maximum moment of **F** about point A is given by

$$(M_A)_{\max} = 36\cos 26.57° + 18\sin 26.57° = 40.2\ \text{kN}\cdot\text{m}$$

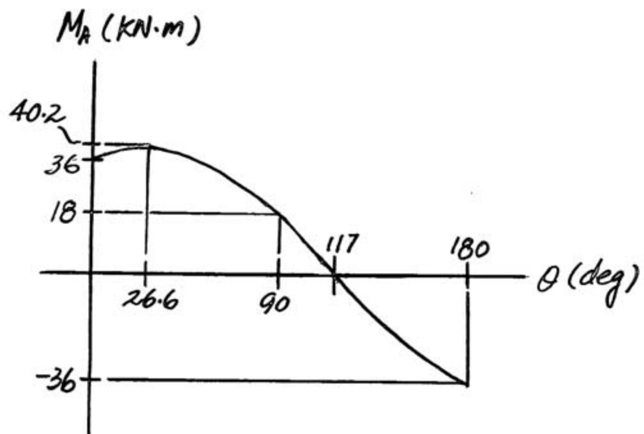
Also,

$$M_A\big|_{\theta=0°} = 36\cos 0° + 18\sin 0° = 36\ \text{kN}\cdot\text{m}$$
$$M_A\big|_{\theta=90°} = 36\cos 90° + 18\sin 90° = 18\ \text{kN}\cdot\text{m}$$
$$M_A\big|_{\theta=180°} = 36\cos 180° + 18\sin 180° = -36\ \text{kN}\cdot\text{m}$$

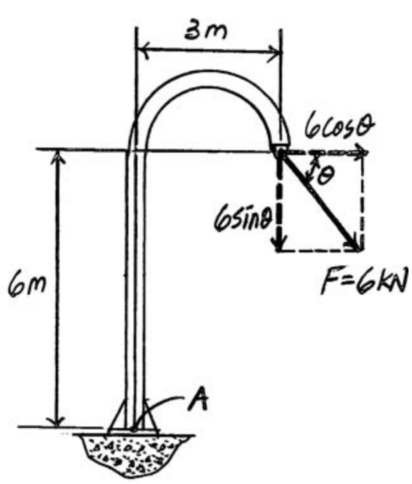
When $M_A = 0$,

$$0 = 36\cos\theta + 18\sin\theta \qquad \theta = 117°$$

The plot of M_A versus θ is shown in Fig. *b*.

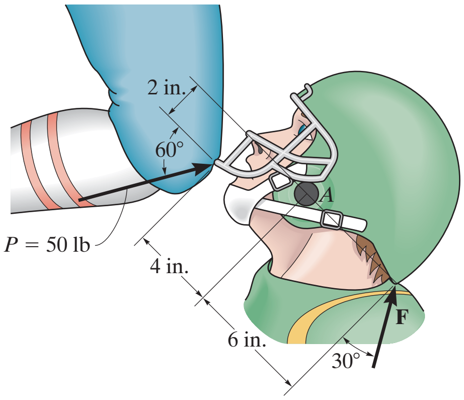


(b)



(a)

4–14. Serious neck injuries can occur when a football player is struck in the face guard of his helmet in the manner shown, giving rise to a guillotine mechanism. Determine the moment of the knee force $P = 50$ lb about point A. What would be the magnitude of the neck force **F** so that it gives the counterbalancing moment about A?

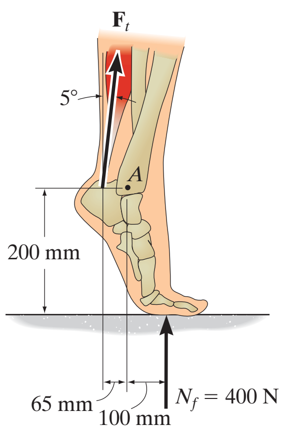


$\curvearrowleft + M_A = 50\sin 60°(4) - 50\cos 60°(2) = 123.2 = 123\ \text{lb}\cdot\text{in.}\ \curvearrowright$ **Ans**

$123.2 = F\cos 30°(6)$

$F = 23.7$ lb **Ans**

4–15. The Achilles tendon force of $F_t = 650$ N is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of $N_f = 400$ N. Determine the resultant moment of $\mathbf{F}_t$ and $\mathbf{N}_f$ about the ankle joint A.

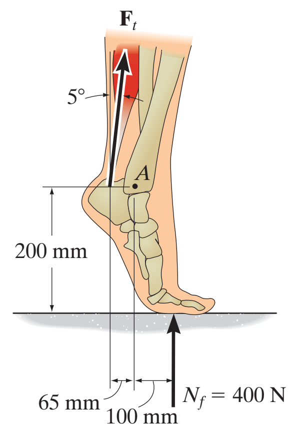


Referring to Fig. *a*,

$\curvearrowleft +(M_R)_A = \Sigma Fd; \qquad (M_R)_A = 400(0.1) - 650(0.65)\cos 5°$

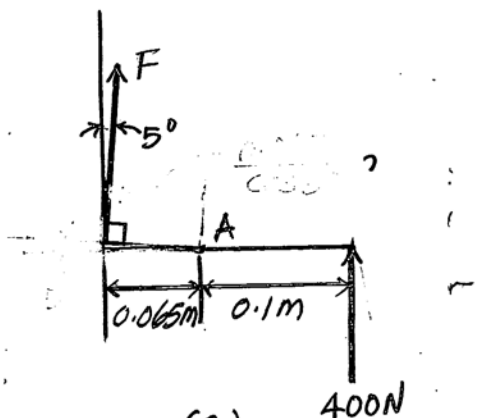
$= -2.09\ \text{N}\cdot\text{m} = 2.09\ \text{N}\cdot\text{m}$ (clockwise) **Ans.**

***4–16.** The Achilles tendon force $\mathbf{F}_t$ is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of $N_t = 400$ N. If the resultant moment produced by forces $\mathbf{F}_t$ and $\mathbf{N}_t$ about the ankle joint A is required to be zero, determine the magnitude of $\mathbf{F}_t$.



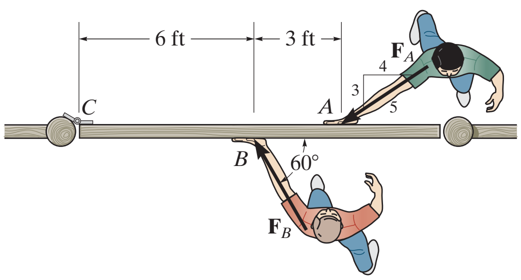
Referring to Fig. a,

$$\circlearrowleft+(M_R)_A = \Sigma Fd; \qquad 0 = 400(0.1) - F\cos 5°(0.065)$$
$$F = 618\text{N} \qquad \textbf{Ans.}$$



(a)

•4–17. The two boys push on the gate with forces of $F_A = 30$ lb and as shown. Determine the moment of each force about C. Which way will the gate rotate, clockwise or counterclockwise? Neglect the thickness of the gate.

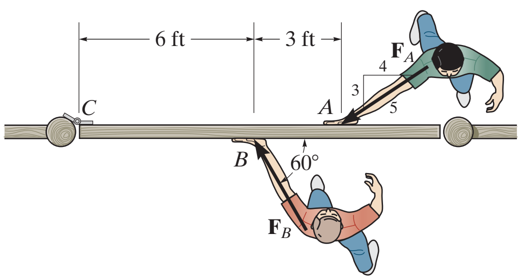


$$\circlearrowleft+ \quad (M_{F_A})_C = -30\left(\frac{3}{5}\right)(9)$$
$$= -162 \text{ lb}\cdot\text{ft} = 162 \text{ lb}\cdot\text{ft} \quad \textbf{(Clockwise)} \qquad \textbf{Ans}$$

$$\circlearrowleft+ \quad (M_{F_B})_C = 50(\sin 60°)(6)$$
$$= 260 \text{ lb}\cdot\text{ft} \quad \textbf{(Counterclockwise)} \qquad \textbf{Ans}$$

Since $(M_{F_B})_C > (M_{F_A})_C$, the gate will rotate ***Counterclockwise.*** **Ans**

4–18. Two boys push on the gate as shown. If the boy at B exerts a force of $F_B = 30$ lb, determine the magnitude of the force $\mathbf{F}_A$ the boy at A must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

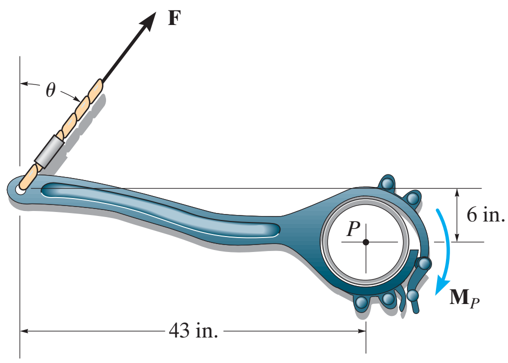


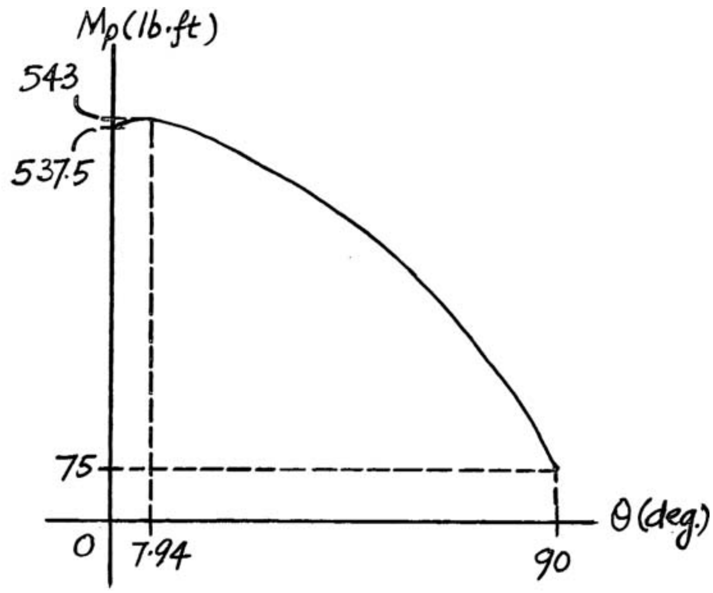
In order to prevent the gate from turning, the resultant moment about point C must be equal to zero.

$$\circlearrowleft+ M_{R_C} = \Sigma Fd; \qquad M_{R_C} = 0 = 30\sin 60°(6) - F_A\left(\frac{3}{5}\right)(9)$$

$$F_A = 28.9 \text{ lb} \qquad \textbf{Ans}$$

4–19. The tongs are used to grip the ends of the drilling pipe P. Determine the torque (moment) M_P that the applied force $F = 150$ lb exerts on the pipe about point P as a function of θ. Plot this moment M_P versus θ for $0 \leq \theta \leq 90°$.





$$M_P = 150\cos\theta(43) + 150\sin\theta(6)$$

$$= (6450\cos\theta + 900\sin\theta)\ \text{lb}\cdot\text{in.}$$

$$= (537.5\cos\theta + 75\sin\theta)\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

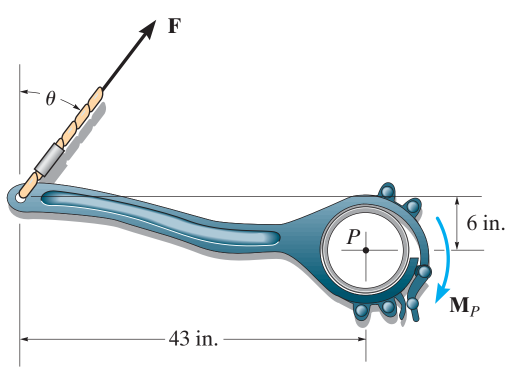
$$\frac{dM_P}{d\theta} = -537.5\sin\theta + 75\cos\theta = 0 \qquad \tan\theta = \frac{75}{537.5} \qquad \theta = 7.943°$$

At $\theta = 7.943°$, M_P is maximum.

$$(M_P)_{max} = 538\cos 7.943° + 75\sin 7.943° = 543\ \text{lb}\cdot\text{ft}$$

Also $(M_P)_{max} = 150\ \text{lb}\left(\left(\frac{43}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right)^{\frac{1}{2}} = 543\ \text{lb}\cdot\text{ft}$

*__4–20.__ The tongs are used to grip the ends of the drilling pipe P. If a torque (moment) of $M_P = 800$ lb·ft is needed at P to turn the pipe, determine the cable force F that must be applied to the tongs. Set $\theta = 30°$.

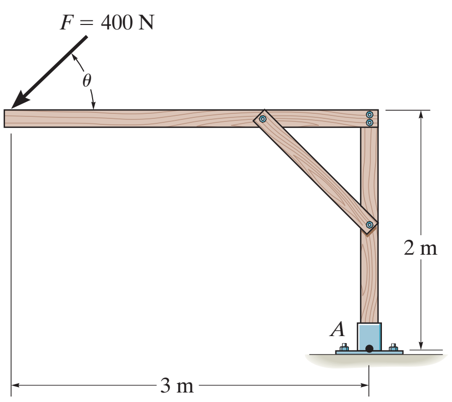


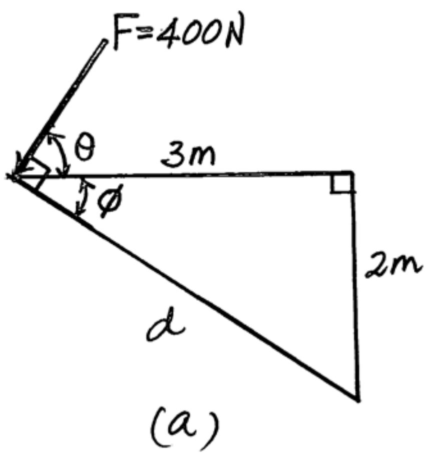
$$M_P = F\cos 30°(43) + F\sin 30°(6) \qquad \text{Set } M_P = 800(12)\ \text{lb}\cdot\text{in.}$$

$$800(12) = F\cos 30°(43) + F\sin 30°(6)$$

$$F = 239\ \text{lb} \qquad \textbf{Ans}$$

•4–21. Determine the direction θ for $0° \leq \theta \leq 180°$ of the force **F** so that it produces the maximum moment about point A. Calculate this moment.



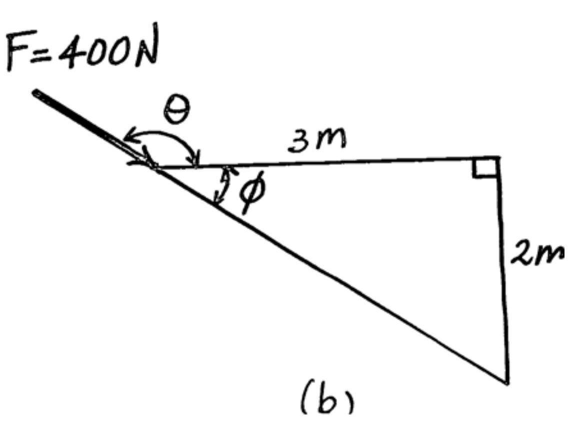


$$\circlearrowleft + M_A = 400\sqrt{(3)^2 + (2)^2} = 1442 \text{ N}\cdot\text{m}$$

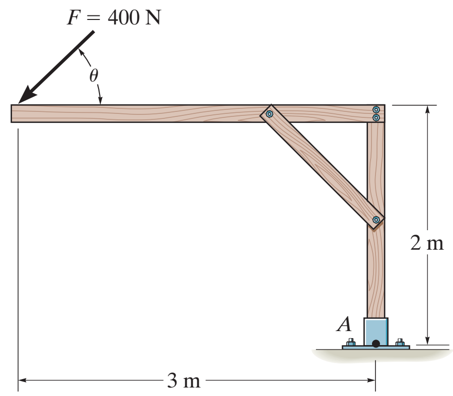
$M_A = 1.44 \text{ kN}\cdot\text{m}$ **Ans**

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$$

$\theta = 90° - 33.69° = 56.3°$ **Ans**



4–22. Determine the moment of the force **F** about point A as a function of θ. Plot the results of M (ordinate) versus θ (abscissa) for $0° \leq \theta \leq 180°$.



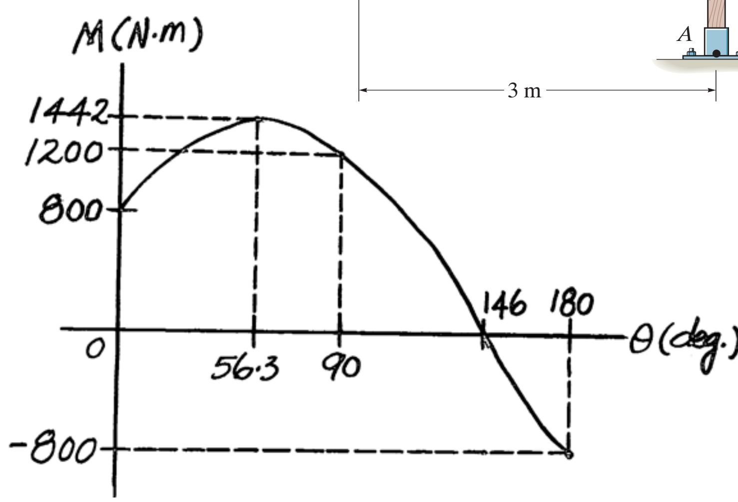
$$\curvearrowleft + M_A = 400 \sin\theta(3) + 400 \cos\theta(2)$$

$$= 1200 \sin\theta + 800 \cos\theta \qquad \textbf{Ans}$$

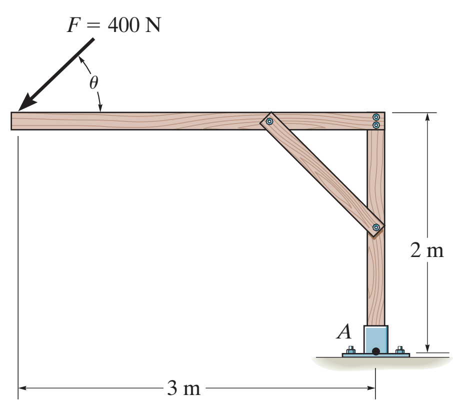
$$\frac{dM_A}{d\theta} = 1200 \cos\theta - 800 \sin\theta = 0$$

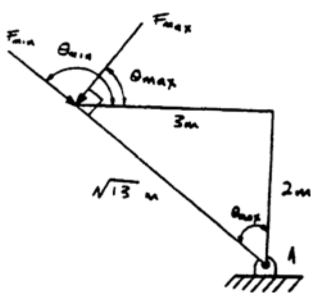
$$\theta = \tan^{-1}\left(\frac{1200}{800}\right) = 56.3°$$

$$(M_A)_{max} = 1200 \sin 56.3° + 800 \cos 56.3° = 1442 \text{ N} \cdot \text{m}$$



4–23. Determine the minimum moment produced by the force **F** about point A. Specify the angle θ ($0° \leq \theta \leq 180°$).

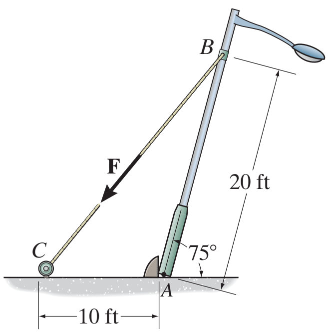




$$M_{min} = 400(0) = 0 \qquad \textbf{Ans}$$

$$\theta_{min} = 90° + 56.3° = 146° \qquad \textbf{Ans}$$

***4–24.** In order to raise the lamp post from the position shown, force **F** is applied to the cable. If $F = 200$ lb, determine the moment produced by **F** about point A.



Geometry: Applying the law of cosines to Fig. *a*,

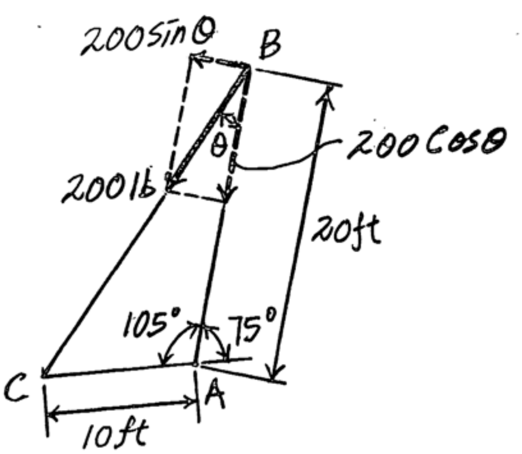
$$BC^2 = 10^2 + 20^2 - 2(10)(20)\cos 105°$$
$$BC = 24.57 \text{ ft}$$

Then, applying the law of sines,

$$\frac{\sin\theta}{10} = \frac{\sin 105°}{24.57} \qquad \theta = 23.15°$$

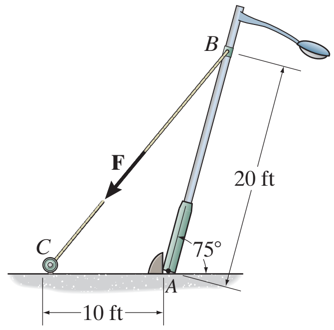
Moment About Point A: By resolving force **F** into components parallel and perpendicular to the lamp pole, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + (M_R)_A = \Sigma Fd; \qquad M_A = 200\sin 23.15°(20) + 200\cos 23.15°(0)$$
$$= 1572.73 \text{ lb}\cdot\text{ft} = 1.57 \text{ kip}\cdot\text{ft (counterclockwise)} \qquad \textbf{Ans.}$$



(a)

•**4–25.** In order to raise the lamp post from the position shown, the force **F** on the cable must create a counterclockwise moment of 1500 lb · ft about point A. Determine the magnitude of **F** that must be applied to the cable.



Geometry: Applying the law of cosines to Fig. a,

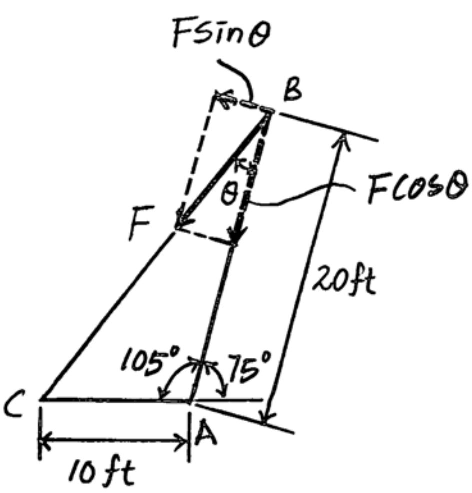
$$BC^2 = 10^2 + 20^2 - 2(10)(20)\cos 105°$$
$$BC = 24.57 \text{ ft}$$

Then, applying the law of sines,

$$\frac{\sin\theta}{10} = \frac{\sin 105°}{24.57} \qquad \theta = 23.15°$$

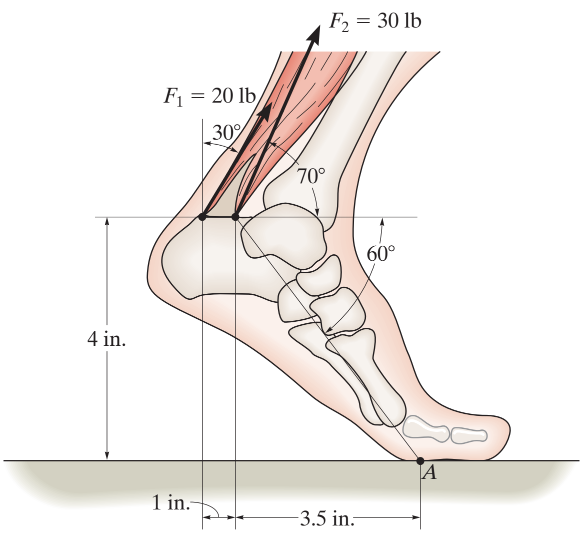
Moment About Point A: By resolving force **F** into components parallel and perpendicular to the lamp pole, Fig. a, and applying the principle of moments,

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad 1500 = F\sin 23.15°(20)$$
$$F = 191 \text{ lb}$$ **Ans.**



(a)

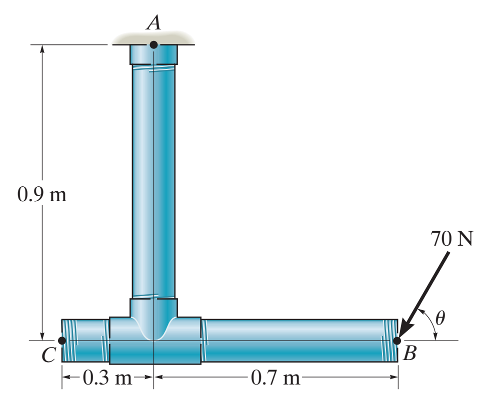
4–26. The foot segment is subjected to the pull of the two plantarflexor muscles. Determine the moment of each force about the point of contact A on the ground.



$(M_A)_1 = 20\cos 30°(4.5) + 20\sin 30°(4) = 118\ \text{lb}\cdot\text{in.}$ **Ans**

$(M_A)_2 = 30\cos 70°(4) + 30\sin 70°(3.5) = 140\ \text{lb}\cdot\text{in.}$ **Ans**

4–27. The 70-N force acts on the end of the pipe at B. Determine (a) the moment of this force about point A, and (b) the magnitude and direction of a horizontal force, applied at C, which produces the same moment. Take $\theta = 60°$.



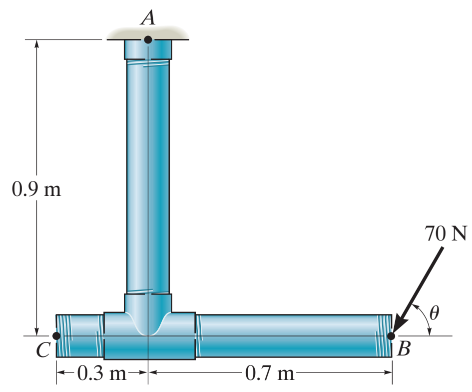
(a) $\curvearrowleft +M_A = 70\sin 60°(0.7) + 70\cos 60°(0.9)$

$M_A = 73.94 = 73.9\ \text{N}\cdot\text{m}$ **Ans**

(b) $F_C(0.9) = 73.94$

$F_C = 82.2\ \text{N} \leftarrow$ **Ans**

*__4–28.__ The 70-N force acts on the end of the pipe at B. Determine the angles θ $(0^\circ \leq \theta \leq 180^\circ)$ of the force that will produce maximum and minimum moments about point A. What are the magnitudes of these moments?



$$\curvearrowleft +M_A = 70\sin\theta(0.7) + 70\cos\theta(0.9)$$

$$M_A = 49\sin\theta + 63\cos\theta$$

For maximum moment $\frac{dM_A}{d\theta} = 0$

$$\frac{dM_A}{d\theta} = 0; \qquad 49\cos\theta - 63\sin\theta = 0$$

$$\theta = \tan^{-1}\left(\frac{49}{63}\right) = 37.9^\circ \qquad \textbf{Ans}$$

$$(M_A)_{max} = 49\sin 37.9^\circ + 63\cos 37.9^\circ$$

$$= 79.8\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

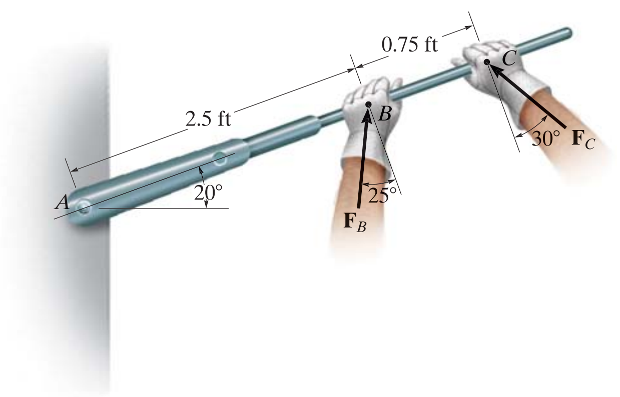
For minimum moment $M_A = 0$

$$M_A = 0; \qquad 49\sin\theta + 63\cos\theta = 0$$

$$\theta = 180^\circ + \tan^{-1}\left(\frac{-63}{49}\right) = 128^\circ \qquad \textbf{Ans}$$

$$(M_A)_{min} = 49\sin 128^\circ + 63\cos 128^\circ = 0 \qquad \textbf{Ans}$$

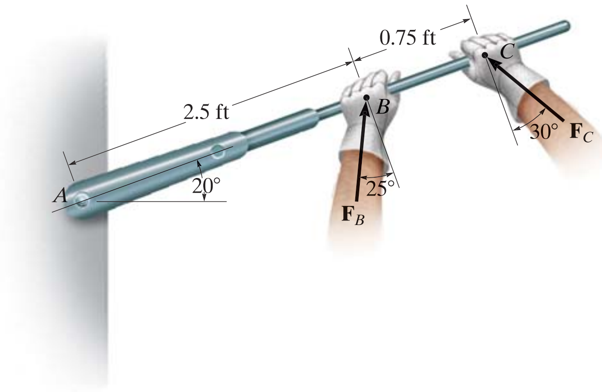
•**4–29.** Determine the moment of each force about the bolt located at A. Take $F_B = 40$ lb, $F_C = 50$ lb.



$$\curvearrowleft +M_B = 40\cos 25^\circ(2.5) = 90.6\ \text{lb}\cdot\text{ft} \circlearrowleft \qquad \textbf{Ans}$$

$$\curvearrowleft +M_C = 50\cos 30^\circ(3.25) = 141\ \text{lb}\cdot\text{ft} \circlearrowleft \qquad \textbf{Ans}$$

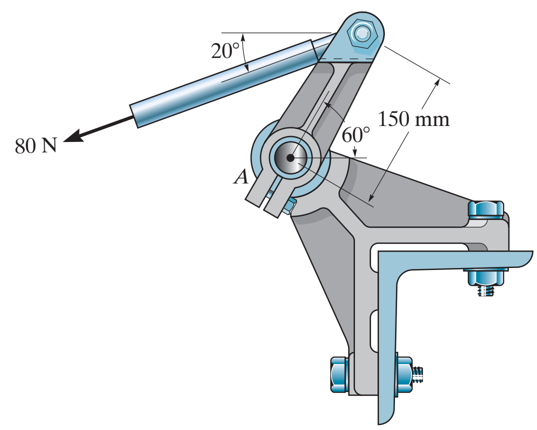
4–30. If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at A.



$$\circlearrowleft +M_A = 30\cos 25°(2.5) + 45\cos 30°(3.25)$$

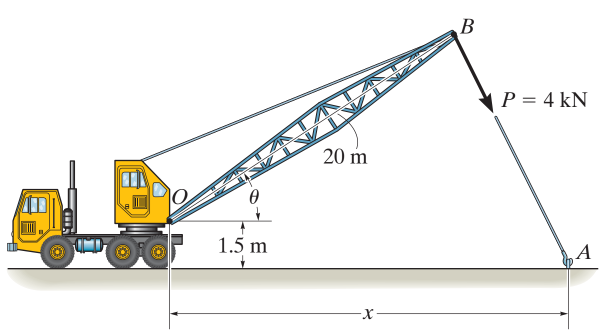
$$= 195 \text{ lb}\cdot\text{ft} \circlearrowleft \qquad \textbf{Ans}$$

4–31. The rod on the power control mechanism for a business jet is subjected to a force of 80 N. Determine the moment of this force about the bearing at A.



$$\circlearrowleft + M_A = 80\cos 20° (0.15 \sin 60°) - 80 \sin 20° (0.15\cos 60°) = 7.71 \text{ N}\cdot\text{m} \circlearrowleft \quad \textbf{Ans}$$

***4–32.** The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at A so that this force creates a maximum moment about point O. What is this moment?



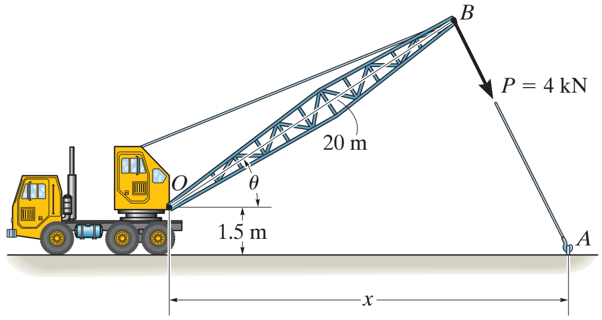
Maximum moment, $OB \perp BA$

$$\circlearrowright + (M_O)_{max} = 4 \text{ kN}(20) = 80 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textbf{Ans}$$

$$4 \text{ kN} \sin 60°(x) - 4 \text{ kN} \cos 60°(1.5) = 80 \text{ kN}\cdot\text{m}$$

$$x = 24.0 \text{ m} \qquad \textbf{Ans}$$

•4–33. The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $x = 25$ m, determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?



Maximum moment, $OB \perp BA$

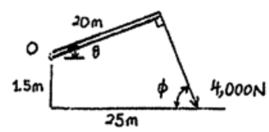
$\circlearrowleft + (M_O)_{max} = 4000(20) = 80\,000 \text{ N}\cdot\text{m} = 80.0 \text{ kN}\cdot\text{m}$ **Ans**

$4000 \sin\phi(25) - 4000 \cos\phi(1.5) = 80\,000$

$25\sin\phi - 1.5\cos\phi = 20$

$\phi = 56.43°$

$\theta = 90° - 56.43° = 33.6°$ **Ans**



Also,

$(1.5)^2 + z^2 = y^2$

$2.25 + z^2 = y^2$

Similar triangles

$$\frac{20+y}{z} = \frac{25+z}{y}$$

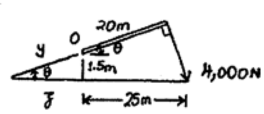
$20y + y^2 = 25z + z^2$

$20(\sqrt{2.25 + z^2}) + 2.25 + z^2 = 25z + z^2$

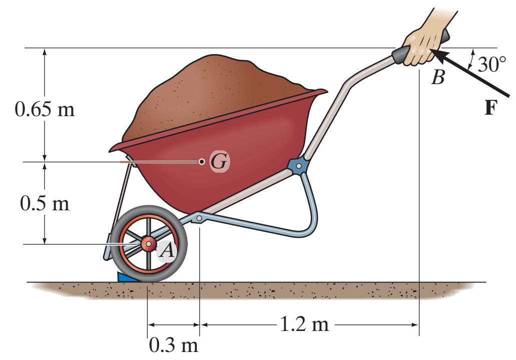
$z = 2.259$ m

$y = 2.712$ m

$\theta = \cos^{-1}\left(\frac{2.259}{2.712}\right) = 33.6°$ **Ans**



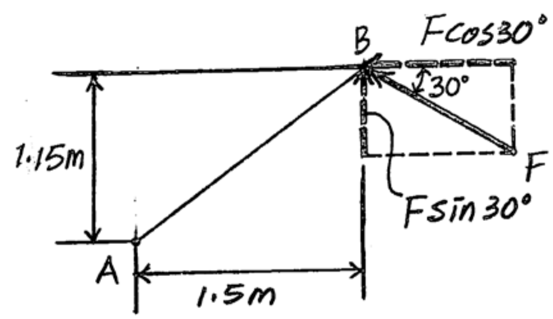
4–34. In order to hold the wheelbarrow in the position shown, force **F** must produce a counterclockwise moment of 200 N·m about the axle at A. Determine the required magnitude of force **F**.



Resolving force **F** into its horizontal and vertical components, Fig. *a*, and applying the principle of moments,

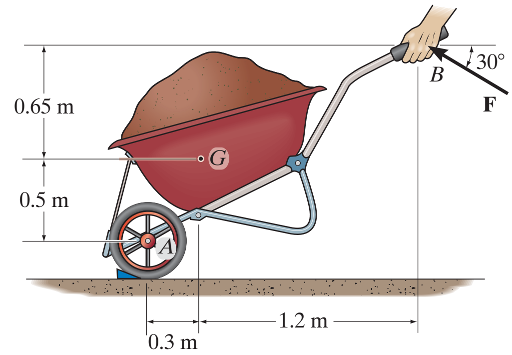
$$\curvearrowleft + M_A = 200 = F \sin 30°(1.5) + F \cos 30°(1.15)$$

$$F = 115 \text{ N} \qquad \textbf{Ans.}$$



(a)

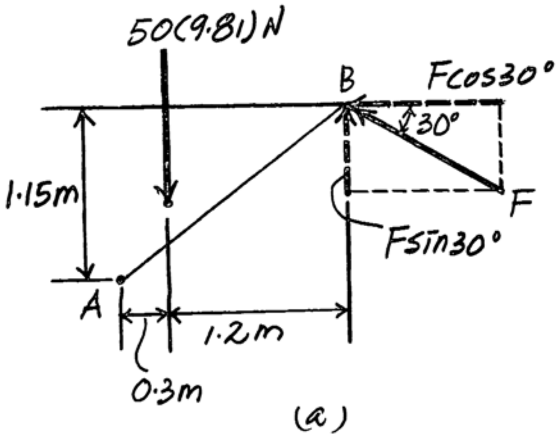
4–35. The wheelbarrow and its contents have a mass of 50 kg and a center of mass at G. If the resultant moment produced by force **F** and the weight about point A is to be zero, determine the required magnitude of force **F**.



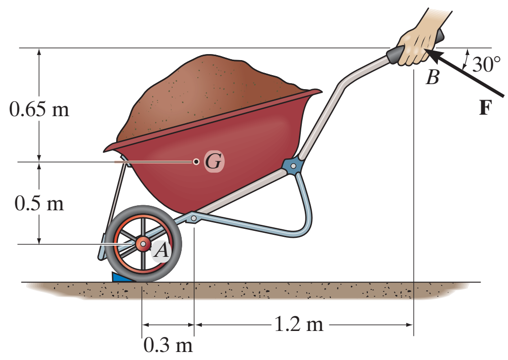
Resolving force **F** into its horzontal and vertical components, Fig. a, and applying the principle of moments,

$$\curvearrowleft +(M_R)_A = \Sigma Fd; \qquad 0 = F\sin 30°(1.5) + F\cos 30°(1.15) - 50(9.81)(0.3)$$

$$F = 84.3\text{ N} \qquad \textbf{Ans.}$$



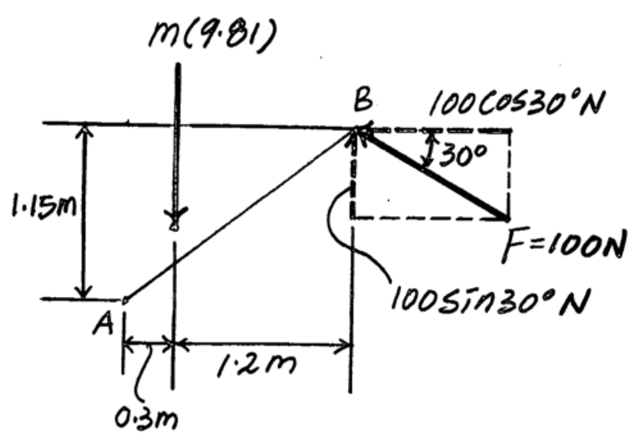
***4–36.** The wheelbarrow and its contents have a center of mass at G. If $F = 100$ N and the resultant moment produced by force **F** and the weight about the axle at A is zero, determine the mass of the wheelbarrow and its contents.



Resolving force **F** into its horzontal and vertical components, Fig. a, and applying the principle of moments,

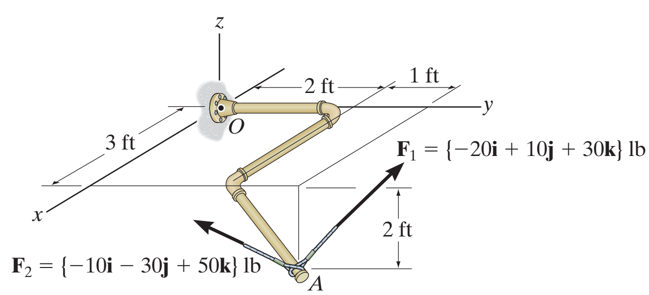
$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad 0 = 100\cos 30°(1.15) + 100\sin 30°(1.5) - M(9.81)(0.3)$$

$$M = 59.3 \text{ kg} \qquad \textbf{Ans.}$$



(a)

•**4–37.** Determine the moment produced by $\mathbf{F}_1$ about point O. Express the result as a Cartesian vector.

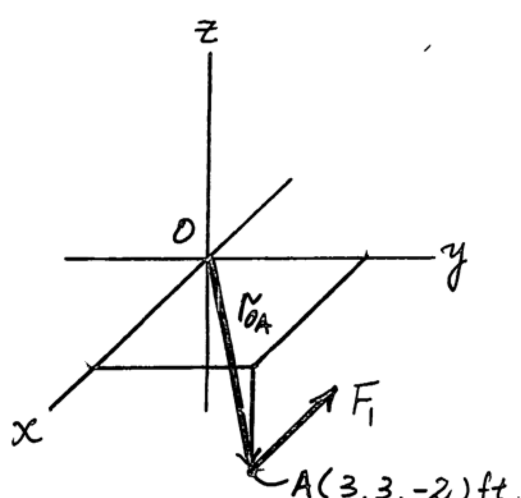


Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. a, must be determined first.

$$\mathbf{r}_{OA} = (3-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k} = [3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]\text{ft}$$

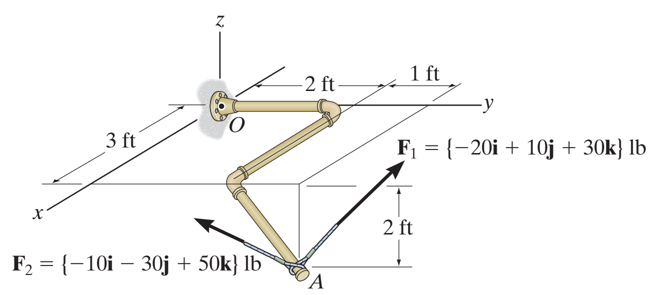
Vector Cross Product: The moment of $\mathbf{F}_1$ about point O is

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -20 & 10 & 30 \end{vmatrix} = [110\mathbf{i} - 50\mathbf{j} + 90\mathbf{k}]\,\text{lb}\cdot\text{ft}$$ **Ans.**



(a)

4–38. Determine the moment produced by $\mathbf{F}_2$ about point O. Express the result as a Cartesian vector.

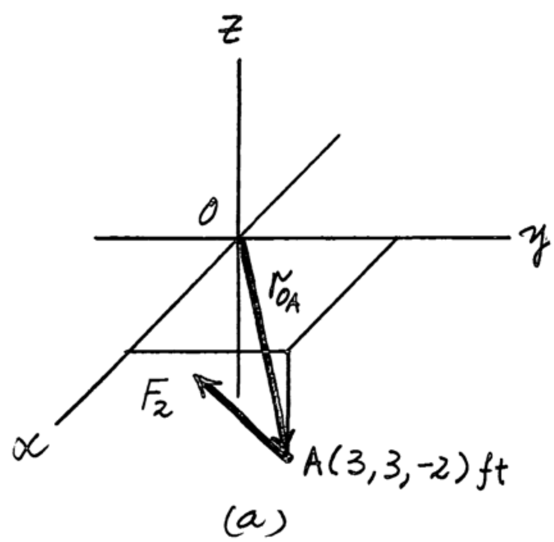


Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. a, must be determined first.

$$\mathbf{r}_{OA} = (3-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k} = [3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]\text{ft}$$

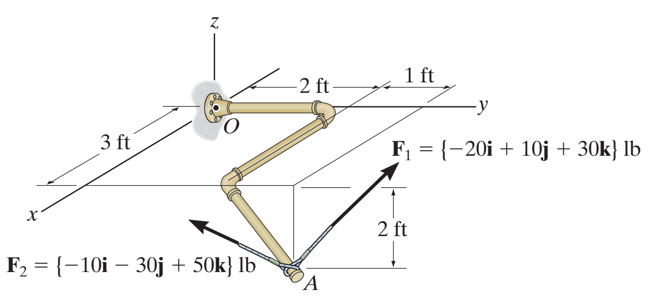
Vector Cross Product: The moment of $\mathbf{F}_2$ about point O is

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -10 & -30 & 50 \end{vmatrix} = [90\mathbf{i} - 130\mathbf{j} - 60\mathbf{k}]\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



(a)

4–39. Determine the resultant moment produced by the two forces about point O. Express the result as a Cartesian vector.



Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. a, must be determined first.

$$\mathbf{r}_{OA} = (3-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k} = [3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]\text{ft}$$

Resultant Moment: The resultant moment of $\mathbf{F}_1$ and $\mathbf{F}_2$ about point O can be determined by

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OA} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -20 & 10 & 30 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -10 & -30 & 50 \end{vmatrix}$$

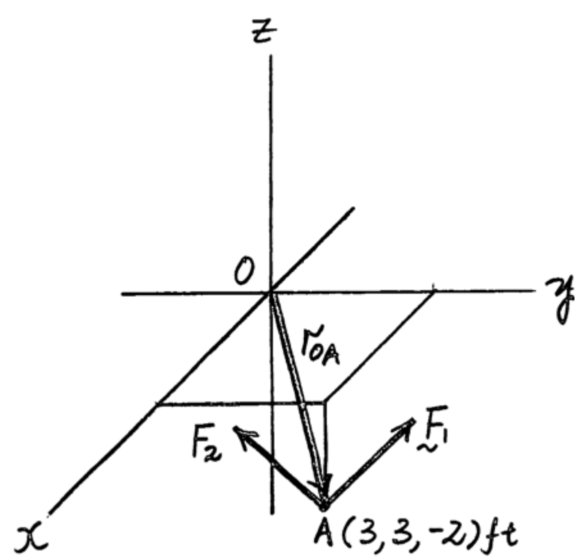
$$= [200\mathbf{i} - 180\mathbf{j} + 30\mathbf{k}]\ \text{lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

Or we can apply the principle of moments which gives

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times (\mathbf{F}_1 + \mathbf{F}_2)$$

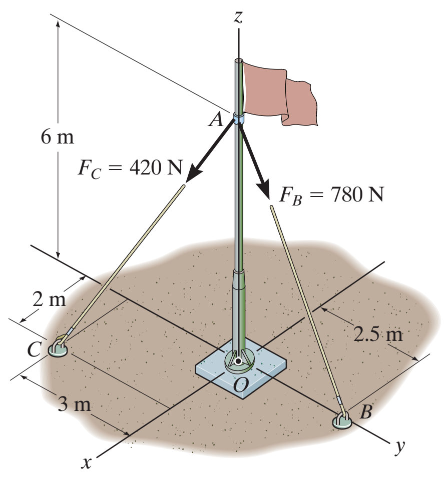
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -30 & -20 & 80 \end{vmatrix}$$

$$= \{200\mathbf{i} - 180\mathbf{j} + 30\mathbf{k}\}\ \text{lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



(a)

***4–40.** Determine the moment produced by force $\mathbf{F}_B$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OB}$ can be used to determine the moment of $\mathbf{F}_B$ about point O.

$\mathbf{r}_{OA} = \{6\mathbf{k}\}$ m $\qquad \mathbf{r}_{OB} = \{2.5\mathbf{j}\}$ m

The force vector $\mathbf{F}_B$ is given by

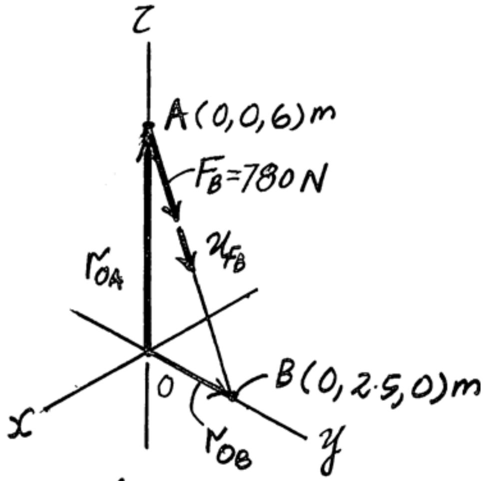
$$\mathbf{F}_B = F_B\mathbf{u}_{FB} = 780\left[\frac{(0-0)\mathbf{i}+(2.5-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(2.5-0)^2+(0-6)^2}}\right] = \{300\mathbf{j}-720\mathbf{k}\}\text{N}$$

Vector Cross Product: The moment of $\mathbf{F}_B$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}_B = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 0 & 300 & -720\end{vmatrix} = \{-1800\mathbf{i}\}\ \text{N}\cdot\text{m} = \{-1.80\mathbf{i}\}\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

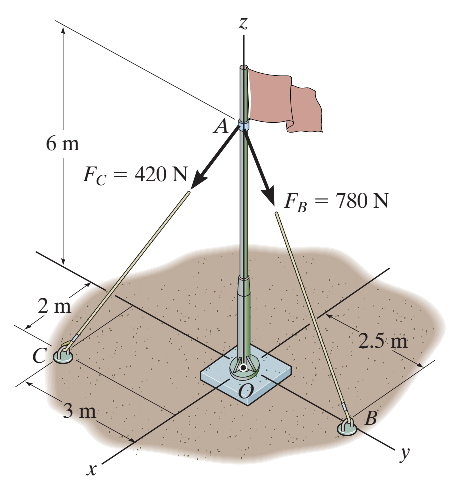
or

$$\mathbf{M}_O = \mathbf{r}_{OB}\times\mathbf{F}_B = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 2.5 & 0\\ 0 & 300 & -720\end{vmatrix} = \{-1800\mathbf{i}\}\ \text{N}\cdot\text{m} = \{-1.80\mathbf{i}\}\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

•4–41. Determine the moment produced by force $\mathbf{F}_C$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OC}$ can be used to determine the moment of $\mathbf{F}_C$ about point O.

$\mathbf{r}_{OA} = \{6\mathbf{k}\}$ m

$\mathbf{r}_{OC} = (2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-0)\mathbf{k} = [2\mathbf{i} - 3\mathbf{j}]$ m

The force vector $\mathbf{F}_C$ is given by

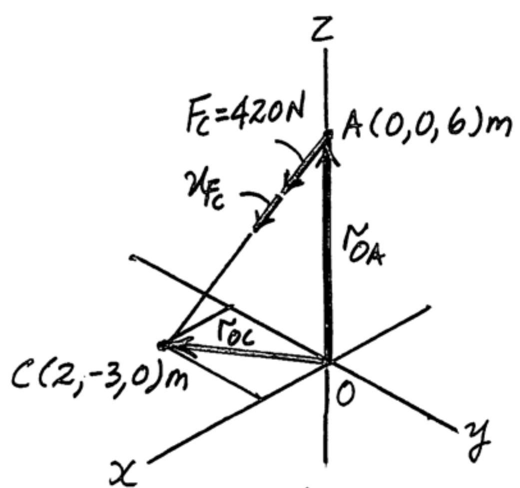
$$\mathbf{F}_C = F_C\mathbf{u}_{FC} = 420\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}}\right] = [120\mathbf{i} - 180\mathbf{j} - 360\mathbf{k}]\text{N}$$

Vector Cross Product: The moment of $\mathbf{F}_C$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix} = [1080\mathbf{i} + 720\mathbf{j}]\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

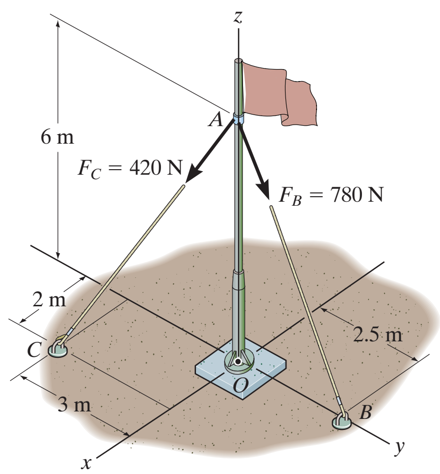
or

$$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 0 \\ 120 & -180 & -360 \end{vmatrix} = [1080\mathbf{i} + 720\mathbf{j}]\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–42. Determine the resultant moment produced by forces $\mathbf{F}_B$ and $\mathbf{F}_C$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: The position vector $\mathbf{r}_{OA}$ and force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = \{6\mathbf{k}\}\text{ m}$$

$$\mathbf{F}_B = F_B\mathbf{u}_{F_B} = 780\left[\frac{(0-0)\mathbf{i}+(2.5-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(2.5-0)^2+(0-6)^2}}\right] = [300\mathbf{j}-720\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{F_C} = 420\left[\frac{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(2-0)^2+(-3-0)^2+(0-6)^2}}\right] = [120\mathbf{i}-180\mathbf{j}-360\mathbf{k}]\text{N}$$

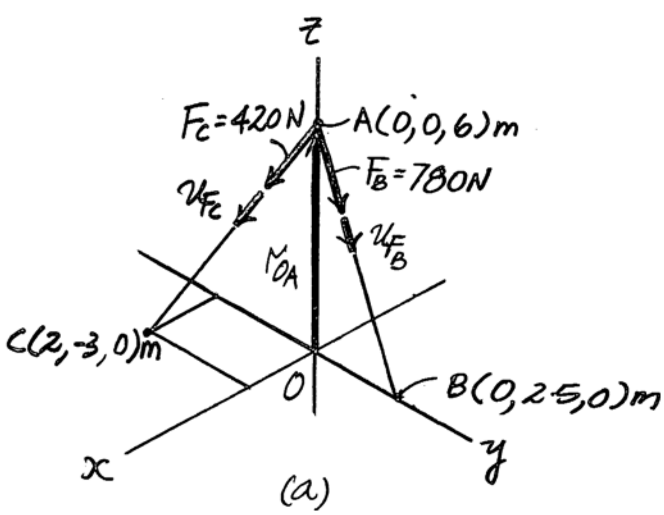
Resultant Moment: The resultant moment of $\mathbf{F}_B$ and $\mathbf{F}_C$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}_B + \mathbf{r}_{OA}\times\mathbf{F}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 0 & 300 & -720 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix}$$

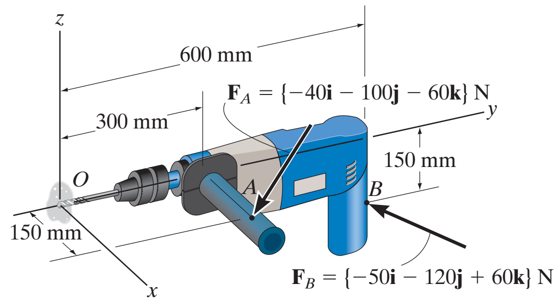
$$= [-720\mathbf{i} + 720\mathbf{j}]\ \text{N}\cdot\text{m}$$

Ans.



(a)

4–43. Determine the moment produced by each force about point O located on the drill bit. Express the results as Cartesian vectors.



Position Vector: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = (0.15-0)\mathbf{i} + (0.3-0)\mathbf{j} + (0-0)\mathbf{k} = [0.15\mathbf{i} + 0.3\mathbf{j}]\text{ m}$$

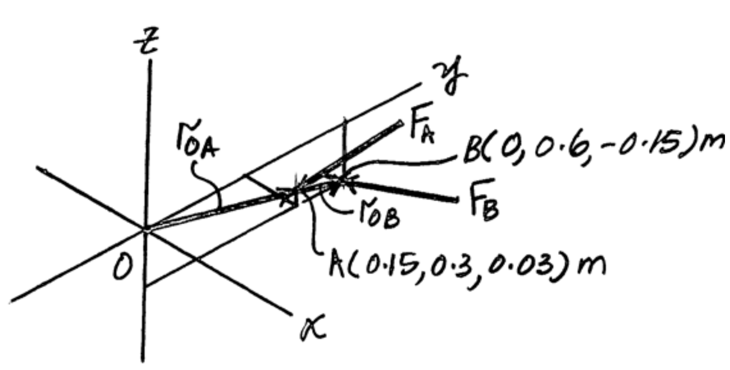
$$\mathbf{r}_{OB} = (0-0)\mathbf{i} + (0.6-0)\mathbf{j} + (-0.15-0)\mathbf{k} = [0.6\mathbf{j} - 0.15\mathbf{k}]\text{ m}$$

Vector Cross Product: The moment of $\mathbf{F}_A$ about point O is

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.3 & 0 \\ -40 & -100 & -60 \end{vmatrix} = [-18\mathbf{i} + 9\mathbf{j} - 3\mathbf{k}]\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The moment of $\mathbf{F}_B$ about point O is

$$(\mathbf{M}_R)_O = \mathbf{r}_{OB} \times \mathbf{F}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.6 & -0.15 \\ -50 & -120 & 60 \end{vmatrix} = [18\mathbf{i} + 7.5\mathbf{j} + 30\mathbf{k}]\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

***4–44.** A force of $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produces a moment of $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m about the origin of coordinates, point O. If the force acts at a point having an x coordinate of $x = 1$ m, determine the y and z coordinates.

$$\mathbf{M}_{RO} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix} = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}\,\text{kN}\cdot\text{m}$$

$y + 2z = 4$

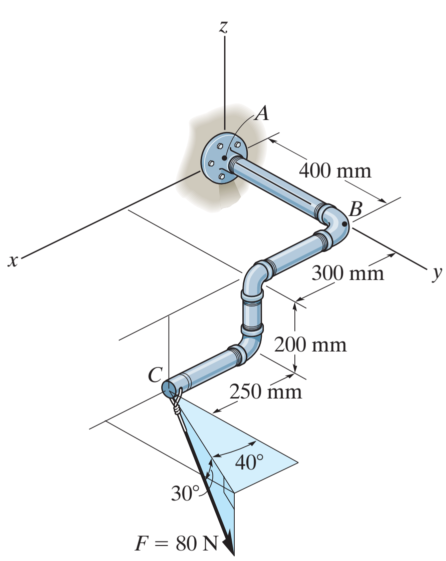
$-1 + 6z = 5$

$-2 - 6y = -14$

$y = 2$ m **Ans.**

$z = 1$ m **Ans.**

•4–45. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.



Position Vector And Force Vector :

$$\mathbf{r}_{AC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$
$$= \{0.55\mathbf{i} + 0.4\mathbf{j} - 0.2\mathbf{k}\}\text{ m}$$

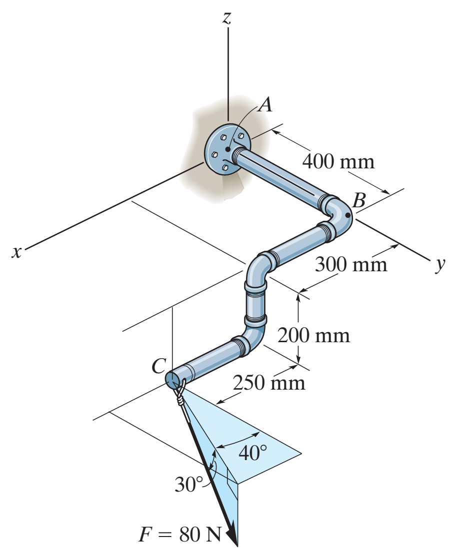
$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k})\text{ N}$$
$$= \{44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

***Moment of Force* F *About Point A* :** Applying Eq. 4 – 7, we have

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans**

4–46. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point *B*.



Position Vector And Force Vector :

$$\mathbf{r}_{BC} = \{(0.55-0)\mathbf{i}+(0.4-0.4)\mathbf{j}+(-0.2-0)\mathbf{k}\}\ \mathrm{m}$$
$$= \{0.55\mathbf{i}-0.2\mathbf{k}\}\ \mathrm{m}$$

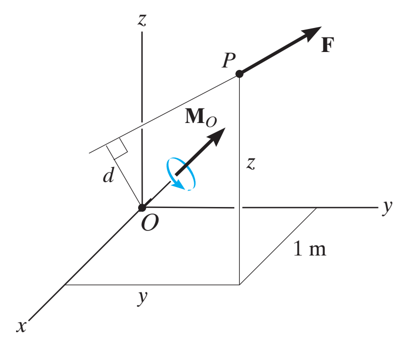
$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ \mathbf{i}+\cos 30^\circ \cos 40^\circ \mathbf{j}-\sin 30^\circ \mathbf{k})\ \mathrm{N}$$
$$= \{44.53\mathbf{i}+53.07\mathbf{j}-40.0\mathbf{k}\}\ \mathrm{N}$$

Moment of Force F About Point B : Applying Eq. 4 – 7, we have

$$\mathbf{M}_B = \mathbf{r}_{BC}\times\mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{10.6\mathbf{i}+13.1\mathbf{j}+29.2\mathbf{k}\}\ \mathrm{N\cdot m} \qquad \textbf{Ans}$$

4–47. The force $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N creates a moment about point *O* of $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}\ \mathrm{N\cdot m}$. If the force passes through a point having an *x* coordinate of 1 m, determine the *y* and *z* coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance *d* from point *O* to the line of action of **F**.



$$-14\mathbf{i}+8\mathbf{j}+2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$$-14 = 10y - 8z$$

$$8 = -10 + 6z$$

$$2 = 8 - 6y$$

$$y = 1\ \mathrm{m} \qquad \textbf{Ans}$$

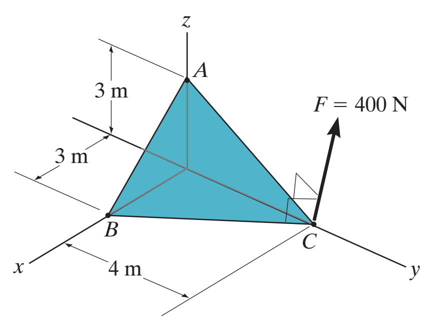
$$z = 3\ \mathrm{m} \qquad \textbf{Ans}$$

$$M_O = \sqrt{(-14)^2+(8)^2+(2)^2} = 16.25\ \mathrm{N\cdot m}$$

$$F = \sqrt{(6)^2+(8)^2+(10)^2} = 14.14\ \mathrm{N}$$

$$d = \frac{16.25}{14.14} = 1.15\ \mathrm{m} \qquad \textbf{Ans}$$

***4–48.** Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point A. Express the result as a Cartesian vector.



Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. a. Here

$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]$ m

$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}]$ m

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$= [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\ \text{m}^2$$

Then,

$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

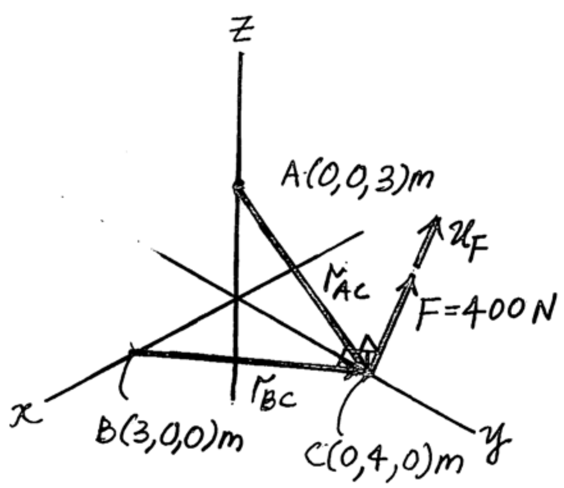
$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$

$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\ \text{N}$$

Vector Cross Product: The moment of **F** about point A is

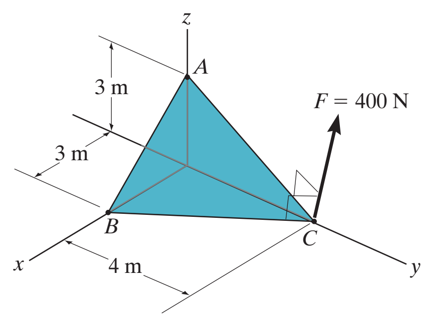
$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$

$$= [1.56\mathbf{i} - 0.750\mathbf{j} - 1\mathbf{k}]\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

•4–49. Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point B. Express the result as a Cartesian vector.



Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. a. Here

$$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}]\text{ m}$$

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix} = [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\text{ m}^2$$

Then,

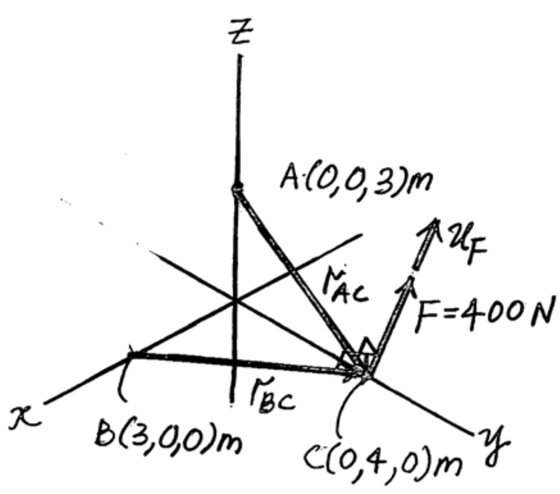
$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$
$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\text{ N}$$

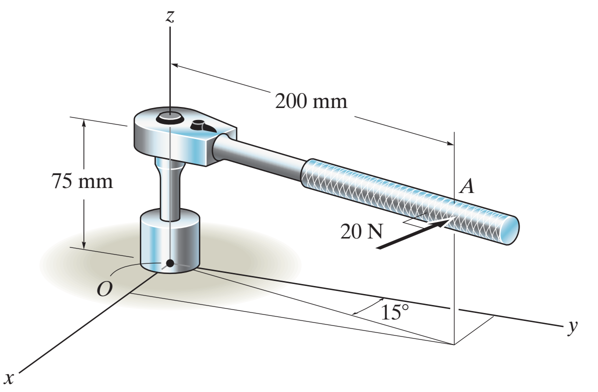
Vector Cross Product: The moment of **F** about point B is

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$
$$= [1\mathbf{i} + 0.750\mathbf{j} - 1.56\mathbf{k}]\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–50. A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and the coordinate direction angles of the moment created by this force about point O.



$$\mathbf{r}_A = 0.2 \sin 15° \mathbf{i} + 0.2 \cos 15° \mathbf{j} + 0.075 \mathbf{k}$$

$$= 0.05176 \mathbf{i} + 0.1932 \mathbf{j} + 0.075 \mathbf{k}$$

$$\mathbf{F} = -20 \cos 15° \mathbf{i} + 20 \sin 15° \mathbf{j}$$

$$= -19.32 \mathbf{i} + 5.176 \mathbf{j}$$

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.05176 & 0.1932 & 0.075 \\ -19.32 & 5.176 & 0 \end{vmatrix}$$

$$= \{-0.3882 \mathbf{i} - 1.449 \mathbf{j} + 4.00 \mathbf{k}\} \text{ N} \cdot \text{m}$$

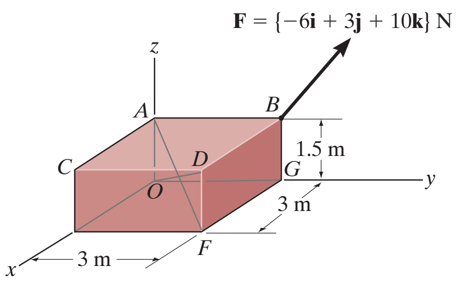
$$M_O = 4.272 = 4.27 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{-0.3882}{4.272}\right) = 95.2° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-1.449}{4.272}\right) = 110° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{4}{4.272}\right) = 20.6° \qquad \textbf{Ans}$$

4–51. Determine the moment produced by force **F** about the diagonal AF of the rectangular block. Express the result as a Cartesian vector.



Moment About Diagonal AF: Either position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{FB}$, Fig. *a*, can be used to find the moment of **F** about diagonal AF.

$$\mathbf{r}_{AB} = (0-0)\mathbf{i} + (3-0)\mathbf{j} + (1.5-1.5)\mathbf{k} = [3\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{FB} = (0-3)\mathbf{i} + (3-3)\mathbf{j} + (1.5-0)\mathbf{k} = [-3\mathbf{i} + 1.5\mathbf{k}]\text{ m}$$

The unit vector $\mathbf{u}_{AF}$, Fig. *a*, that specifies the direction of diagonal AF is given by

$$\mathbf{u}_{AF} = \frac{(3-0)\mathbf{i} + (3-0)\mathbf{j} + (0-1.5)\mathbf{k}}{\sqrt{(3-0)^2 + (3-0)^2 + (0-1.5)^2}} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

The magnitude of the moment of **F** about diagonal AF axis is

$$M_{AF} = \mathbf{u}_{AF} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 3 & 0 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[3(10) - (3)(0)] - \frac{2}{3}[0(10) - (-6)(0)] + \left(-\frac{1}{3}\right)[0(3) - (-6)(3)]$$

$$= 14\text{ N}\cdot\text{m}$$

or

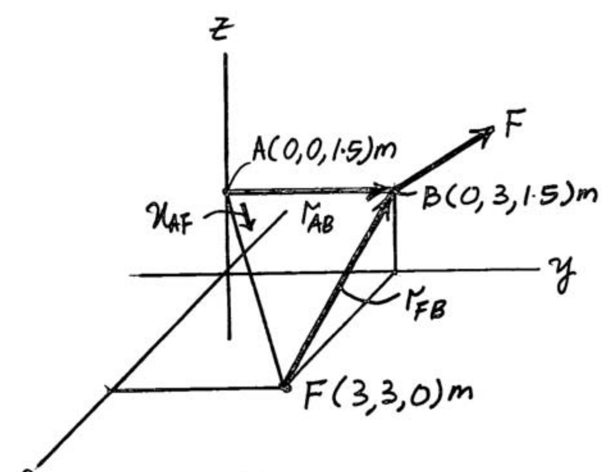
$$M_{AF} = \mathbf{u}_{AF} \cdot \mathbf{r}_{FB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -3 & 0 & 1.5 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[0(10) - (3)(1.5)] - \frac{2}{3}[(-3)(10) - (-6)(1.5)] + \left(-\frac{1}{3}\right)[(-3)(3) - (-6)(0)]$$

$$= 14\text{ N}\cdot\text{m}$$

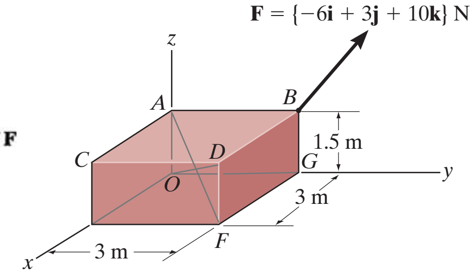
Thus, $\mathbf{M}_{AF}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{AF} = M_{AF}\mathbf{u}_{AF} = 14\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) = [9.33\mathbf{i} + 9.33\mathbf{j} - 4.67\mathbf{k}]\text{ N}\cdot\text{m}$$ **Ans.**



(a)

***4–52.** Determine the moment produced by force **F** about the diagonal *OD* of the rectangular block. Express the result as a Cartesian vector.



Moment About Diagonal OD: Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{DB}$, Fig. *a*, can be used to find the moment of **F** about diagonal *OD*.

$$\mathbf{r}_{OB} = (0-0)\mathbf{i} + (3-0)\mathbf{j} + (1.5-0)\mathbf{k} = [3\mathbf{j} + 1.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{DB} = (0-3)\mathbf{i} + (3-3)\mathbf{j} + (1.5-1.5)\mathbf{k} = [-3\mathbf{i}]\text{ m}$$

The unit vector $\mathbf{u}_{OD}$, Fig. *a*, that specifies the direction of diagonal *OD* is given by

$$\mathbf{u}_{AF} = \frac{(3-0)\mathbf{i} + (3-0)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(3-0)^2 + (3-0)^2 + (0-1.5)^2}} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

The magnitude of the moment of **F** about diagonal *OD* is

$$M_{OD} = \mathbf{u}_{OD} \cdot \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 3 & 1.5 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[3(10) - (3)(1.5)] - \frac{2}{3}[0(10) - (-6)(1.5)] + \frac{1}{3}[0(3) - (-6)(3)]$$

$$= 17\text{ N}\cdot\text{m}$$

or

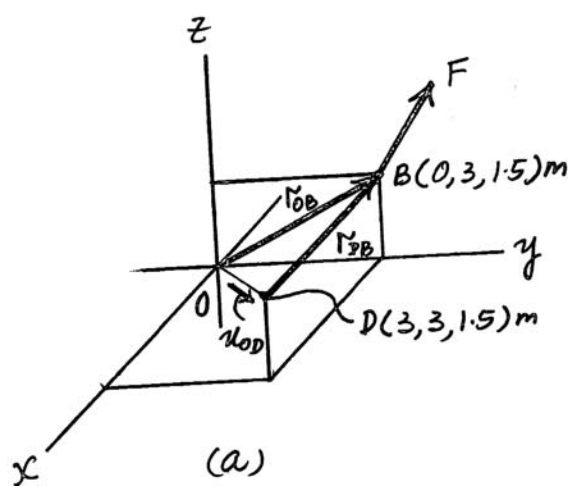
$$M_{OD} = \mathbf{u}_{OD} \cdot \mathbf{r}_{DB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -3 & 0 & 0 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[0(10) - (3)(0)] - \frac{2}{3}[-3(10) - (-6)(0)] + \frac{1}{3}[-3(3) - (-6)(0)]$$

$$= 17\text{ N}\cdot\text{m}$$

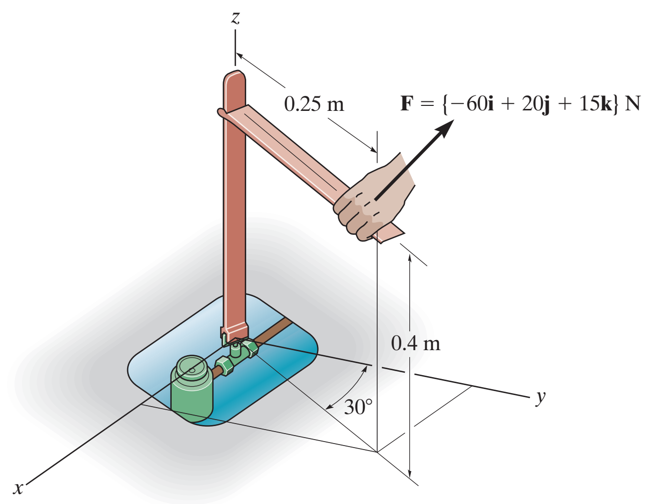
Thus, $\mathbf{M}_{OD}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{OD} = M_{OD}\mathbf{u}_{OD} = 17\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right) = [11.3\mathbf{i} + 11.3\mathbf{j} + 5.67\mathbf{k}]\text{ N}\cdot\text{m}$$ **Ans.**



(a)

•4–53. The tool is used to shut off gas valves that are difficult to access. If the force **F** is applied to the handle, determine the component of the moment created about the z axis of the valve.



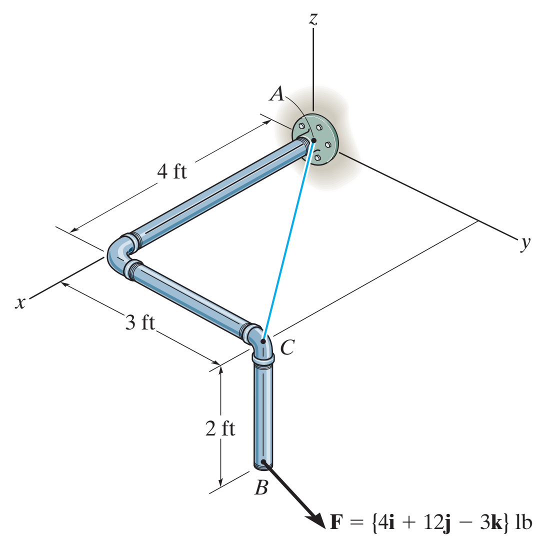
$$\mathbf{u} = \mathbf{k}$$

$$\mathbf{r} = 0.25\sin 30^\circ\,\mathbf{i} + 0.25\cos 30^\circ\,\mathbf{j}$$

$$= 0.125\,\mathbf{i} + 0.2165\,\mathbf{j}$$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix} = 15.5\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4–54. Determine the magnitude of the moments of the force **F** about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



a) *Vector Analysis*

Position Vector :

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

Moment of Force* F *About x, y and z Axes : The unit vectors along x, y and z axes are $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ respectively. Applying Eq. 4–7, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$
$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$
$$= \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - 4(3)] = 36.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

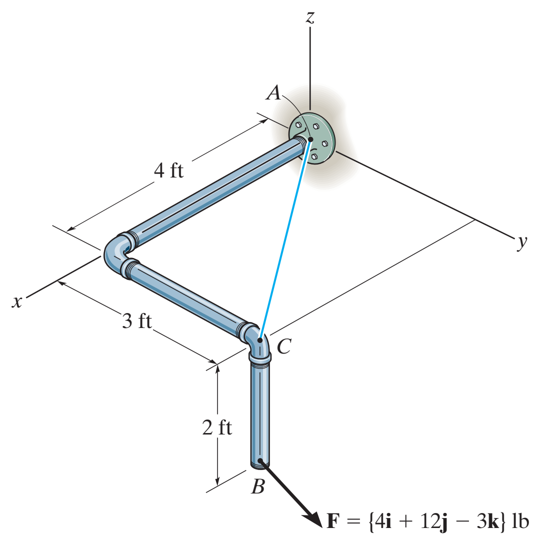
b) *Scalar Analysis*

$$M_x = \Sigma M_x;\quad M_x = 12(2) - 3(3) = 15.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_y = \Sigma M_y;\quad M_y = -4(2) + 3(4) = 4.00\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_z = \Sigma M_z;\quad M_z = -4(3) + 12(4) = 36.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

4–55. Determine the moment of the force **F** about an axis extending between *A* and *C*. Express the result as a Cartesian vector.



Position Vector :

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\ \text{ft}$$
$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

Unit Vector Along AC Axis :

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

Moment of Force F About AC Axis : With $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$ lb. applying Eq. 4–11, we have

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{CB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$
$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4\ \text{lb} \cdot \text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

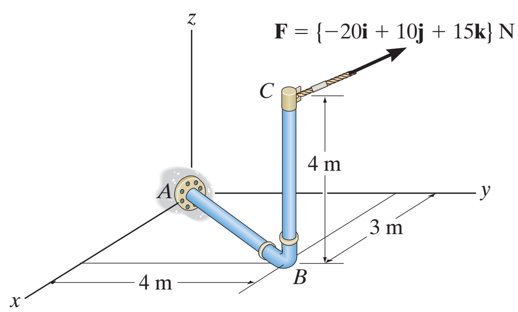
$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$
$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4\ \text{lb} \cdot \text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

$$\mathbf{M}_{AC} = M_{AC}\mathbf{u}_{AC}$$
$$= 14.4(0.8\mathbf{i} + 0.6\mathbf{j})$$
$$= \{11.5\mathbf{i} + 8.64\mathbf{j}\}\ \text{lb} \cdot \text{ft} \qquad \textbf{Ans}$$

***4–56.** Determine the moment produced by force **F** about segment AB of the pipe assembly. Express the result as a Cartesian vector.



Moment About Line AB: Either position vector $\mathbf{r}_{AC}$ or $\mathbf{r}_{BC}$ can be conveniently used to determine the moment of **F** about line AB.

$$\mathbf{r}_{AC} = (3-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k} = [3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (3-3)\mathbf{i} + (4-4)\mathbf{j} + (4-0)\mathbf{k} = [4\mathbf{k}]\text{ m}$$

The unit vector $\mathbf{u}_{AB}$, Fig. a, that specifies the direction of line AB is given by

$$\mathbf{u}_{AB} = \frac{(3-0)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (0-0)^2}} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$$

Thus, the magnitude of the moment of **F** about line AB is given by

$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 3 & 4 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[4(15) - 10(4)] - \frac{4}{5}[3(15) - (-20)(4)] + 0$$

$$= -88\text{ N}\cdot\text{m}$$

or

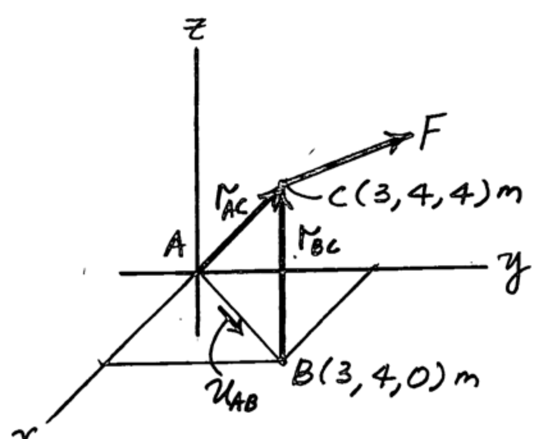
$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 0 & 0 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[0(15) - 10(4)] - \frac{4}{5}[0(15) - (-20)(4)] + 0$$

$$= -88\text{ N}\cdot\text{m}$$

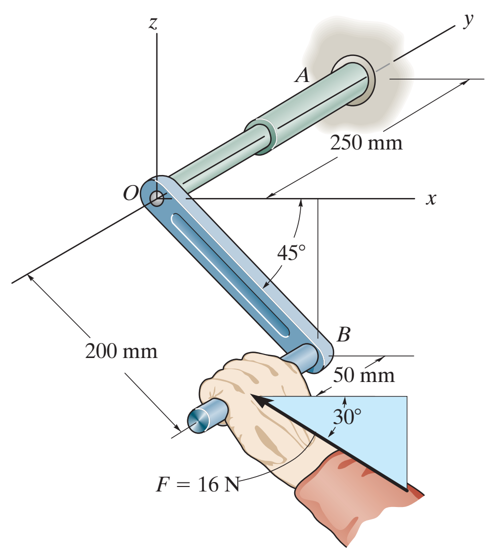
Thus, $\mathbf{M}_{AB}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{AB} = M_{AB}\mathbf{u}_{AB} = -88\left(\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}\right) = [-52.8\mathbf{i} - 70.4\mathbf{j}]\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

•4–57. Determine the magnitude of the moment that the force **F** exerts about the y axis of the shaft. Solve the problem using a Cartesian vector approach and using a scalar approach.



a) *Vector Analysis*

Position Vector and Force Vector :

$$\mathbf{r}_{OB} = \{0.2\cos 45°\mathbf{i} - 0.2\sin 45°\mathbf{k}\}\ \text{m} = \{0.1414\mathbf{i} - 0.1414\mathbf{k}\}\ \text{m}$$

$$\mathbf{F} = 16\{-\cos 30°\mathbf{i} + \sin 30°\mathbf{k}\}\ \text{N} = \{-13.856\mathbf{i} + 8.00\mathbf{k}\}\ \text{N}$$

Moment of Force F About y Axis : The unit vector along the y axis is $\mathbf{j}$. Applying Eq. 4 – 11, we have

$$M_y = \mathbf{j} \cdot (\mathbf{r}_{OB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 0.1414 & 0 & -0.1414 \\ -13.856 & 0 & 8 \end{vmatrix}$$

$$= 0 - 1[0.1414(8) - (-13.856)(-0.1414)] + 0$$

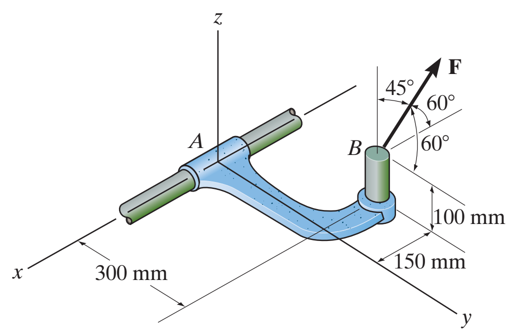
$$= 0.828\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

b) *Scalar Analysis*

$$M_y = \Sigma M_y; \quad M_y = 16\cos 30°(0.2\sin 45°) - 16\sin 30°(0.2\cos 45°)$$

$$= 0.828\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4–58. If $F = 450$ N, determine the magnitude of the moment produced by this force about the x axis.



Moment About the x axis: Either position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{CB}$ can be used to determine the moment of **F** about the x axis.

$\mathbf{r}_{AB} = (-0.15-0)\mathbf{i}+(0.3-0)\mathbf{j}+(0.1-0)\mathbf{k} = [-0.15\mathbf{i}+0.3\mathbf{j}+0.1\mathbf{k}]$ m

$\mathbf{r}_{CB} = [(-1.5-(-0.15)]\mathbf{i}+(0.3-0)\mathbf{j}+(0.1-0)\mathbf{k} = [0.3\mathbf{j}+0.1\mathbf{k}]$ m

The force vector **F** is given by

$\mathbf{F} = 450(-\cos 60°\mathbf{i}+\cos 60°\,\mathbf{j}+\cos 45°\mathbf{k}) = [-225\mathbf{i}+225\mathbf{j}+318.20\mathbf{k}]$ N

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

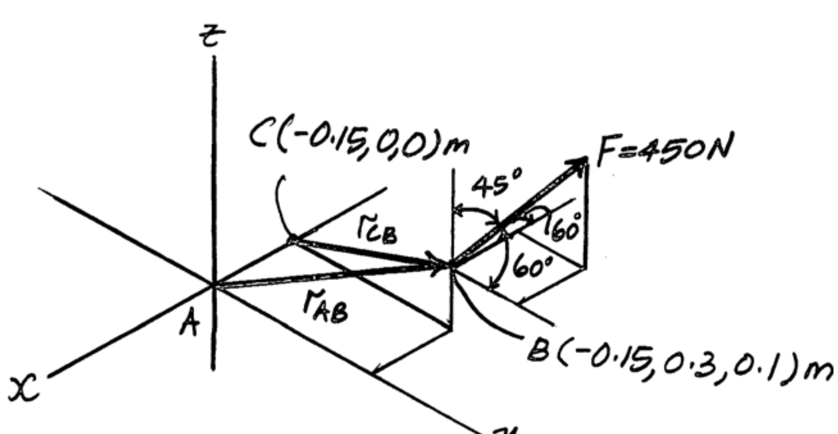
$$M_x = \mathbf{i}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ -0.15 & 0.3 & 0.1 \\ -225 & 225 & 318.20 \end{vmatrix}$$

$$= 1[0.3(318.20)-(225)(0.1)]+0+0 = 73.0\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

or

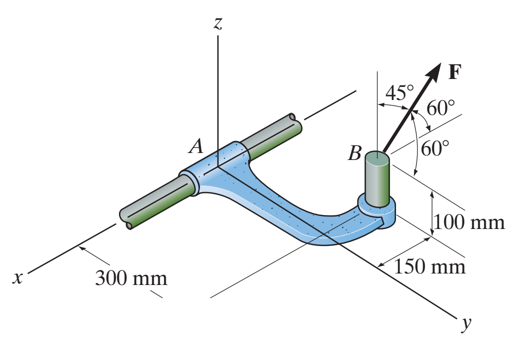
$$M_x = \mathbf{i}\cdot\mathbf{r}_{CB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.1 \\ -225 & 225 & 318.20 \end{vmatrix}$$

$$= 1[0.3(318.20)-(225)(0.1)]+0+0 = 73.0\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–59. The friction at sleeve A can provide a maximum resisting moment of 125 N · m about the x axis. Determine the largest magnitude of force **F** that can be applied to the bracket so that the bracket will not turn.



Moment About the x axis: The position vector $\mathbf{r}_{AB}$, Fig. *a*, will be used to determine the moment of **F** about the x axis.

$\mathbf{r}_{AB} = (-0.15 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.1 - 0)\mathbf{k} = [-0.15\mathbf{i} + 0.3\mathbf{j} + 0.1\mathbf{k}]$ m

The force vector **F** is given by

$\mathbf{F} = F(-\cos 60°\,\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\,\mathbf{k}) = -0.5F\mathbf{i} + 0.5F\mathbf{j} + 0.7071F\mathbf{k}$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

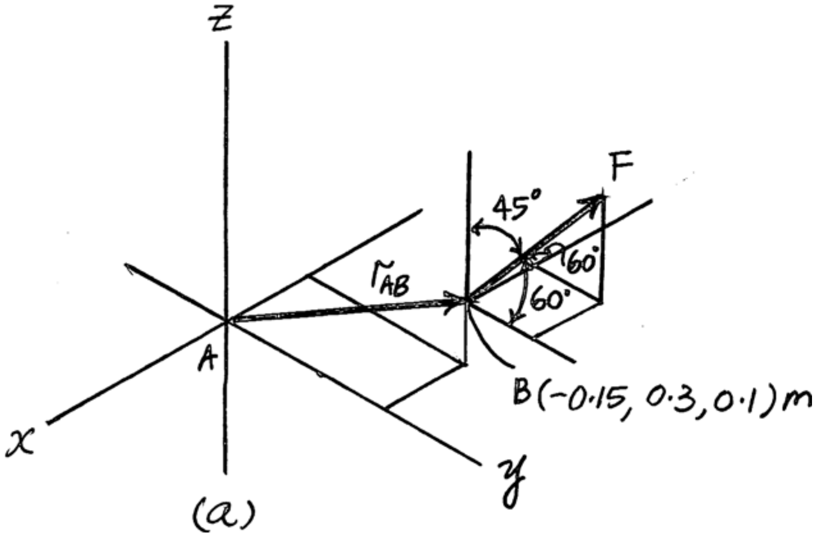
$$M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ -0.15 & 0.3 & 0.1 \\ -0.5F & 0.5F & 0.7071F \end{vmatrix}$$

$$= 1[0.3(0.7071F) - 0.5F(0.1)] + 0 + 0 = 0.1621F$$

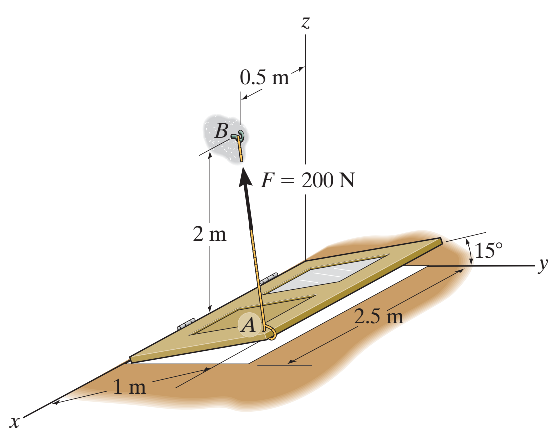
Since the friction at sleeve A can resist a moment of $M_x = 125$ N · m, the maximum allowable magnitude of **F** is given by

$$125 = 0.1621F$$

$$F = 771 \text{ N} \qquad \textbf{Ans.}$$



***4–60.** Determine the magnitude of the moment produced by the force of $F = 200$ N about the hinged axis (the x axis) of the door.



Moment About the x axis: Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{CA}$ can be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{CA} = (2.5-2.5)\mathbf{i}+(0.9659-0)\mathbf{j}+(0.2588-0)\mathbf{k} = [0.9659\mathbf{j}+0.2588\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{OB} = (0.5-0)\mathbf{i}+(0-0)\mathbf{j}+(2-0)\mathbf{k} = [0.5\mathbf{i}+2\mathbf{k}]\text{ m}$$

The force vector **F** is given by

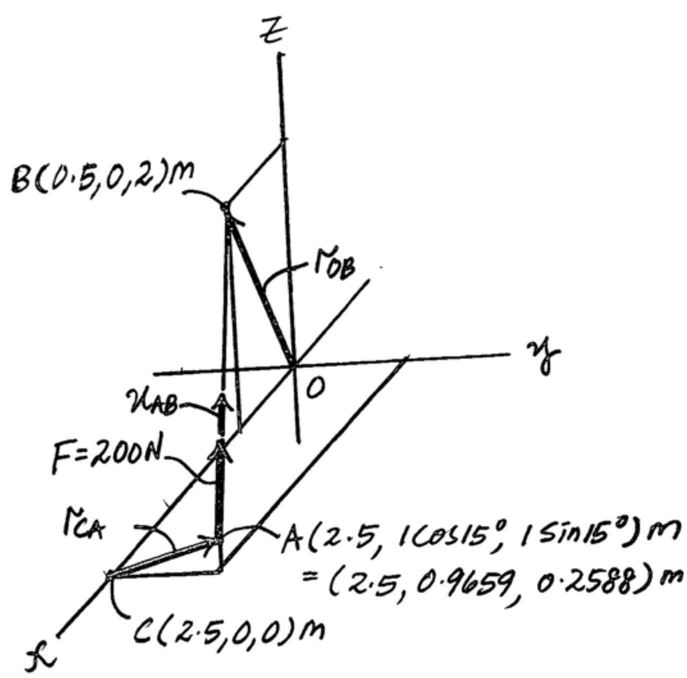
$$\mathbf{F} = F\mathbf{u}_{AB} = 200\left[\frac{(0.5-2.5)\mathbf{i}+(0-0.9659)\mathbf{j}+(2-0.2588)\mathbf{k}}{\sqrt{(0.5-2.5)^2+(0-0.9659)^2+(2-0.2588)^2}}\right] = [-141.73\mathbf{i}-68.45\mathbf{j}+123.39\mathbf{k}]\text{ N}$$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i}\cdot\mathbf{r}_{CA}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.9659 & 0.2588 \\ -141.73 & -68.45 & 123.39 \end{vmatrix} = 137\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

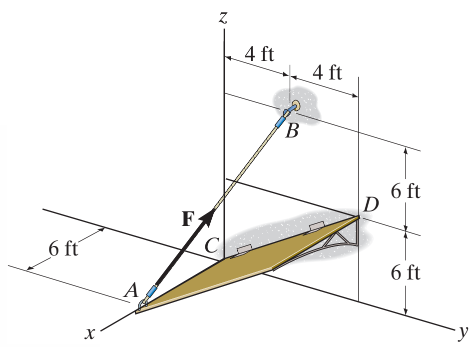
or

$$M_x = \mathbf{i}\cdot\mathbf{r}_{OB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0.5 & 0 & 2 \\ -141.73 & -68.45 & 123.39 \end{vmatrix} = 137\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

•4–61. If the tension in the cable is $F = 140$ lb, determine the magnitude of the moment produced by this force about the hinged axis, CD, of the panel.



Moment About the CD axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{DB}$, Fig. a, can be used to determine the moment of $\mathbf{F}$ about the CD axis.

$$\mathbf{r}_{CA} = (6-0)\mathbf{i} + (0-0)\mathbf{j} + (0-0)\mathbf{k} = [6\mathbf{i}]\text{ ft}$$

$$\mathbf{r}_{DB} = (0-0)\mathbf{i} + (4-8)\mathbf{j} + (12-6)\mathbf{k} = [-4\mathbf{j} + 6\mathbf{k}]\text{ ft}$$

Referring to Fig. a, the force vector $\mathbf{F}$ can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = 140\left[\frac{(0-6)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k}}{\sqrt{(0-6)^2 + (4-0)^2 + (12-0)^2}}\right] = [-60\mathbf{i} + 40\mathbf{j} + 120\mathbf{k}]\text{ lb}$$

The unit vector $\mathbf{u}_{CD}$, Fig. a, that specifies the direction of the CD axis is given by

$$\mathbf{u}_{CD} = \frac{(0-0)\mathbf{i} + (8-0)\mathbf{j} + (6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (8-0)^2 + (6-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of $\mathbf{F}$ about the CD axis is given by

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[6(120) - (-60)(0)] + \frac{3}{5}[6(40) - (-60)(0)]$$

$$= -432 \text{ lb}\cdot\text{ft}$$ **Ans.**

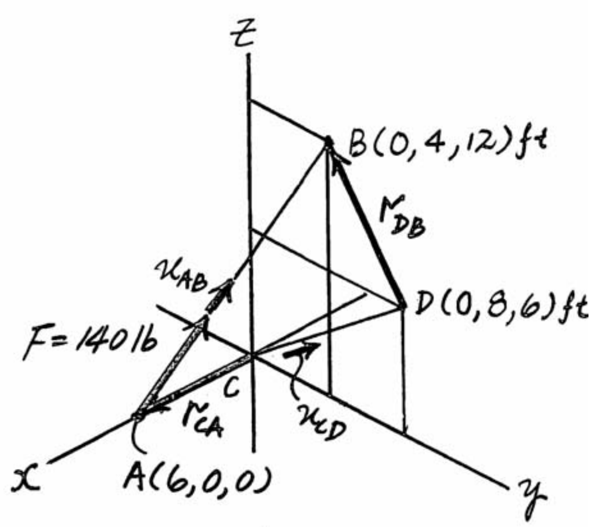
or

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{DB} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & -4 & 6 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[0(120) - (-60)(6)] + \frac{3}{5}[0(40) - (-60)(-4)]$$

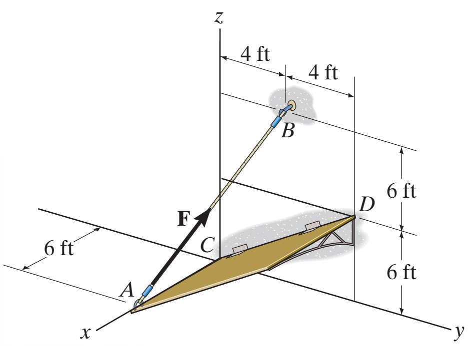
$$= -432 \text{ lb}\cdot\text{ft}$$ **Ans.**

The negative sign indicates that $\mathbf{M}_{CD}$ acts in the opposite sense to that of $\mathbf{u}_{CD}$.



(a)

4–62. Determine the magnitude of force **F** in cable *AB* in order to produce a moment of 500 lb · ft about the hinged axis *CD*, which is needed to hold the panel in the position shown.



Moment About the CD axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{CB}$, Fig. *a*, can be used to determine the moment of **F** about the *CD* axis.

$\mathbf{r}_{CA} = (6-0)\mathbf{i} + (0-0)\mathbf{j} + (0-0)\mathbf{k} = [6\mathbf{i}]\text{ft}$

$\mathbf{r}_{CB} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k} = [4\mathbf{j} + 12\mathbf{k}]\text{ft}$

Referring to Fig. *a*, the force vector **F** can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = F\left[\frac{(0-6)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k}}{\sqrt{(0-6)^2 + (4-0)^2 + (12-0)^2}}\right] = -\frac{3}{7}F\mathbf{i} + \frac{2}{7}F\mathbf{j} + \frac{6}{7}F\mathbf{k}$$

The unit vector $\mathbf{u}_{CD}$, Fig. *a*, that specifies the direction of the *CD* axis is given by

$$\mathbf{u}_{CD} = \frac{(0-0)\mathbf{i} + (8-0)\mathbf{j} + (6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (8-0)^2 + (6-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the *CD* axis is required to be $\mathbf{M}_{CD} = |500|\ \text{lb}\cdot\text{ft}$. Thus,

$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F}$

$$|500| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix}$$

$$-500 = 0 - \frac{4}{5}\left[6\left(\frac{6}{7}F\right) - \left(-\frac{3}{7}F\right)(0)\right] + \frac{3}{5}\left[6\left(\frac{2}{7}F\right) - \left(-\frac{3}{7}F\right)(0)\right]$$

$F = 162\ \text{lb}$ **Ans.**

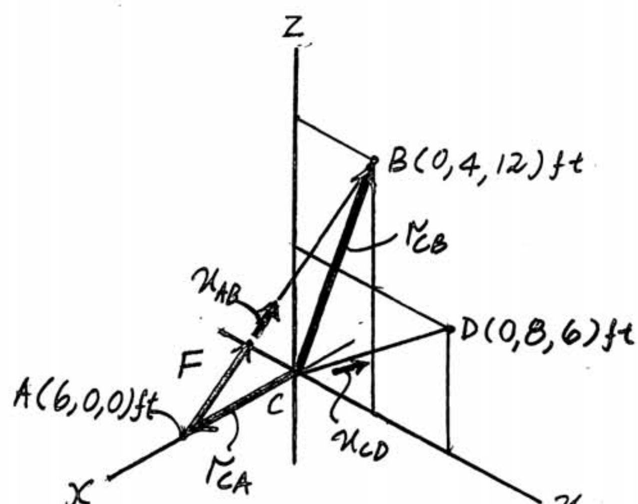
or

$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CB} \times \mathbf{F}$

$$|500| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & 4 & 12 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix}$$

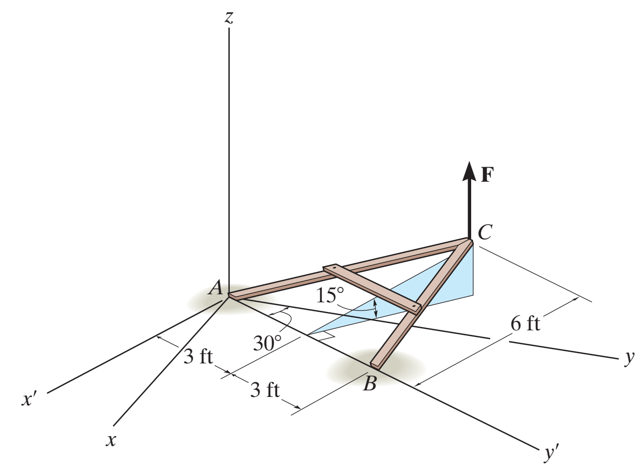
$$-500 = 0 - \frac{4}{5}\left[0\left(\frac{6}{7}F\right) - \left(-\frac{3}{7}F\right)(12)\right] + \frac{3}{5}\left[0\left(\frac{2}{7}F\right) - \left(-\frac{3}{7}F\right)(4)\right]$$

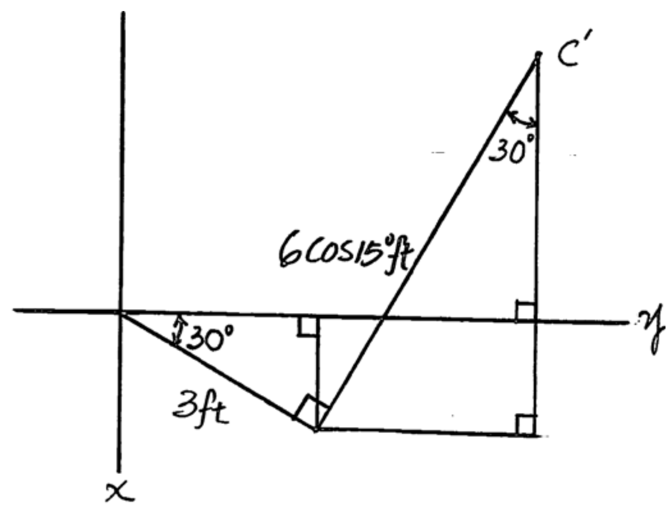
$F = 162\ \text{lb}$ **Ans.**



(a)

4–63. The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y' axis passing through points A and B when the frame is in the position shown.





Scalar analysis :

$M_{y'} = 80\,(6 \cos 15°) = 464 \text{ lb}\cdot\text{ft}$ **Ans**

Vector analysis :

$\mathbf{u}_{AB} = \sin 30°\, \mathbf{i} + \cos 30°\, \mathbf{j}$

Coordinates of point C :

$x = 3 \sin 30° - 6 \cos 15° \cos 30° = -3.52$ ft

$y = 3 \cos 30° + 6 \cos 15° \sin 30° = 5.50$ ft

$z = 6 \sin 15° = 1.55$ ft

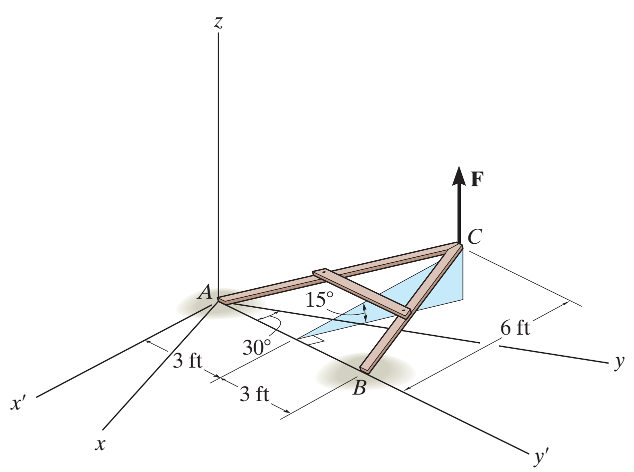
$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$

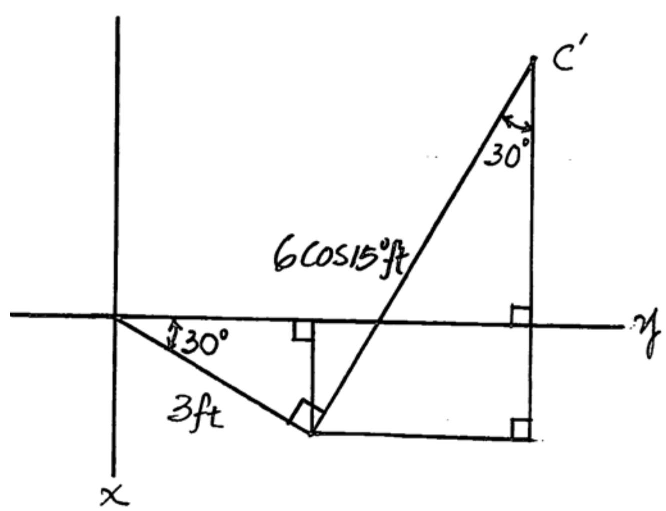
$\mathbf{F} = 80\,\mathbf{k}$

$$M_{y'} = \begin{vmatrix} \sin 30° & \cos 30° & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix}$$

$M_{y'} = 464 \text{ lb}\cdot\text{ft}$ **Ans**

***4–64.** The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the x axis when the frame is in the position shown.





Using x', y', z:

$\mathbf{u}_x = \cos 30°\,\mathbf{i}' + \sin 30°\,\mathbf{j}'$

$\mathbf{r}_{AC} = -6\cos 15°\,\mathbf{i}' + 3\,\mathbf{j}' + 6\sin 15°\,\mathbf{k}$

$\mathbf{F} = 80\,\mathbf{k}$

$$M_x = \begin{vmatrix} \cos 30° & \sin 30° & 0 \\ -6\cos 15° & 3 & 6\sin 15° \\ 0 & 0 & 80 \end{vmatrix} = 207.85 + 231.82 + 0$$

$M_x = 440$ lb · ft **Ans**

Also, using x, y, z.

Coordinates of point C :

$x = 3\sin 30° - 6\cos 15°\cos 30° = -3.52$ ft

$y = 3\cos 30° + 6\cos 15°\sin 30° = 5.50$ ft

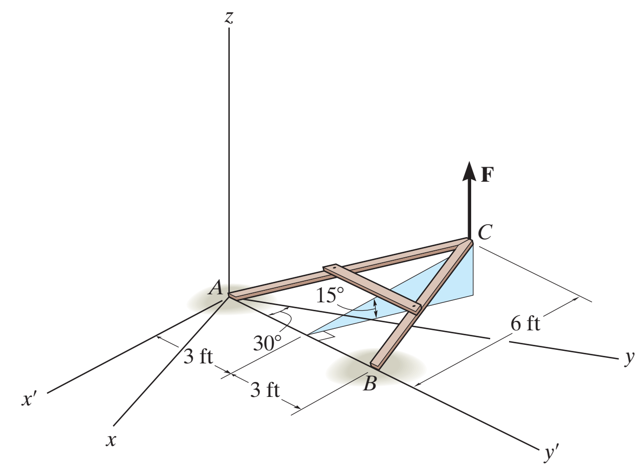
$z = 6\sin 15° = 1.55$ ft

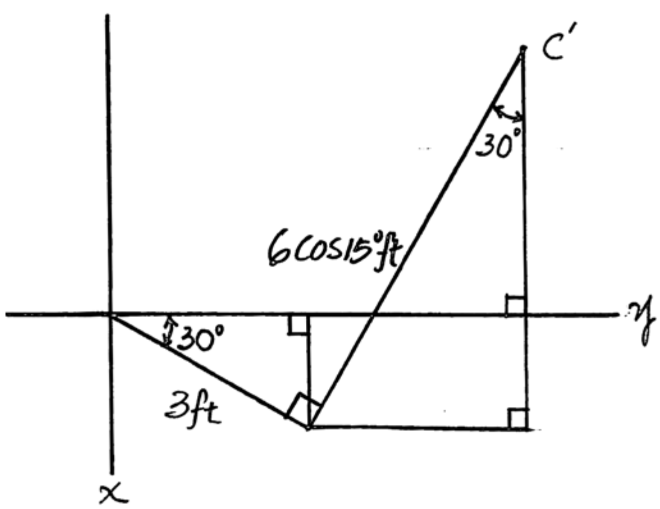
$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$

$\mathbf{F} = 80\,\mathbf{k}$

$$M_x = \begin{vmatrix} 1 & 0 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 440 \text{ lb} \cdot \text{ft} \quad \textbf{Ans}$$

•4–65. The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y axis when the frame is in the position shown.





Using x', y', z:

$\mathbf{u}_y = -\sin 30°\ \mathbf{i}' + \cos 30°\ \mathbf{j}'$

$\mathbf{r}_{AC} = -6\cos 15°\ \mathbf{i}' + 3\ \mathbf{j}' + 6\sin 15°\ \mathbf{k}$

$\mathbf{F} = 80\ \mathbf{k}$

$$M_y = \begin{vmatrix} -\sin 30° & \cos 30° & 0 \\ -6\cos 15° & 3 & 6\sin 15° \\ 0 & 0 & 80 \end{vmatrix} = -120 + 401.52 + 0$$

$M_y = 282$ lb · ft **Ans**

Also, using x, y, z:

Coordinates of point C:

$x = 3\sin 30° - 6\cos 15°\cos 30° = -3.52$ ft

$y = 3\cos 30° + 6\cos 15°\sin 30° = 5.50$ ft

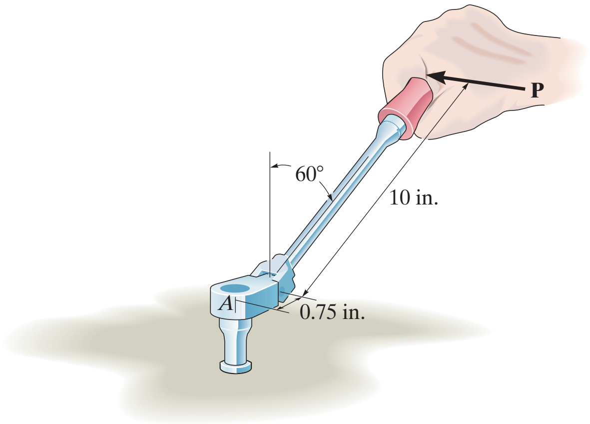
$z = 6\sin 15° = 1.55$ ft

$\mathbf{r}_{AC} = -3.52\ \mathbf{i} + 5.50\ \mathbf{j} + 1.55\ \mathbf{k}$

$\mathbf{F} = 80\ \mathbf{k}$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 282 \text{ lb} \cdot \text{ft}$$ **Ans**

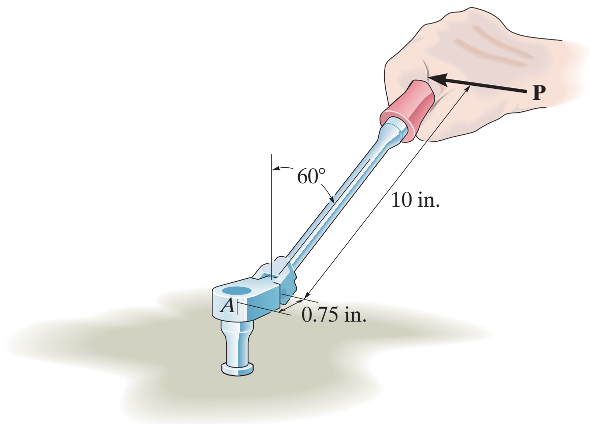
4–66. The flex-headed ratchet wrench is subjected to a force of $P = 16$ lb, applied perpendicular to the handle as shown. Determine the moment or torque this imparts along the vertical axis of the bolt at A.



$$M = 16(0.75 + 10\sin 60°)$$

$$M = 151 \text{ lb}\cdot\text{in.} \qquad \textbf{Ans}$$

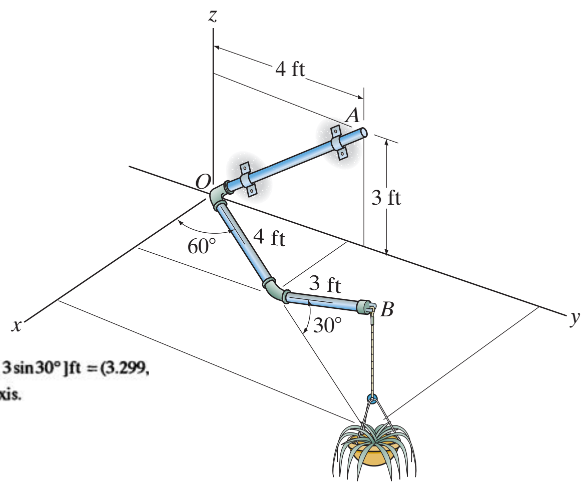
4–67. If a torque or moment of 80 lb · in. is required to loosen the bolt at A, determine the force P that must be applied perpendicular to the handle of the flex-headed ratchet wrench.



$$80 = P(0.75 + 10\sin 60°)$$

$$P = \frac{80}{9.41} = 8.50 \text{ lb} \qquad \textbf{Ans}$$

***4–68.** The pipe assembly is secured on the wall by the two brackets. If the flower pot has a weight of 50 lb, determine the magnitude of the moment produced by the weight about the *OA* axis.



Moment About the OA axis: The coordinates of point B are $[(4+3\cos 30°)\cos 60°, (4+3\cos 30°)\sin 60°, 3\sin 30°]$ ft $= (3.299, 5.714, 1.5)$ ft. Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{AB}$ can be used to determine the moment of $\mathbf{W}$ about the OA axis.

$\mathbf{r}_{OB} = (3.299-0)\mathbf{i} + (5.714-0)\mathbf{j} + (1.5-0)\mathbf{k} = [3.299\mathbf{i} + 5.714\mathbf{j} + 1.5\mathbf{k}]$ ft

$\mathbf{r}_{AB} = (3.299-0)\mathbf{i} + (5.714-4)\mathbf{j} + (1.5-3)\mathbf{k} = [3.299\mathbf{i} + 1.714\mathbf{j} - 1.5\mathbf{k}]$ ft

Since $\mathbf{W}$ is directed towards the negative z axis, we can write

$\mathbf{W} = [-50\mathbf{k}]$ lb

The unit vector $\mathbf{u}_{OA}$, Fig. *a*, that specifies the direction of the OA axis is given by

$$\mathbf{u}_{OA} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

The magnitude of the moment of $\mathbf{W}$ about the OA axis is given by

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{OB} \times \mathbf{W} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 1.5 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[3.299(-50) - 0(1.5)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

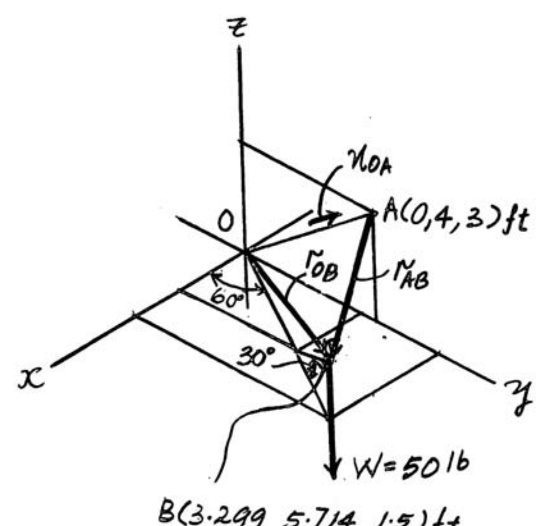
$$= 132 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

or

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{AB} \times \mathbf{W} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 1.714 & -1.5 \\ 0 & 0 & -50 \end{vmatrix}$$

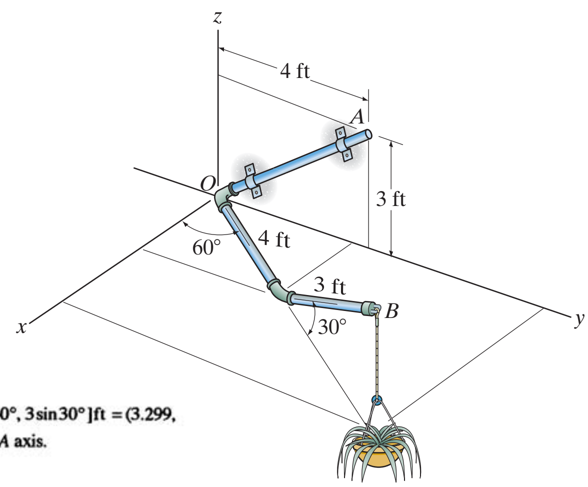
$$= 0 - \frac{4}{5}[3.299(-50) - 0(-1.5)] + \frac{3}{5}[3.299(0) - 0(1.714)]$$

$$= 132 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



(a)

•4–69. The pipe assembly is secured on the wall by the two brackets. If the frictional force of both brackets can resist a maximum moment of 150 lb·ft, determine the largest weight of the flower pot that can be supported by the assembly without causing it to rotate about the OA axis.



Moment About the OA axis: The coordinates of point B are $[(4+3\cos 30°)\cos 60°, (4+3\cos 30°)\sin 60°, 3\sin 30°]$ ft $= (3.299, 5.714, 1.5)$ ft. Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{OC}$ can be used to determine the moment of $\mathbf{W}$ about the OA axis.

$$\mathbf{r}_{OB} = (3.299-0)\mathbf{i}+(5.714-0)\mathbf{j}+(1.5-0)\mathbf{k} = [3.299\mathbf{i}+5.714\mathbf{j}+1.5\mathbf{k}]\text{ft}$$

$$\mathbf{r}_{AB} = (3.299-0)\mathbf{i}+(5.714-4)\mathbf{j}+(1.5-3)\mathbf{k} = [3.299\mathbf{i}+1.714\mathbf{j}-1.5\mathbf{k}]\text{ft}$$

Since $\mathbf{W}$ is directed towards the negative z axis, we can write

$$\mathbf{W} = -W\mathbf{k}$$

The unit vector $\mathbf{u}_{OA}$, Fig. a, that specifies the direction of the OA axis is given by

$$\mathbf{u}_{OA} = \frac{(0-0)\mathbf{i}+(4-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(0-0)^2+(4-0)^2+(3-0)^2}} = \frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$$

Since it is required that the magnitude of the moment of $\mathbf{W}$ about the OA axis not exceed 150 ft·lb, we can write

$$M_{OA} = \mathbf{u}_{OA}\cdot\mathbf{r}_{OB}\times\mathbf{W}$$

$$|150| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 1.5 \\ 0 & 0 & -W \end{vmatrix}$$

$$150 = 0-\frac{4}{5}[3.299(-W)-0(1.5)]+\frac{3}{5}[3.299(0)-0(5.714)]$$

$W = 56.8$ lb **Ans.**

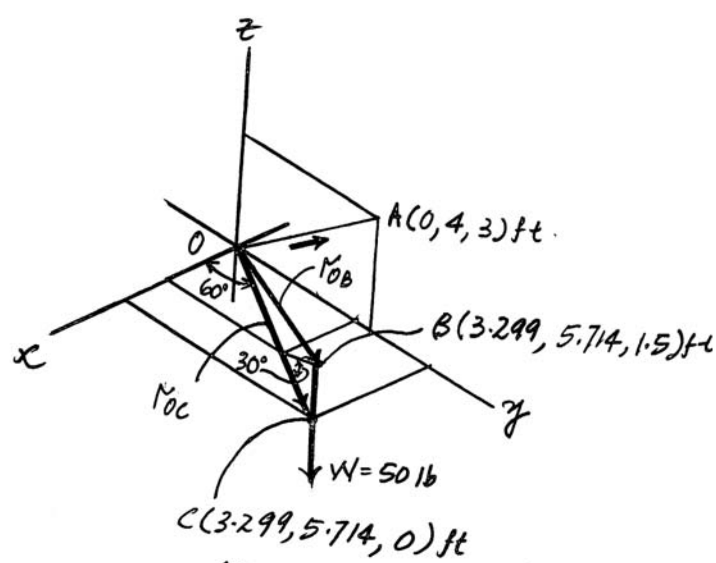
or

$$M_{OA} = \mathbf{u}_{OA}\cdot\mathbf{r}_{OB}\times\mathbf{W}$$

$$|150| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 0 \\ 0 & 0 & -W \end{vmatrix}$$

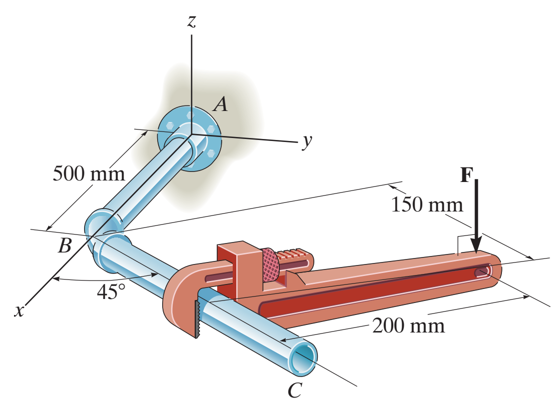
$$150 = 0-\frac{4}{5}[3.299(-W)-0(0)]+\frac{3}{5}[3.299(0)-0(5.714)]$$

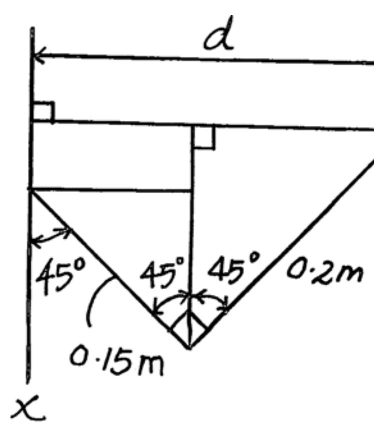
$W = 56.8$ lb **Ans.**



(a)

4–70. A vertical force of $F = 60$ N is applied to the handle of the pipe wrench. Determine the moment that this force exerts along the axis AB (x axis) of the pipe assembly. Both the wrench and pipe assembly ABC lie in the $x-y$ plane. *Suggestion:* Use a scalar analysis.





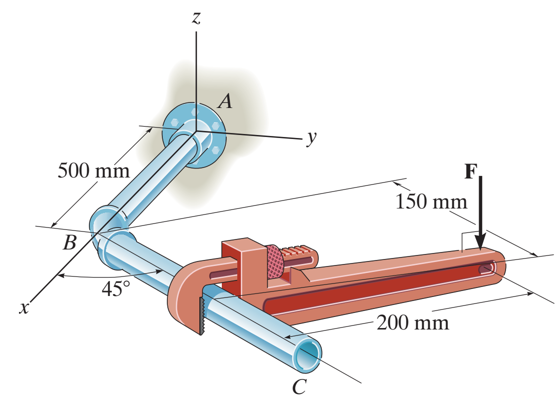
Scalar Analysis : From the geometry, the perpendicular distance from x axis to force **F** is $d = 0.15\sin 45° + 0.2\sin 45° = 0.2475$ m.

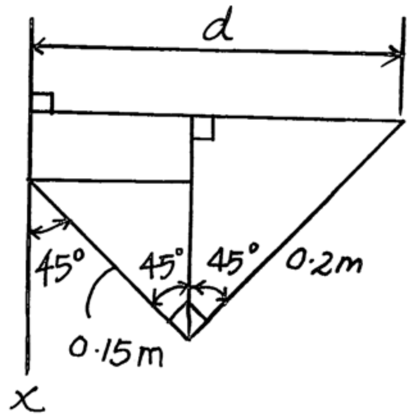
$$M_x = \Sigma M_x; \quad M_x = -Fd = -60(0.2475) = -14.8 \text{ N}\cdot\text{m}$$

Negative sign indicates that $\mathbf{M}_x$ is directed toward negative x axis.

$\mathbf{M}_x = 14.8$ N·m **Ans**

4–71. Determine the magnitude of the vertical force **F** acting on the handle of the wrench so that this force produces a component of moment along the AB axis (x axis) of the pipe assembly of $(M_A)_x = \{-5\mathbf{i}\}$ N · m. Both the pipe assembly ABC and the wrench lie in the x–y plane. *Suggestion:* Use a scalar analysis.

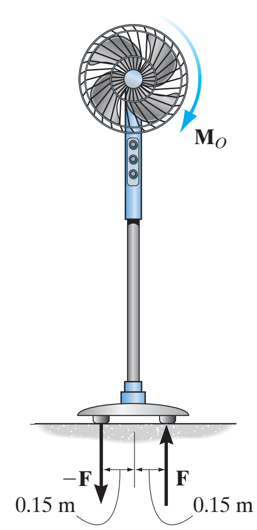




Scalar Analysis **: From the geometry, the perpendicular distance from x axis to F is $d = 0.15\sin 45° + 0.2\sin 45° = 0.2475$ m.**

$$M_x = \Sigma M_x; \quad -5 = -F(0.2475)$$
$$F = 20.2 \text{ N} \qquad \textbf{Ans}$$

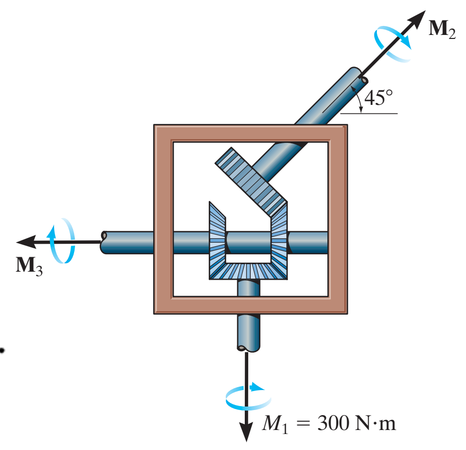
***4–72.** The frictional effects of the air on the blades of the standing fan creates a couple moment of $M_O = 6$ N · m on the blades. Determine the magnitude of the couple forces at the base of the fan so that the resultant couple moment on the fan is zero.



Couple Moment: The couple moment of **F** produces a counterclockwise moment of $M_C = F(0.15 + 0.15) = 0.3F$. Since the resultant couple moment about the axis perpendicular to the page is required to be zero,

$$\circlearrowleft + (M_c)_R = \Sigma M; \qquad 0 = 0.3F - 6 \qquad F = 20\text{ N} \qquad \textbf{Ans.}$$

•**4–73.** Determine the required magnitude of the couple moments $\mathbf{M}_2$ and $\mathbf{M}_3$ so that the resultant couple moment is zero.



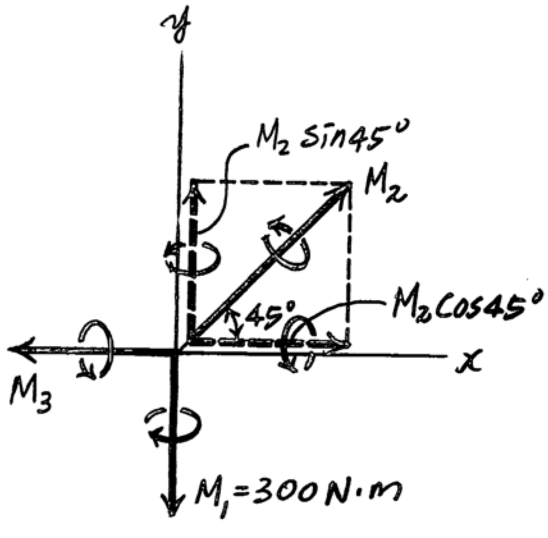
Since the couple moment is the free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ can be simplified as shown in Fig. *a*. Since the resultant of $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ is required to be zero,

$(M_R)_y = \Sigma M_y;$ $\quad 0 = M_2 \sin 45° - 300$

$M_2 = 424.26 \text{ N} \cdot \text{m} = 424 \text{ N} \cdot \text{m}$ **Ans.**

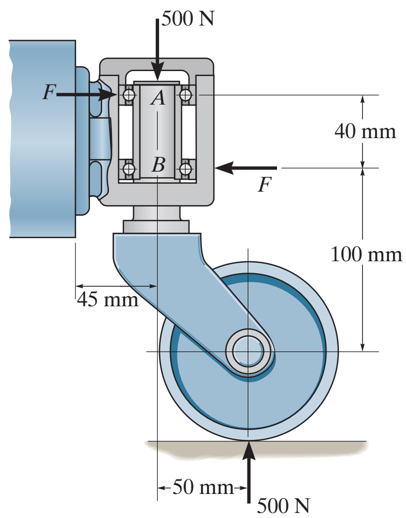
$(M_R)_x = \Sigma M_x;$ $\quad 0 = 424.26 \cos 45° - M_3$

$M_3 = 300 \text{ N} \cdot \text{m}$ **Ans.**



(a)

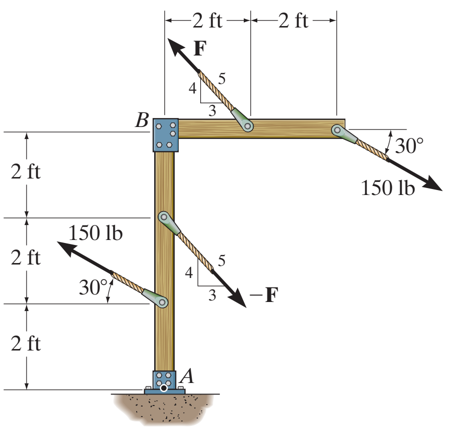
4–74. The caster wheel is subjected to the two couples. Determine the forces F that the bearings exert on the shaft so that the resultant couple moment on the caster is zero.



$\circlearrowleft + \Sigma M_A = 0; \quad 500(50) - F(40) = 0$

$F = 625 \text{ N}$ **Ans**

4–75. If $F = 200$ lb, determine the resultant couple moment.



a) By resolving the 150 - lb and 200 - lb couples into their x and y components, Fig. a, the couple moments $(M_C)_1$ and $(M_C)_2$ produced by the 150 - lb and 200 - lb couples, respectively, are given by

$$\circlearrowleft + (M_c)_1 = -150\cos 30°(4) - 150\sin 30°(4) = -819.62 \text{ lb}\cdot\text{ft} = 819.62 \text{ lb}\cdot\text{ft}$$

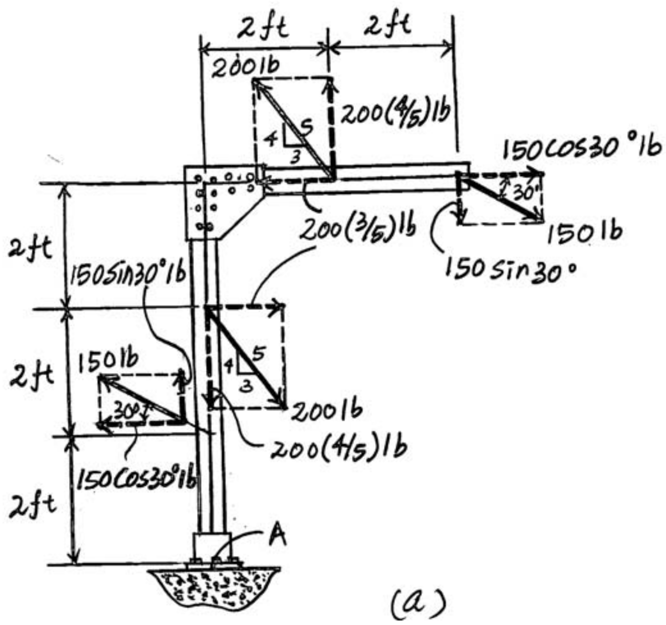
$$\circlearrowleft + (M_c)_2 = 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(2) = 560 \text{ lb}\cdot\text{ft}$$

Thus, the resultant couple moment can be determined from

$$\circlearrowleft + (M_c)_R = (M_c)_1 + (M_c)_2$$
$$= -819.62 + 560 = -259.62 \text{ lb}\cdot\text{ft} = 260 \text{ lb}\cdot\text{ft (clockwise)}$$ **Ans.**

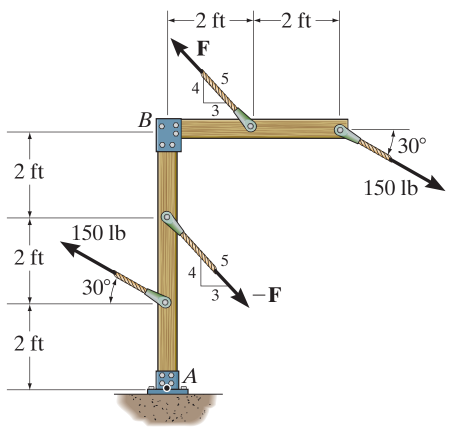
b) By resolving the 150 - lb and 200 - lb couples into their x and y components, Fig. a, and summing the moments of these force components algebraically about point A,

$$\circlearrowleft + (M_C)_R = \Sigma M_A;\ (M_C)_R = -150\sin 30°(4) - 150\cos 30°(6) + 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(6)$$
$$- 200\left(\frac{3}{5}\right)(4) + 200\left(\frac{4}{5}\right)(0) + 150\cos 30°(2) + 150\sin 30°(0)$$
$$= -259.62 \text{ lb}\cdot\text{ft} = 260 \text{ lb}\cdot\text{ft (clockwise)}$$ **Ans.**



(a)

***4–76.** Determine the required magnitude of force **F** if the resultant couple moment on the frame is 200 lb · ft, clockwise.



By resolving **F** and the 150 - lb couple into their x and y components, Fig. a, the couple moments $(M_C)_1$ and $(M_C)_2$ produced by **F** and the 5 - kN couple, respectively, are given by

$$\curvearrowleft +(M_c)_1 = F\left(\frac{4}{5}\right)(2) + F\left(\frac{3}{5}\right)(2) = 2.8F$$

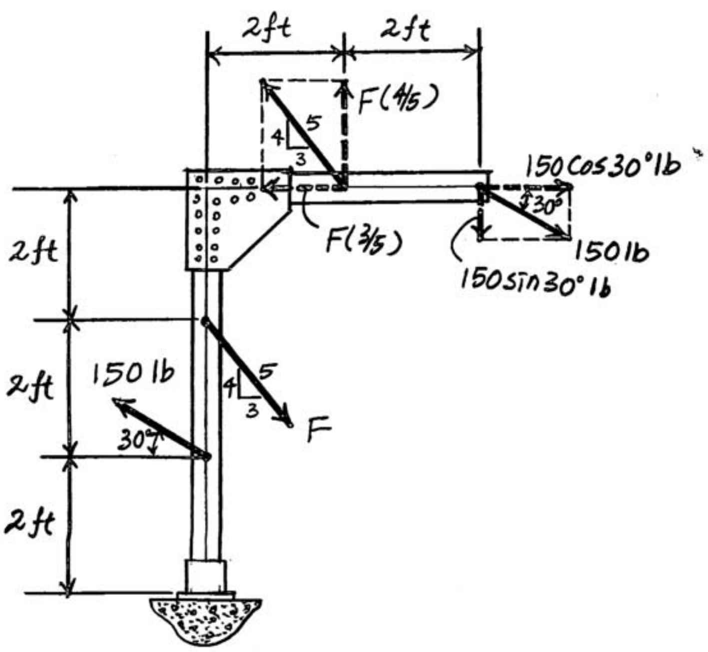
$$\curvearrowleft +(M_c)_2 = -150\cos 30°(4) - 150\sin 30°(4) = -819.62 \text{ lb}\cdot\text{ft} = 819.62 \text{ lb}\cdot\text{ft} \curvearrowright$$

The resultant couple moment acting on the beam is required to be 200 lb·ft, clockwise. Thus,

$$\curvearrowleft +(M_c)_R = (M_c)_1 + (M_c)_2$$

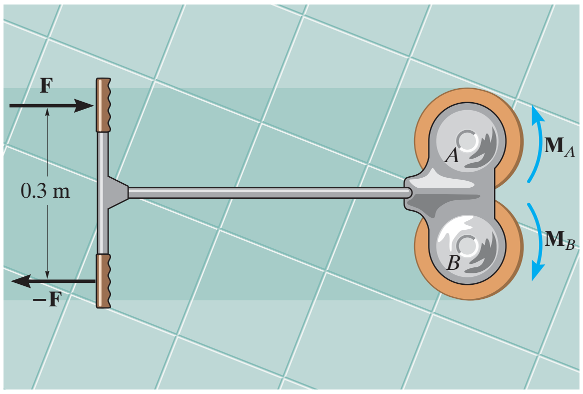
$$-200 = 2.8F - 819.62$$

$$F = 221 \text{ lb} \qquad \textbf{Ans.}$$



(a)

•4–77. The floor causes a couple moment of $M_A = 40\ \text{N}\cdot\text{m}$ and $M_B = 30\ \text{N}\cdot\text{m}$ on the brushes of the polishing machine. Determine the magnitude of the couple forces that must be developed by the operator on the handles so that the resultant couple moment on the polisher is zero. What is the magnitude of these forces if the brush at B suddenly stops so that $M_B = 0$?



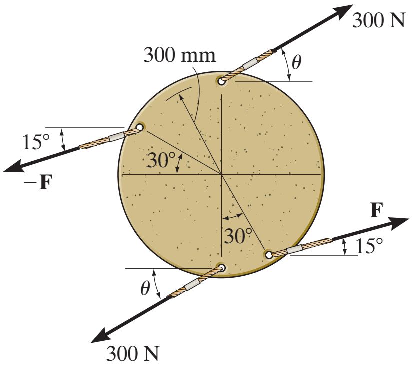
$$\circlearrowleft + \; M_R = 40 - 30 - F'(0.3) = 0$$

$$F' = 33.3\ \text{N} \qquad \textbf{Ans}$$

$$\circlearrowleft + \; M_R = 40 - F(0.3) = 0$$

$$F = 133\ \text{N} \qquad \textbf{Ans}$$

4–78. If $\theta = 30°$, determine the magnitude of force **F** so that the resultant couple moment is $100\ \text{N}\cdot\text{m}$, clockwise.

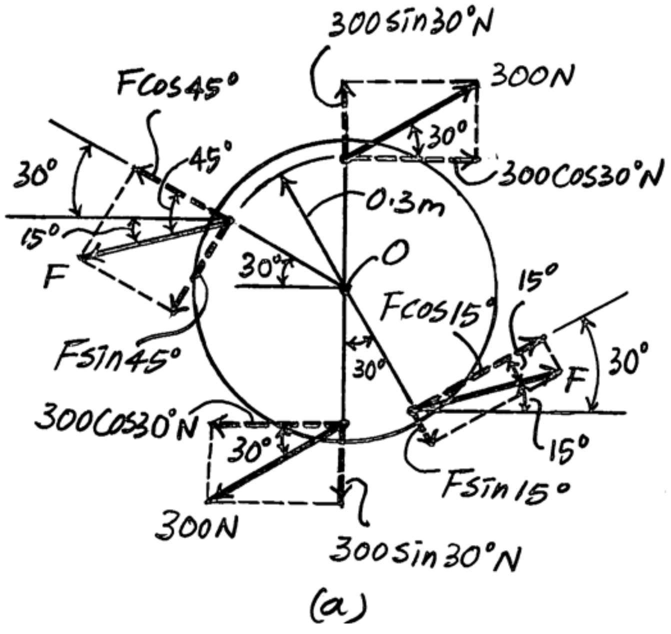


By resolving **F** and the 300 -N couple into their radial and tangential components, Fig. *a*, and summing the moment of these two force components about point O,

$$\circlearrowleft +(M_c)_R = \Sigma M_O; \qquad -100 = F\sin 45°(0.3) + F\cos 15°(0.3) - 2(300\cos 30°)(0.3)$$

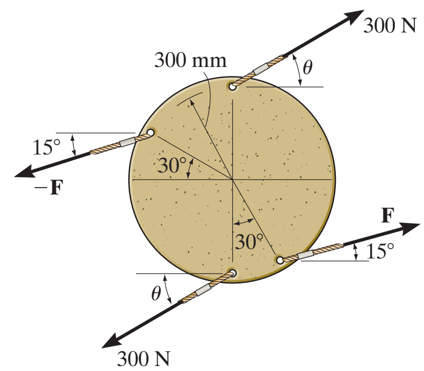
$$F = 111\ \text{N} \qquad \textbf{Ans.}$$

Note: Since the line of action of the radial component of the forces pass through point O, no moment is produced about this point.



(a)

4–79. If $F = 200$ N, determine the required angle θ so that the resultant couple moment is zero.

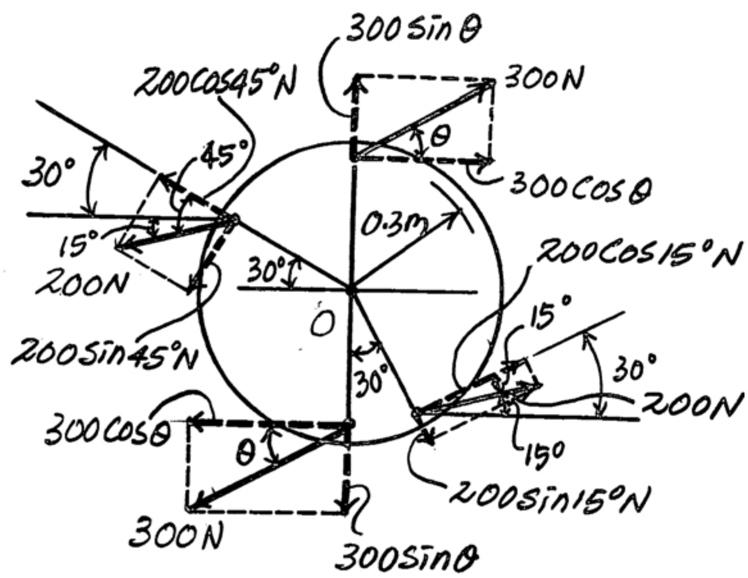


By resolving the 300 - N and 200 -N couples into their radial and tangential components, Fig. *a*, and summing the moment of these two force components about point O,

$$\curvearrowleft +(M_c)_R = \Sigma M_O; \qquad 0 = 200\sin 45°(0.3) + 200\cos 15°(0.3) - 300\cos\theta(0.3) - 300\cos\theta(0.3)$$

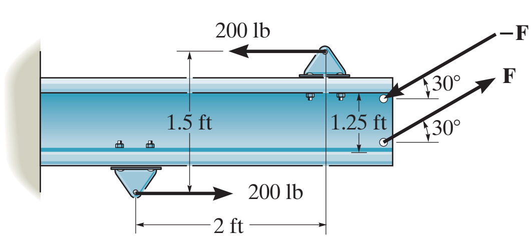
$$\theta = 56.1° \quad \textbf{Ans.}$$

Note: Since the line of action of the radial component of the forces pass through point O, no moment is produced about this point.



(a)

***4–80.** Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?

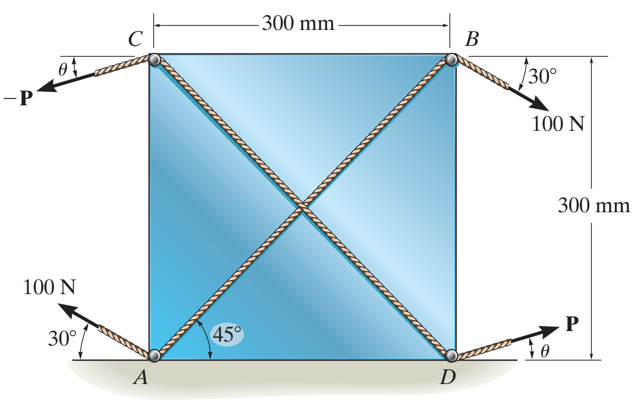


$$\curvearrowleft + M_R = \Sigma M; \qquad 450 = 200(1.5) + F\cos 30°(1.25)$$

$$F = 139 \text{ lb} \qquad \textbf{Ans}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

•**4–81.** The cord passing over the two small pegs A and B of the square board is subjected to a tension of 100 N. Determine the required tension P acting on the cord that passes over pegs C and D so that the resultant couple produced by the two couples is 15 N · m acting clockwise. Take $\theta = 15°$.



$$\curvearrowleft +M_R = 100\cos 30°(0.3) + 100\sin 30°(0.3) - P\sin 15°(0.3) - P\cos 15°(0.3) = -15$$

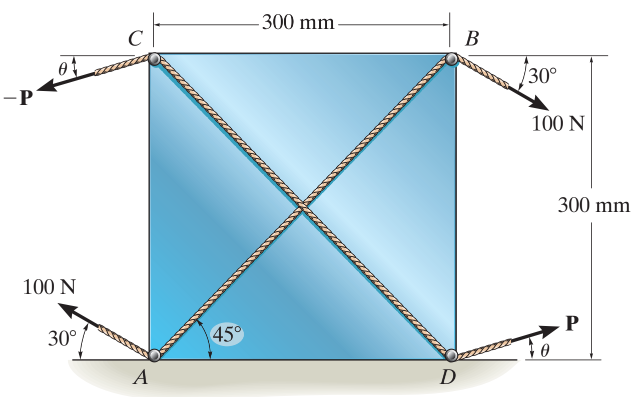
$P = 70.7$ N **Ans**

4–82. The cord passing over the two small pegs A and B of the board is subjected to a tension of 100 N. Determine the *minimum* tension P and the orientation θ of the cord passing over pegs C and D, so that the resultant couple moment produced by the two cords is 20 N · m, clockwise.

For minimum P require $\theta = 45°$ **Ans**

$$\curvearrowleft +M_R = 100\cos 30°(0.3) + 100\sin 30°(0.3) - P\left(\frac{0.3}{\cos 45°}\right) = 20$$

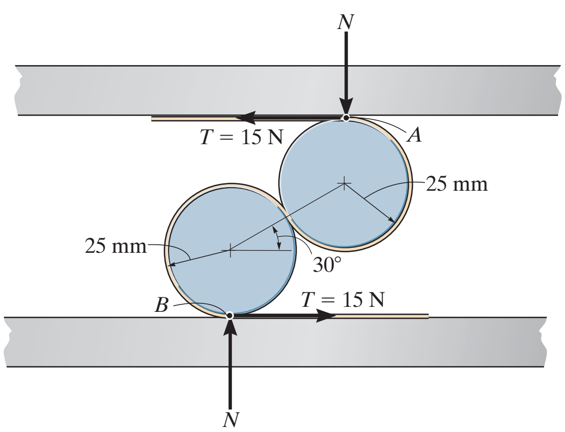
$P = 49.5$ N **Ans**



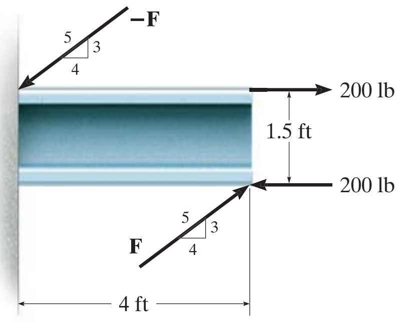
4–83. A device called a rolamite is used in various ways to replace slipping motion with rolling motion. If the belt, which wraps between the rollers, is subjected to a tension of 15 N, determine the reactive forces N of the top and bottom plates on the rollers so that the resultant couple acting on the rollers is equal to zero.

$$\curvearrowleft +\Sigma M_A = 0; \quad 15(50 + 50\sin 30°) - N(50\cos 30°) = 0$$

$N = 26.0$ N **Ans**



***4–84.** Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is 300 lb · ft counterclockwise. Where on the beam does the resultant couple act?

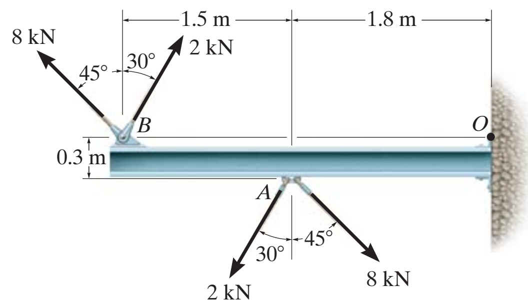


$$\circlearrowleft +(M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$$

$F = 167$ lb **Ans**

Resultant couple can act anywhere. **Ans**

•**4–85.** Determine the resultant couple moment acting on the beam. Solve the problem two ways: (a) sum moments about point O; and (b) sum moments about point A.



(a)

$\circlearrowleft + M_R = \Sigma M_O;$ $M_R = 8\cos 45°(1.8) + 8\sin 45°(0.3) + 2\cos 30°(1.8)$

$- 2\sin 30°(0.3) - 2\cos 30°(3.3) - 8\cos 45°(3.3)$

$M_R = -9.69 \text{ kN}\cdot\text{m} = 9.69 \text{ kN}\cdot\text{m} \circlearrowright$ **Ans**

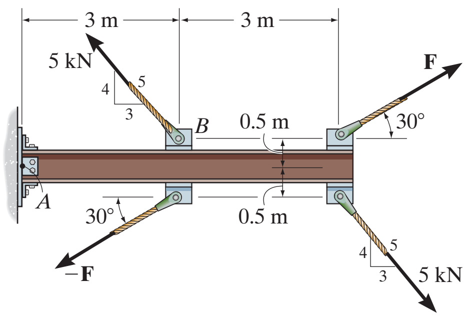
(b)

$\circlearrowleft + M_R = \Sigma M_A;$ $M_R = 8\sin 45°(0.3) - 8\cos 45°(1.5)$

$- 2\cos 30°(1.5) - 2\sin 30°(0.3)$

$= -9.69 \text{ kN}\cdot\text{m} = 9.69 \text{ kN}\cdot\text{m} \circlearrowright$ **Ans**

4–86. Two couples act on the cantilever beam. If $F = 6$ kN, determine the resultant couple moment.



a) By resolving the 6 - kN and 5 - kN couples into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by the 6 - kN and 5 - kN couples, respectively, are given by

$$\circlearrowleft +(M_c)_1 = 6\sin 30°(3) - 6\cos 30°(0.5 + 0.5) = 3.804 \text{ kN}\cdot\text{m}$$

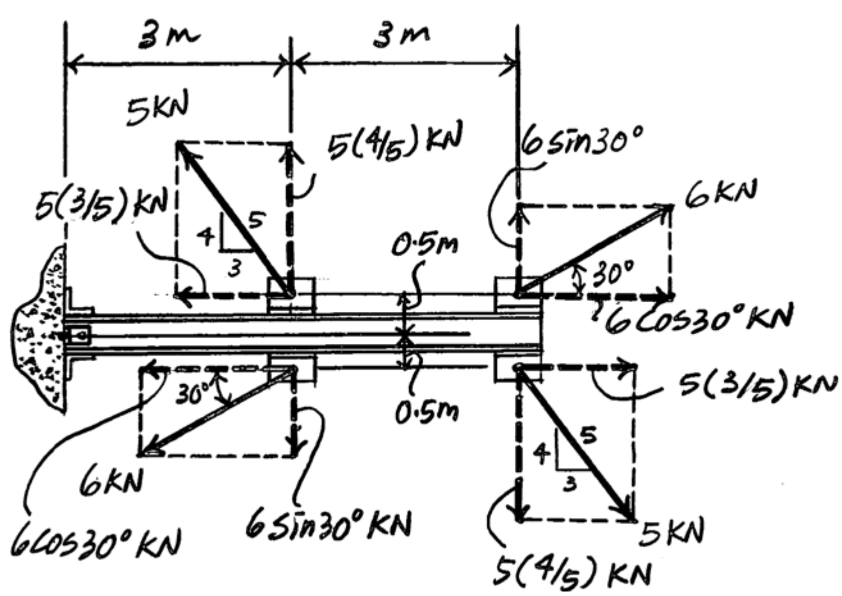
$$\circlearrowleft +(M_c)_2 = 5\left(\frac{3}{5}\right)(0.5 + 0.5) - 5\left(\frac{4}{5}\right)(3) = -9 \text{ kN}\cdot\text{m}$$

Thus, the resultant couple moment can be determined from

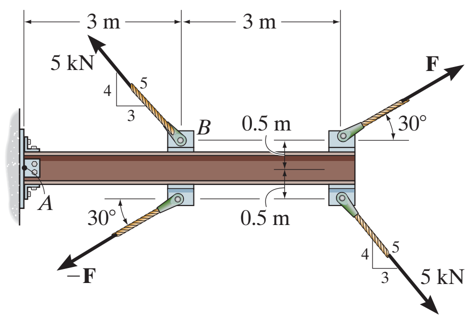
$$(M_c)_R = (M_c)_1 + (M_c)_2$$
$$= 3.804 - 9 = -5.196 \text{ kN}\cdot\text{m} = 5.20 \text{ kN}\cdot\text{m} \text{ (clockwise)} \qquad \textbf{Ans.}$$

b) By resolving the 6 - kN and 5 - kN couples into their x and y components, Fig. a, and summing the moments of these force components about point A, we can write

$$\circlearrowleft +(M_c)_R = \Sigma M_A; \qquad (M_c)_R = 5\left(\frac{3}{5}\right)(0.5) + 5\left(\frac{4}{5}\right)(3) - 6\cos 30°(0.5) - 6\sin 30°(3)$$
$$+ 6\sin 30°(6) - 6\cos 30°(0.5) + 5\left(\frac{3}{5}\right)(0.5) - 5\left(\frac{4}{5}\right)(6)$$
$$= -5.196 \text{ kN}\cdot\text{m} = 5.20 \text{ kN}\cdot\text{m} \text{ (clockwise)} \qquad \textbf{Ans.}$$



4–87. Determine the required magnitude of force **F**, if the resultant couple moment on the beam is to be zero.



By resolving **F** and the 5 - kN couple into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by **F** and the 5 - kN couple, respectively, are given by

$$\circlearrowleft +(M_c)_1 = F\sin 30°(3) - F\cos 30°(1) = 0.6340F$$

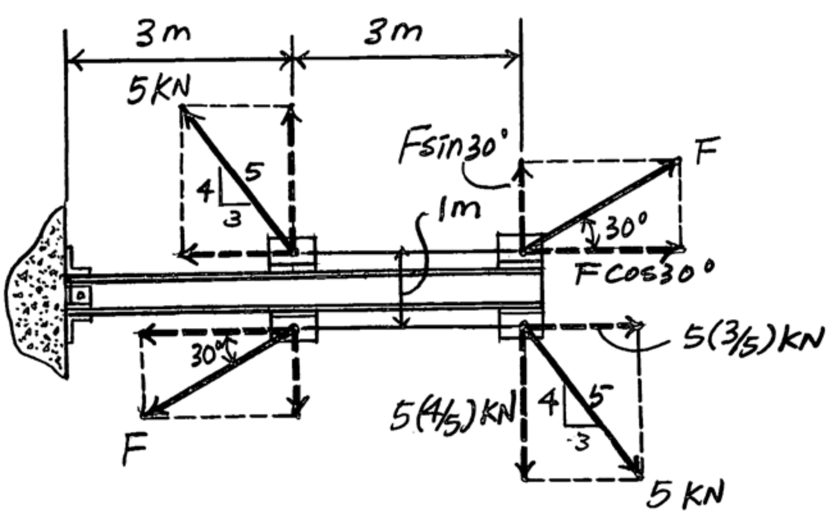
$$\circlearrowleft +(M_c)_2 = 5\left(\frac{3}{5}\right)(1) - 5\left(\frac{4}{5}\right)(3) = -9\text{ kN}\cdot\text{m}$$

The resultant couple moment acting on the beam is required to be zero. Thus,

$$(M_c)_R = (M_c)_1 + (M_c)_2$$

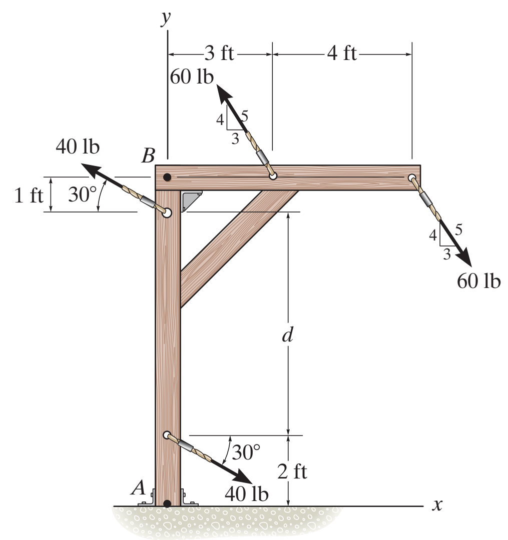
$$0 = 0.6340F - 9$$

$$F = 14.2\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

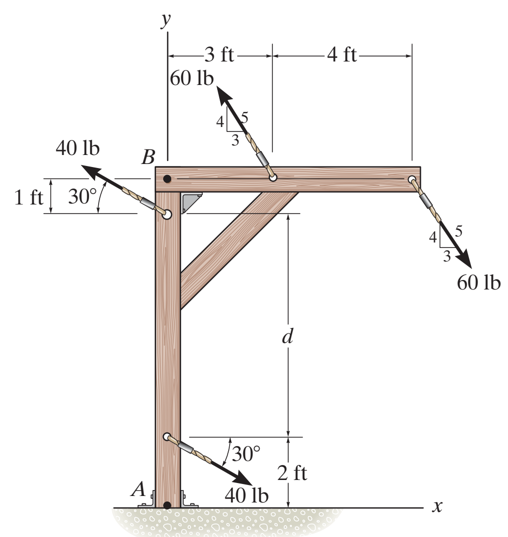
***4–88.** Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the 40-lb couple forces.



$$\circlearrowleft + M_C = 0 = 40\cos 30°(d) - 60\left(\frac{4}{5}\right)(4)$$

$d = 5.54$ ft **Ans**

•4–89. Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point A.



(a)

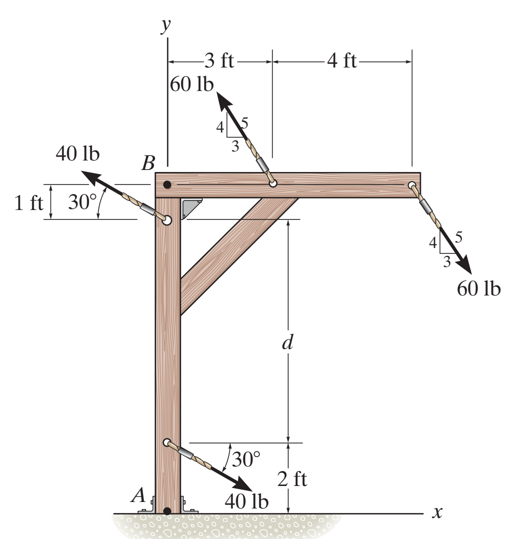
$$\circlearrowleft + M_C = 40\cos 30°(4) - 60\left(\frac{4}{5}\right)(4) = -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \circlearrowright \quad \textbf{Ans}$$

(b)

$$\circlearrowleft + M_C = -40\cos 30°(2) + 40\cos 30°(6) + 60\left(\frac{4}{5}\right)(3) + 60\left(\frac{3}{5}\right)(7) - 60\left(\frac{4}{5}\right)(7) - 60\left(\frac{3}{5}\right)(7)$$

$$= -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \circlearrowright \quad \textbf{Ans}$$

4–90. Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point B.



(a)

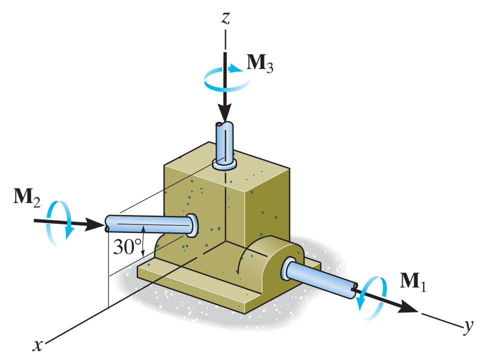
$$\circlearrowleft + M_C = 40\cos 30°(4) - 60\left(\frac{4}{5}\right)(4) = -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \circlearrowright \quad \textbf{Ans}$$

(b)

$$\circlearrowleft + M_C = 40\cos 30°(5) - 40\cos 30°(1) + 60\left(\frac{4}{5}\right)(3) - 60\left(\frac{4}{5}\right)(7)$$

$$= -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \circlearrowright \quad \textbf{Ans}$$

4–91. If $M_1 = 500$ N·m, $M_2 = 600$ N·m, and $M_3 = 450$ N·m, determine the magnitude and coordinate direction angles of the resultant couple moment.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M_1}$, $\mathbf{M_2}$, and $\mathbf{M_3}$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$\mathbf{M_1} = [500\mathbf{j}]$ N · m

$\mathbf{M_2} = 600(-\cos 30°\mathbf{i} - \sin 30°\mathbf{k}) = \{-519.62\mathbf{i} - 300\mathbf{k}\}$ N · m

$\mathbf{M_3} = [-450\mathbf{k}]$ N · m

The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}; \qquad (\mathbf{M}_c)_R = \mathbf{M_1} + \mathbf{M_2} + \mathbf{M_3}$$
$$= 500\mathbf{j} + (-519.62\mathbf{i} - 300\mathbf{k}) + (-450\mathbf{k})$$
$$= [-519.62\mathbf{i} + 500\mathbf{j} - 750\mathbf{k}]\ \text{N} \cdot \text{m}$$

The magnitude of $(\mathbf{M}_c)_R$ is

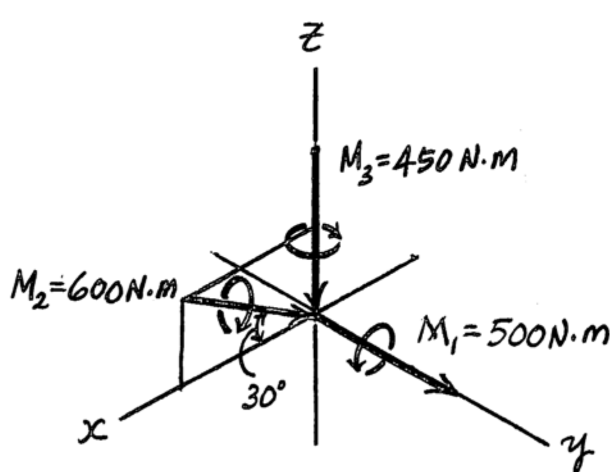
$$(M_c)_R = \sqrt{[(M_c)_R]_x^2 + [(M_c)_R]_y^2 + [(M_c)_R]_z^2}$$
$$= \sqrt{(-519.62)^2 + 500^2 + (-750)^2}$$
$$= 1040.43\ \text{N} \cdot \text{m} = 1.04\ \text{kN} \cdot \text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}_C)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_C)_R]_x}{(M_C)_R}\right) = \cos\left(\frac{-519.62}{1040.43}\right) = 120° \qquad \textbf{Ans.}$$

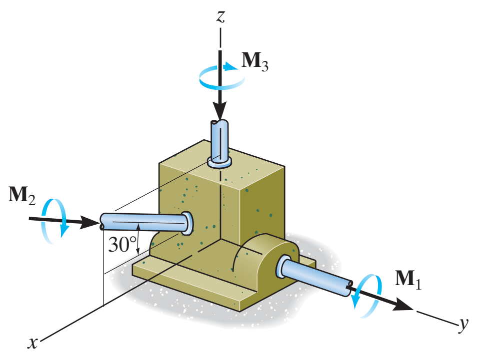
$$\beta = \cos^{-1}\left(\frac{[(M_C)_R]_y}{(M_C)_R}\right) = \cos\left(\frac{500}{1040.43}\right) = 61.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_C)_R]_z}{(M_C)_R}\right) = \cos\left(\frac{-750}{1040.43}\right) = 136° \qquad \textbf{Ans.}$$



(a)

***4–92.** Determine the required magnitude of couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ so that the resultant couple moment is $\mathbf{M}_R = \{-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k}\}\ \mathrm{N\cdot m}$.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$\mathbf{M}_1 = M_1\mathbf{j}$

$\mathbf{M}_2 = M_2(-\cos 30°\,\mathbf{i} - \sin 30°\,\mathbf{k}) = -0.8660M_2\mathbf{i} - 0.5M_2\mathbf{k}$

$\mathbf{M}_3 = -M_3\mathbf{k}$

The resultant couple moment is given by

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};\qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

$$(-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k}) = M_1\mathbf{j} + (-0.8660M_2\mathbf{i} - 0.5M_2\mathbf{k}) + (-M_3\mathbf{k})$$

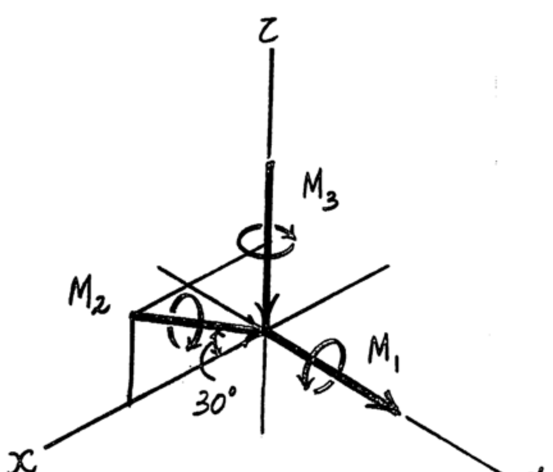
$$-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k} = -0.8660M_2\mathbf{i} + M_1\mathbf{j} - (0.5M_2 + M_3)\mathbf{k}$$

Equating the **i**, **j**, and **k** components yields

$-300 = -0.8660M_2 \qquad M_2 = 346.41\ \mathrm{N\cdot m} = 346\ \mathrm{N\cdot m}$ **Ans.**

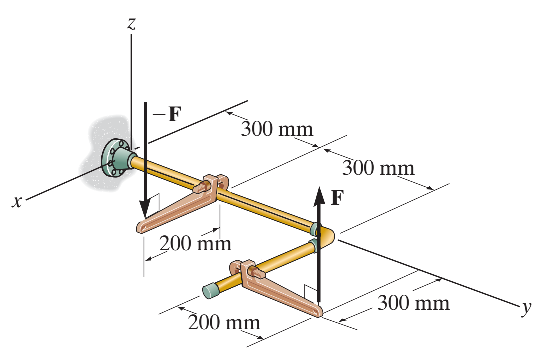
$M_1 = 450\ \mathrm{N\cdot m}$ **Ans.**

$600 = -0.5(346.41) + M_3 \qquad M_3 = 427\ \mathrm{N\cdot m}$ **Ans.**



(a)

•4–93. If $F = 80$ N, determine the magnitude and coordinate direction angles of the couple moment. The pipe assembly lies in the x–y plane.



It is easiest to find the couple moment of **F** by taking the moment of **F** or $-$**F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3-0.2)\mathbf{i} + (0.8-0.3)\mathbf{j} + (0-0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\ \text{m}$$
$$\mathbf{r}_{BA} = (0.2-0.3)\mathbf{i} + (0.3-0.8)\mathbf{j} + (0-0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\ \text{m}$$

The force vectors **F** and $-$**F** can be written as

$\mathbf{F} = \{80\mathbf{k}\}$ N and $-\mathbf{F} = [-80\mathbf{k}]$ N

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\ \text{N}\cdot\text{m}$$

or

$$\mathbf{M}_c = \mathbf{r}_{BA} \times -\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\ \text{N}\cdot\text{m}$$

The magnitude of M_c is given by

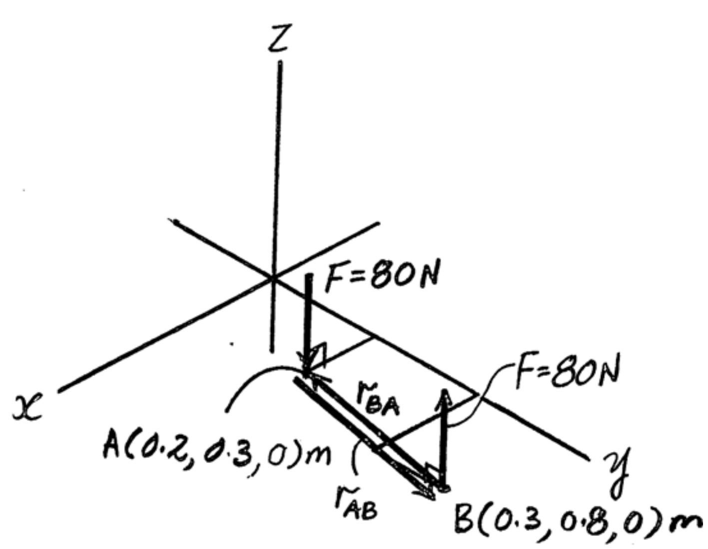
$$M_c = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{40^2 + (-8)^2 + 0^2} = 40.79\ \text{N}\cdot\text{m} = 40.8\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $\mathbf{M}_c$ are

$$\alpha = \cos^{-1}\left(\frac{M_x}{M}\right) = \cos\left(\frac{40}{40.79}\right) = 11.3° \qquad \textbf{Ans.}$$

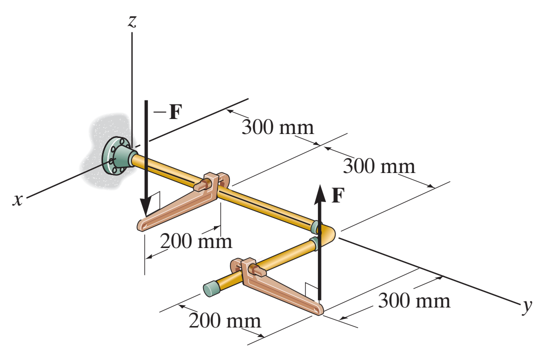
$$\beta = \cos^{-1}\left(\frac{M_y}{M}\right) = \cos\left(\frac{-8}{40.79}\right) = 101° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{M_z}{M}\right) = \cos\left(\frac{0}{40.79}\right) = 90° \qquad \textbf{Ans.}$$



(a)

4–94. If the magnitude of the couple moment acting on the pipe assembly is $50\ \mathrm{N \cdot m}$, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the x–y plane.



It is easiest to find the couple moment of **F** by taking the moment of either **F** or **−F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3-0.2)\mathbf{i} + (0.8-0.3)\mathbf{j} + (0-0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\ \mathrm{m}$$

$$\mathbf{r}_{BA} = (0.2-0.3)\mathbf{i} + (0.3-0.8)\mathbf{j} + (0-0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\ \mathrm{m}$$

The force vectors **F** and **−F** can be written as

$$\mathbf{F} = \{F\mathbf{k}\}\ \mathrm{N} \text{ and } -\mathbf{F} = [-F\mathbf{k}]\ \mathrm{N}$$

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = 0.5F\mathbf{i} - 0.1F\mathbf{j}$$

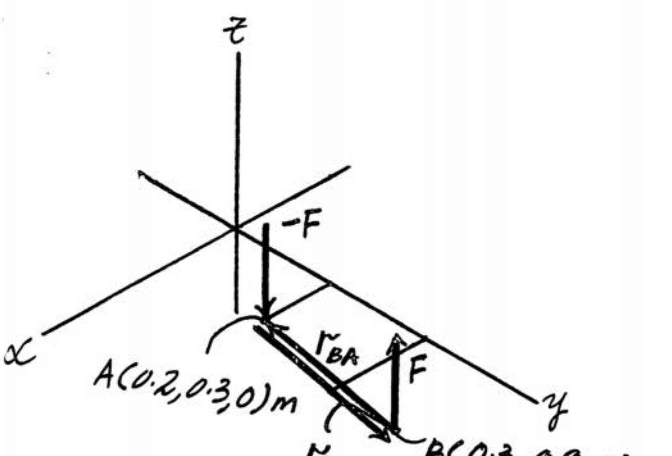
The magnitude of M_c is given by

$$M_c = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(0.5F)^2 + (0.1F)^2 + 0^2} = 0.5099F$$

Since M_c is required to equal $50\ \mathrm{N \cdot m}$,

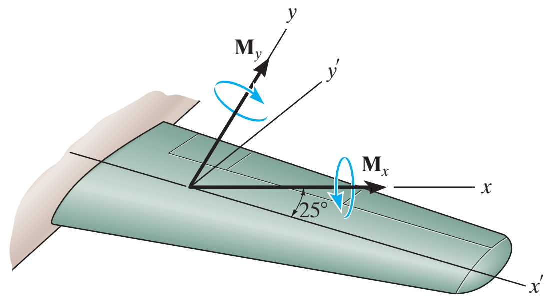
$$50 = 0.5099F$$

$$F = 98.1\ \mathrm{N} \qquad \textbf{Ans.}$$



(a)

4–95. From load calculations it is determined that the wing is subjected to couple moments $M_x = 17$ kip·ft and $M_y = 25$ kip·ft. Determine the resultant couple moments created about the x' and y' axes. The axes all lie in the same horizontal plane.

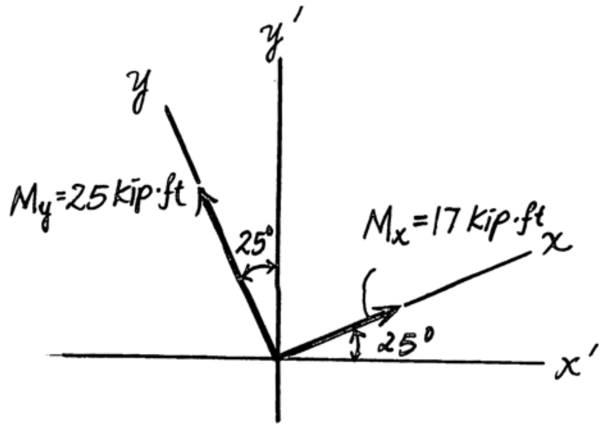


$(M_R)_{x'} = \Sigma M_{x'};\qquad (M_R)_{x'} = 17\cos 25° - 25\sin 25°$

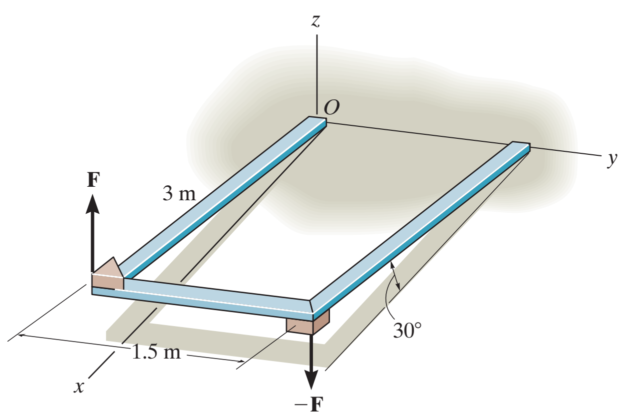
$= 4.84$ kip · ft **Ans**

$(M_R)_{y'} = \Sigma M_{y'};\qquad (M_R)_{y'} = 17\sin 25° + 25\cos 25°$

$= 29.8$ kip · ft **Ans**



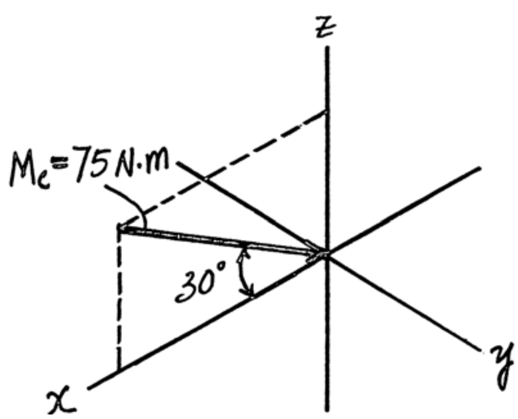
***4–96.** Express the moment of the couple acting on the frame in Cartesian vector form. The forces are applied perpendicular to the frame. What is the magnitude of the couple moment? Take $F = 50$ N.



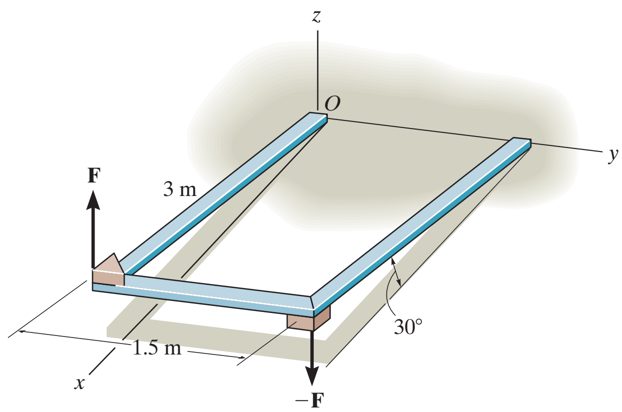
$M_C = 50(1.5) = 75$ N · m **Ans**

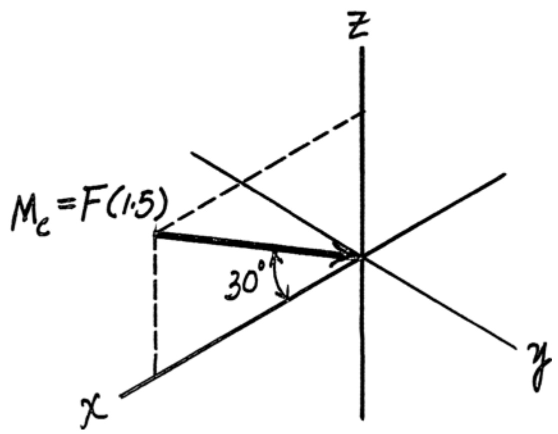
$\mathbf{M}_C = -75(\cos 30°\,\mathbf{i} + \cos 60°\,\mathbf{k})$

$= \{-65.0\mathbf{i} - 37.5\mathbf{k}\}$ N · m **Ans**



•4–97. In order to turn over the frame, a couple moment is applied as shown. If the component of this couple moment along the x axis is $\mathbf{M}_x = \{-20\mathbf{i}\}$ N·m, determine the magnitude F of the couple forces.





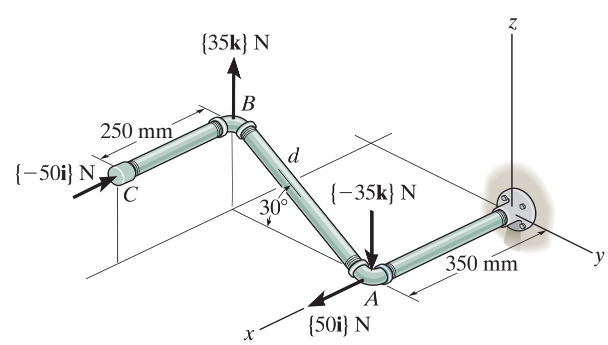
$M_C = F(1.5)$

Thus

$20 = F(1.5)\cos 30°$

$F = 15.4$ N **Ans**

4–98. Determine the resultant couple moment of the two couples that act on the pipe assembly. The distance from A to B is $d = 400$ mm. Express the result as a Cartesian vector.



Vector Analysis

Position Vector :

$$\mathbf{r}_{AB} = \{(0.35 - 0.35)\mathbf{i} + (-0.4\cos 30° - 0)\mathbf{j} + (0.4\sin 30° - 0)\mathbf{k}\} \text{ m}$$
$$= \{-0.3464\mathbf{j} + 0.20\mathbf{k}\} \text{ m}$$

Couple Moments : With $\mathbf{F}_1 = \{35\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-50\mathbf{i}\}$ N, applying Eq. 4 – 15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ 0 & 0 & 35 \end{vmatrix} = \{-12.12\mathbf{i}\} \text{ N}\cdot\text{m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ -50 & 0 & 0 \end{vmatrix} = \{-10.0\mathbf{j} - 17.32\mathbf{k}\} \text{ N}\cdot\text{m}$$

Resultant Couple Moment :

$$\mathbf{M}_R = \Sigma\mathbf{M}; \quad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2$$
$$= \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\} \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

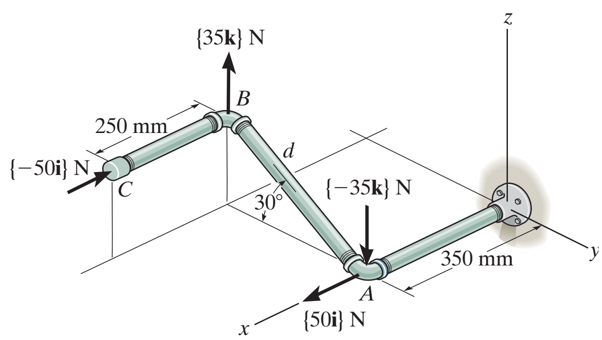
Scalar Analysis : Summing moments about x, y and z axes, we have

$$(M_R)_x = \Sigma M_x; \quad (M_R)_x = -35(0.4\cos 30°) = -12.12 \text{ N}\cdot\text{m}$$
$$(M_R)_y = \Sigma M_y; \quad (M_R)_y = -50(0.4\sin 30°) = -10.0 \text{ N}\cdot\text{m}$$
$$(M_R)_z = \Sigma M_z; \quad (M_R)_z = -50(0.4\cos 30°) = -17.32 \text{ N}\cdot\text{m}$$

Express $\mathbf{M}_R$ as a Cartesian vector, we have

$$\mathbf{M}_R = \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\} \text{ N}\cdot\text{m}$$

4–99. Determine the distance d between A and B so that the resultant couple moment has a magnitude of $M_R = 20\ \text{N}\cdot\text{m}$.



Position Vector :

$$\mathbf{r}_{AB} = \{(0.35-0.35)\mathbf{i} + (-d\cos 30° - 0)\mathbf{j} + (d\sin 30° - 0)\mathbf{k}\}\ \text{m}$$
$$= \{-0.8660d\ \mathbf{j} + 0.50d\ \mathbf{k}\}\ \text{m}$$

Couple Moments : With $\mathbf{F}_1 = \{35\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-50\mathbf{i}\}$ N, applying Eq. 4–15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ 0 & 0 & 35 \end{vmatrix} = \{-30.31d\ \mathbf{i}\}\ \text{N}\cdot\text{m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ -50 & 0 & 0 \end{vmatrix} = \{-25.0d\ \mathbf{j} - 43.30d\ \mathbf{k}\}\ \text{N}\cdot\text{m}$$

Resultant Couple Moment :

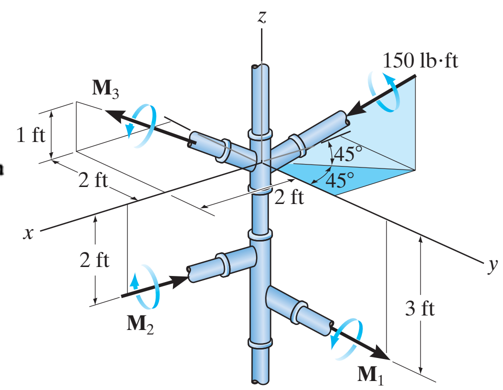
$$\mathbf{M}_R = \Sigma\mathbf{M}; \quad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2$$
$$= \{-30.31d\ \mathbf{i} - 25.0d\ \mathbf{j} - 43.30d\ \mathbf{k}\}\ \text{N}\cdot\text{m}$$

The magnitude of $\mathbf{M}_R$ is 20 N · m thus

$$20 = \sqrt{(-30.31d)^2 + (-25.0d)^2 + (43.30d)^2}$$

$$d = 0.3421\ \text{m} = 342\ \text{mm} \qquad \textbf{Ans}$$

***4–100.** If $M_1 = 180$ lb·ft, $M_2 = 90$ lb·ft, and $M_3 = 120$ lb·ft, determine the magnitude and coordinate direction angles of the resultant couple moment.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, and $\mathbf{M}_4$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$$\mathbf{M}_1 = [180\mathbf{j}]\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_2 = [-90\mathbf{i}]\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_3 = M_3\mathbf{u} = 120\left[\frac{(2-0)\mathbf{i}+(-2-0)\mathbf{j}+(1+0)\mathbf{k}}{\sqrt{(2-0)^2+(-2-0)^2+(1-0)^2}}\right] = [80\mathbf{i}-80\mathbf{j}+40\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_4 = 150[\cos 45^\circ \sin 45^\circ\,\mathbf{i} - \cos 45^\circ \cos 45^\circ\,\mathbf{j} - \sin 45^\circ\,\mathbf{k}] = [75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

The resultant couple moment is given by

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$

$$\begin{aligned}(\mathbf{M}_c)_R &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4\\ &= 180\mathbf{j} - 90\mathbf{i} + (80\mathbf{i} - 80\mathbf{j} + 40\mathbf{k}) + (75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k})\\ &= [65\mathbf{i} + 25\mathbf{j} - 66.07\mathbf{k}]\ \text{lb}\cdot\text{ft}\end{aligned}$$

The magnitude of $(\mathbf{M}_c)_R$ is

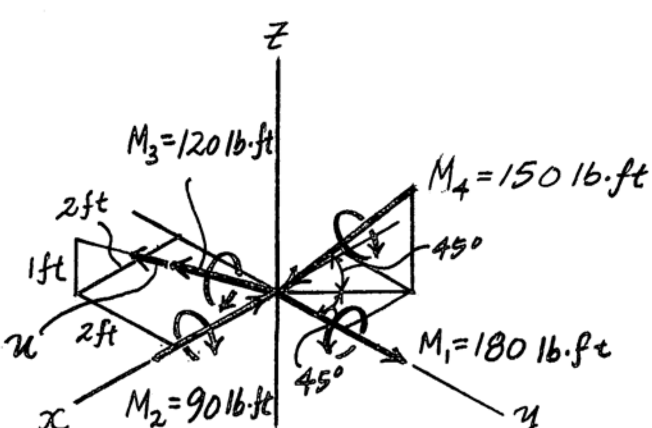
$$\begin{aligned}(M_c)_R &= \sqrt{[(M_c)_R]_x^{\,2} + [(M_c)_R]_y^{\,2} + [(M_c)_R]_z^{\,2}}\\ &= \sqrt{(65)^2 + (25)^2 + (-66.07)^2}\\ &= 95.99\ \text{lb}\cdot\text{ft} = 96.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans.}\end{aligned}$$

The coordinate angles of $(\mathbf{M}_c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{65}{95.99}\right) = 47.4^\circ \qquad \textbf{Ans.}$$

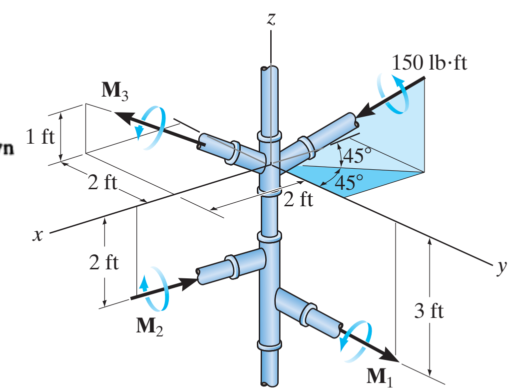
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{25}{95.99}\right) = 74.9^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-66.07}{95.99}\right) = 133^\circ \qquad \textbf{Ans.}$$



(a)

•4–101. Determine the magnitudes of couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ so that the resultant couple moment is zero.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, and $\mathbf{M}_4$ acting on the gear deducer can be simplified, as shown in Fig. a. Expressing each couple moment in Cartesian vector form,

$$\mathbf{M}_1 = M_1\mathbf{j}$$

$$\mathbf{M}_2 = -M_2\mathbf{i}$$

$$\mathbf{M}_3 = M_3\mathbf{u} = M_3\left[\frac{(2-0)\mathbf{i}+(-2-0)\mathbf{j}+(1+0)\mathbf{k}}{\sqrt{(2-0)^2+(-2-0)^2+(1-0)^2}}\right] = \frac{2}{3}M_3\mathbf{i} - \frac{2}{3}M_3\mathbf{j} + \frac{1}{3}M_3\mathbf{k}$$

$$\mathbf{M}_4 = 150[\cos 45^\circ \sin 45^\circ\mathbf{i} - \cos 45^\circ \cos 45^\circ\mathbf{j} - \sin 45^\circ\mathbf{k}] = [75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

The resultant couple moment is required to be zero. Thus,

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$

$$0 = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

$$0 = M_1\mathbf{j} + (-M_2\mathbf{i}) + \left(\frac{2}{3}M_3\mathbf{i} - \frac{2}{3}M_3\mathbf{j} + \frac{1}{3}M_3\mathbf{k}\right) + (75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k})$$

$$0 = \left(-M_2 + \frac{2}{3}M_3 + 75\right)\mathbf{i} + \left(M_1 - \frac{2}{3}M_3 - 75\right)\mathbf{j} + \left(\frac{1}{3}M_3 - 106.07\right)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$0 = -M_2 + \frac{2}{3}M_3 + 75 \qquad (1)$$

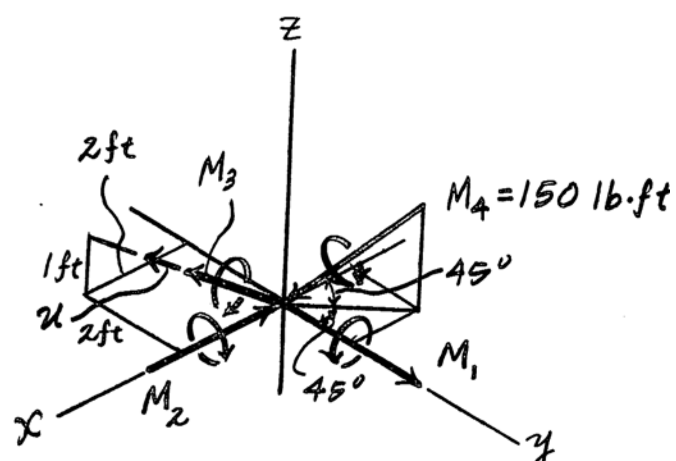
$$0 = M_1 - \frac{2}{3}M_3 - 75 \qquad (2)$$

$$0 = \frac{1}{3}M_3 - 106.07 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

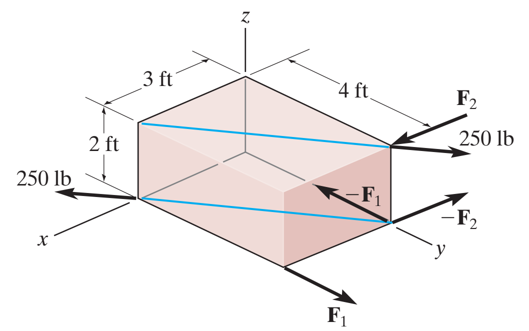
$M_3 = 318\ \text{lb}\cdot\text{ft}$ **Ans.**

$M_1 = M_2 = 287\ \text{lb}\cdot\text{ft}$ **Ans.**



(a)

4–102. If $F_1 = 100$ lb and $F_2 = 200$ lb, determine the magnitude and coordinate direction angles of the resultant couple moment.



Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = [-2\mathbf{k}]$ ft $\quad \mathbf{r}_2 = [2\mathbf{k}]$ ft $\quad \mathbf{r}_3 = [2\mathbf{k}]$ ft

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = [100\mathbf{j}]$ lb $\quad \mathbf{F}_2 = [200\mathbf{i}]$ lb

$$\mathbf{F}_3 = F_3\mathbf{u} = 250\left[\frac{(0-3)\mathbf{i}+(4-0)\mathbf{j}+(2-2)\mathbf{k}}{\sqrt{(0-3)^2+(4-0)^2+(2-2)^2}}\right] = [-150\mathbf{i}+200\mathbf{j}]\text{ lb}$$

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (-2\mathbf{k})\times(100\mathbf{j}) = [200\mathbf{i}]\text{ lb}\cdot\text{ft}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (2\mathbf{k})\times(200\mathbf{i}) = [400\mathbf{j}]\text{ lb}\cdot\text{ft}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (2\mathbf{k})\times(-150\mathbf{i}+200\mathbf{j}) = [-400\mathbf{i}-300\mathbf{j}]\text{ lb}\cdot\text{ft}$$

Resultant Moment: The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$
$$= (200\mathbf{i}) + (400\mathbf{j}) + (-400\mathbf{i} - 300\mathbf{j})$$
$$= [-200\mathbf{i} + 100\mathbf{j}]\text{ lb}\cdot\text{ft}$$

The magnitude of the couple moment is

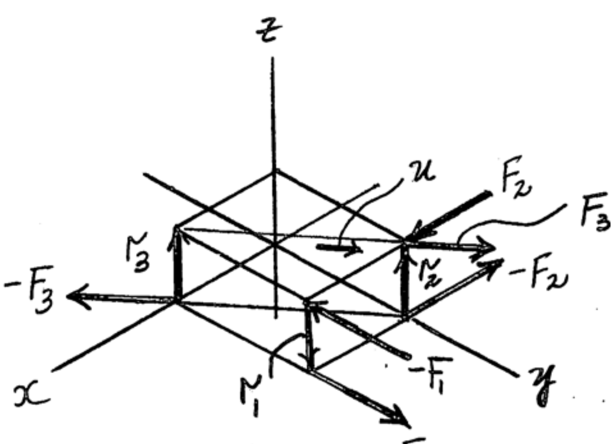
$$(M_c)_R = \sqrt{[(M_c)_R]_x^2 + [(M_c)_R]_y^2 + [(M_c)_R]_z^2}$$
$$= \sqrt{(-200)^2 + (100)^2 + (0)^2}$$
$$= 223.61\text{ N}\cdot\text{m} = 224\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{-200}{223.61}\right) = 153° \qquad \textbf{Ans.}$$

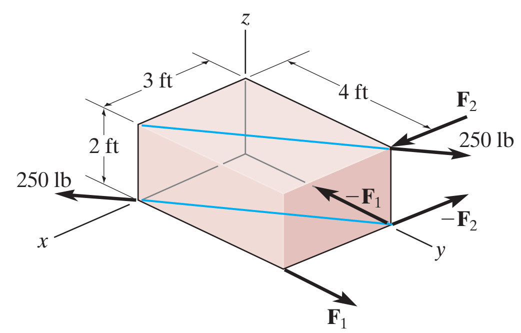
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{100}{223.61}\right) = 63.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{0}{223.61}\right) = 90° \qquad \textbf{Ans.}$$



(a)

4–103. Determine the magnitude of couple forces $\mathbf{F}_1$ and $\mathbf{F}_2$ so that the resultant couple moment acting on the block is zero.



Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, Fig. a, must be determined first.

$\mathbf{r}_1 = [-2\mathbf{k}]$ ft $\qquad \mathbf{r}_2 = [2\mathbf{k}]$ ft $\qquad \mathbf{r}_3 = [2\mathbf{k}]$ ft

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = F_1\mathbf{j} \qquad \mathbf{F}_2 = F_2\mathbf{i}$

$$\mathbf{F}_3 = F_3\mathbf{u} = 250\left[\frac{(0-3)\mathbf{i}+(4-0)\mathbf{j}+(2-2)\mathbf{k}}{\sqrt{(0-3)^2+(4-0)^2+(2-2)^2}}\right] = [-150\mathbf{i}+200\mathbf{j}]\text{ lb}$$

Thus,

$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (-2\mathbf{k})\times(F_1\mathbf{j}) = 2F_1\mathbf{i}$

$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (2\mathbf{k})\times(F_2\mathbf{i}) = 2F_2\mathbf{j}$

$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (2\mathbf{k})\times(-150\mathbf{i}+200\mathbf{j}) = [-400\mathbf{i}-300\mathbf{j}]\text{ lb}\cdot\text{ft}$

Resultant Moment: Since the resultant couple moment is required to be equal to zero,

$(\mathbf{M}_c)_R = \Sigma\mathbf{M}; \qquad \mathbf{0} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

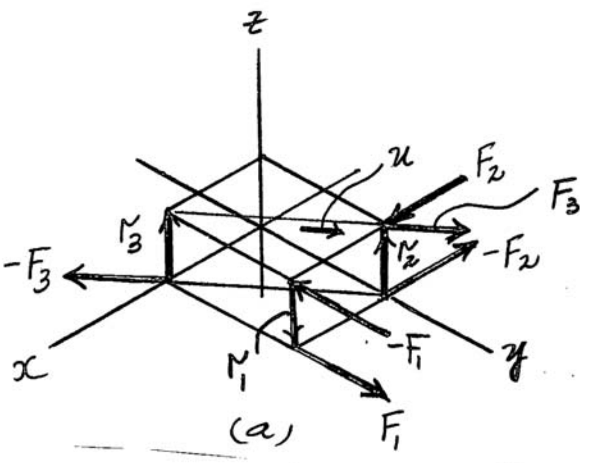
$\mathbf{0} = (2F_1\mathbf{i}) + (2F_2\mathbf{j}) + (-400\mathbf{i} - 300\mathbf{j})$

$\mathbf{0} = (2F_1 - 400)\mathbf{i} + (2F_2 - 300)\mathbf{j}$

Equating the **i**, **j**, and **k** components yields

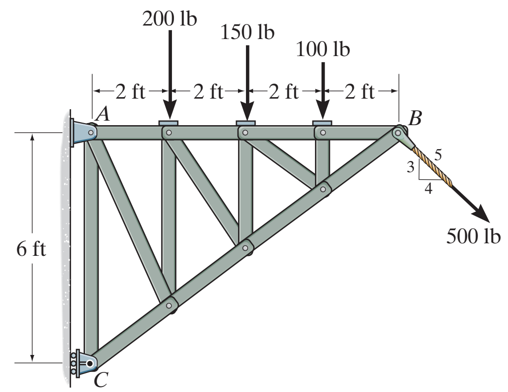
$0 = 2F_1 - 400 \qquad F_1 = 200\text{ lb}$ **Ans.**

$0 = 2F_2 - 300 \qquad F_2 = 150\text{ lb}$ **Ans.**



(a)

***4–104.** Replace the force system acting on the truss by a resultant force and couple moment at point *C*.



Equivalent Resultant Force: The 500-lb force is resolved into its x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\xrightarrow{+}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 500\left(\frac{4}{5}\right) = 400\text{ lb} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -200 - 150 - 100 - 500\left(\frac{3}{5}\right) = -750\text{ lb} = 750\text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{400^2 + 750^2} = 850\text{ lb} \qquad \textbf{Ans.}$$

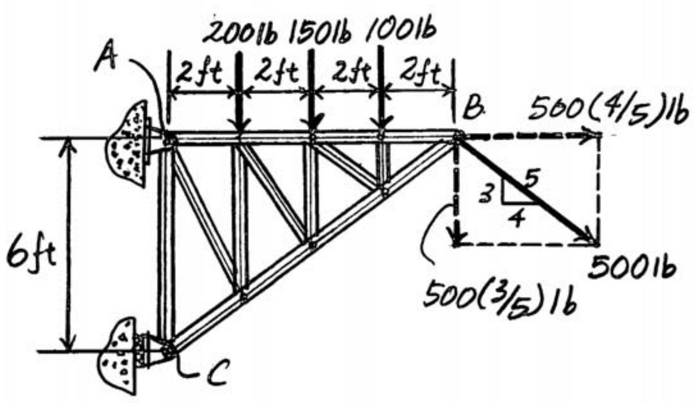
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{750}{400}\right] = 61.93° = 61.9° \qquad \textbf{Ans.}$$

Equivalent Couple Moment: Summing the moment of the forces and force components, Fig. a, algebraically about point C,

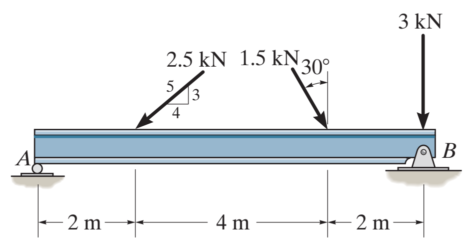
$$\circlearrowleft + (M_R)_C = \Sigma M_C; \qquad (M_R)_C = -200(2) - 150(4) - 100(6) - 500\left(\frac{3}{5}\right)(8) - 500\left(\frac{4}{5}\right)(6)$$

$$= -6400\text{ lb}\cdot\text{ft} = 6.40\text{ kip}\cdot\text{ft (clockwise)} \qquad \textbf{Ans.}$$



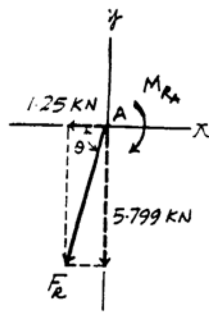
(a)

•4–105. Replace the force system acting on the beam by an equivalent force and couple moment at point A.



$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 1.5\sin 30° - 2.5\left(\frac{4}{5}\right)$$
$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -1.5\cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$
$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

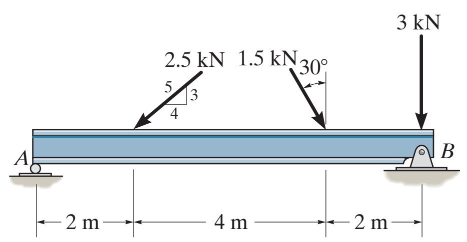
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans}$$

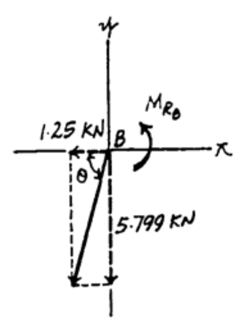
$$\curvearrowleft + M_{R_A} = \Sigma M_A; \quad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5\cos 30°(6) - 3(8)$$
$$= -34.8 \text{ kN}\cdot\text{m} = 34.8 \text{ kN}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

4–106. Replace the force system acting on the beam by an equivalent force and couple moment at point B.



$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 1.5\sin 30° - 2.5\left(\frac{4}{5}\right)$$
$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -1.5\cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$
$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

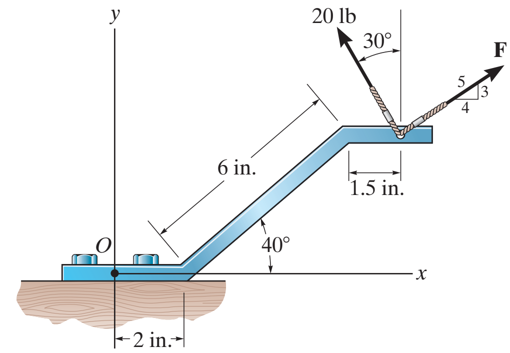
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans}$$

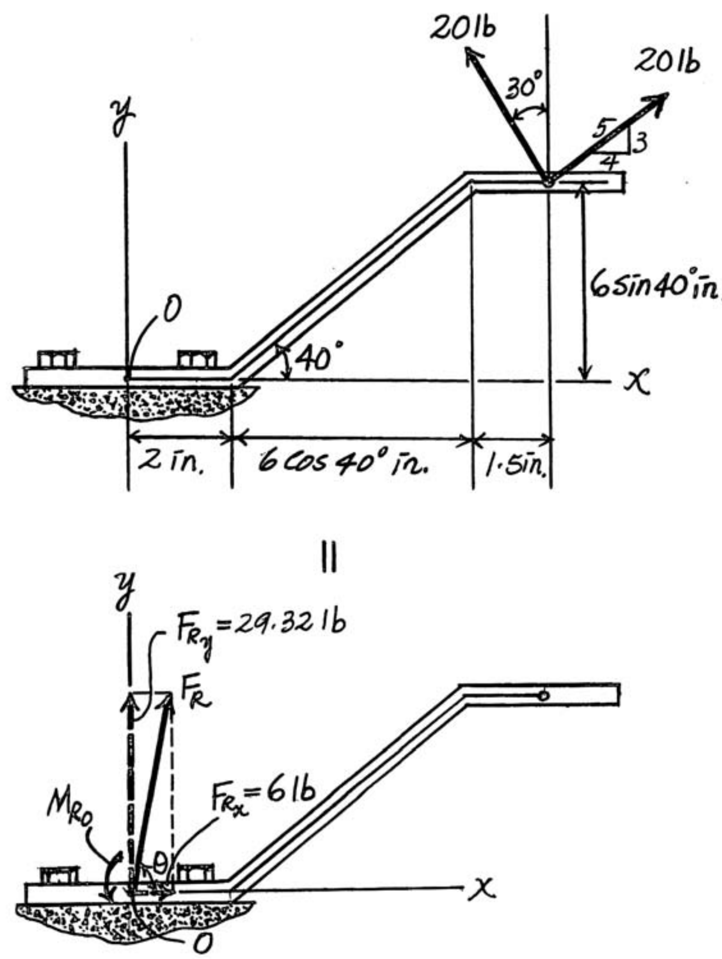
and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans}$$

$$\curvearrowleft + M_{R_B} = \Sigma M_B; \quad M_{R_B} = 1.5\cos 30°(2) + 2.5\left(\frac{3}{5}\right)(6)$$
$$= 11.6 \text{ kN}\cdot\text{m} \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

4–107. Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 20$ lb.





$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = \frac{4}{5}(20) - 20\sin 30° = 6\text{ lb}$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = 20\cos 30° + \frac{3}{5}(20) = 29.32\text{ lb}$

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{6^2 + (29.32)^2} = 29.9\text{ lb} \qquad \textbf{Ans}$$

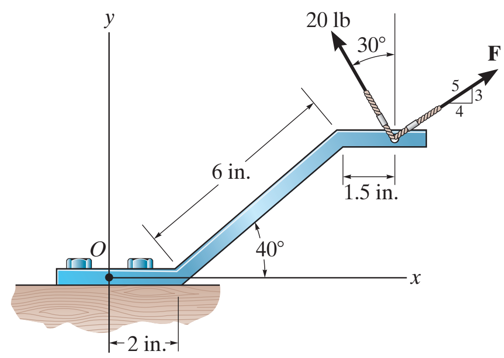
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{29.32}{6}\right) = 78.4° \angle \qquad \textbf{Ans}$$

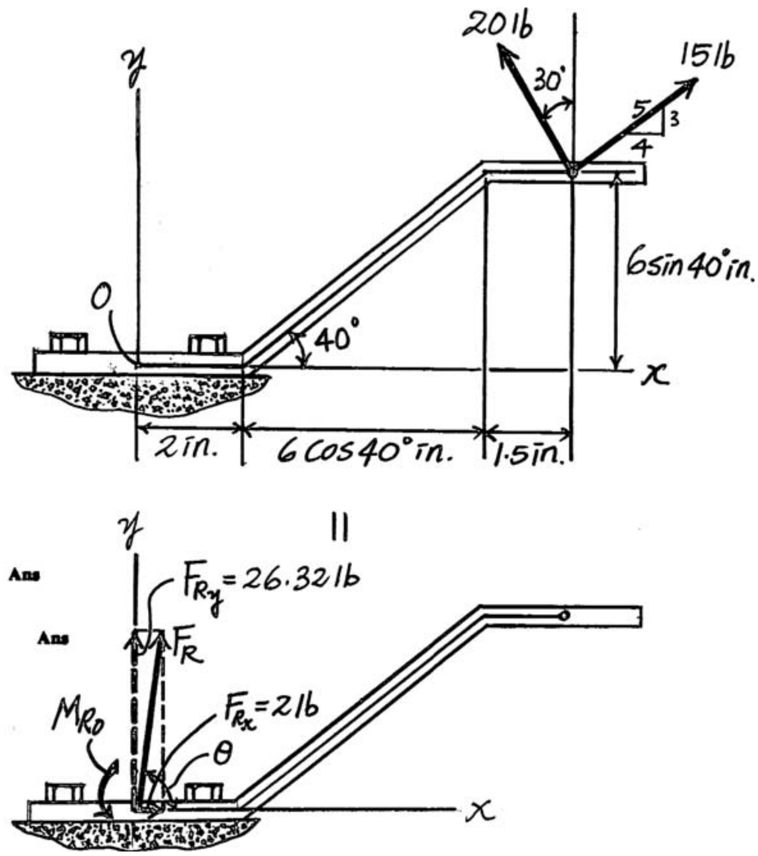
$\curvearrowleft + M_{R_O} = \Sigma M_O;\quad M_{R_O} = 20\sin 30°(6\sin 40°) + 20\cos 30°(3.5 + 6\cos 40°)$

$$-\frac{4}{5}(20)(6\sin 40°) + \frac{3}{5}(20)(3.5 + 6\cos 40°)$$

$$= 214\text{ lb}\cdot\text{in.} \curvearrowright \qquad \textbf{Ans}$$

***4–108.** Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 15$ lb.





$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = \frac{4}{5}(15) - 20 \sin 30° = 2 \text{ lb}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 20 \cos 30° + \frac{3}{5}(15) = 26.32 \text{ lb}$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{2^2 + 26.32^2} = 26.4 \text{ lb} \qquad \textbf{Ans}$$

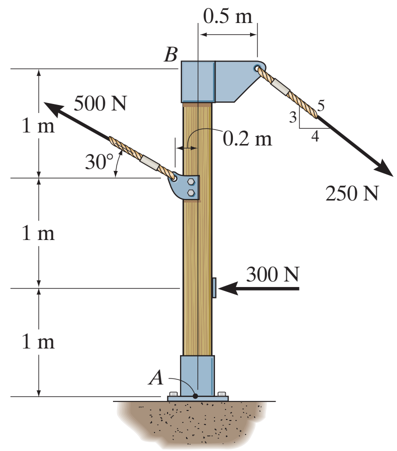
$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\left(\frac{26.32}{2}\right) = 85.7° \qquad \textbf{Ans}$$

$\curvearrowleft + M_{RO} = \Sigma M_O;\quad M_{RO} = 20 \sin 30°(6 \sin 40°) + 20 \cos 30°(3.5 + 6 \cos 40°)$

$$- \frac{4}{5}(15)(6 \sin 40°) + \frac{3}{5}(15)(3.5 + 6 \cos 40°)$$

$$= 205 \text{ lb} \cdot \text{in.} \qquad \textbf{Ans}$$

•4–109. Replace the force system acting on the post by a resultant force and couple moment at point A.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 500\sin 30° - 250\left(\frac{3}{5}\right) = 100\text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N}$$

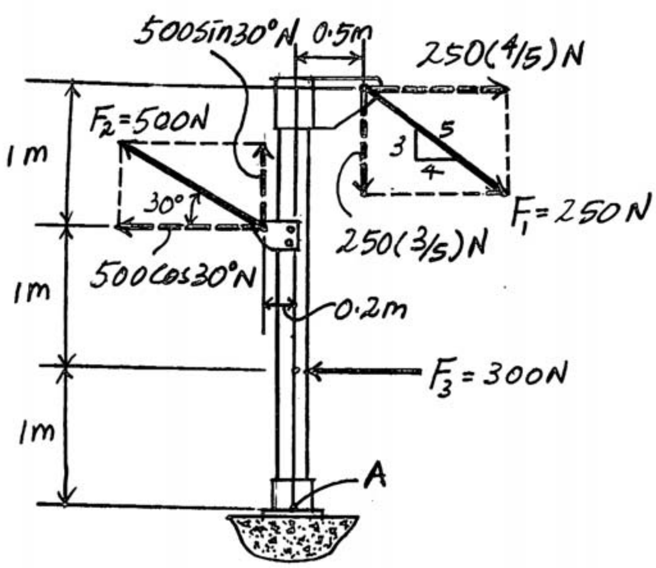
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a, and summing the moments of the force components algebraically about point A,

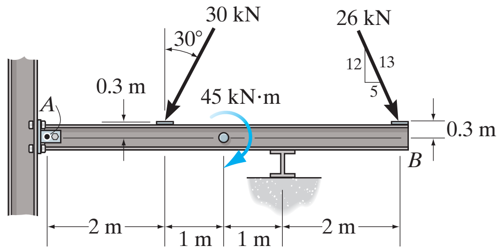
$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_A = 500\cos 30°(2) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$= 441.02\text{ N}\cdot\text{m} = 441\text{ N}\cdot\text{m (counterclockwise)} \qquad \textbf{Ans.}$$



(a)

4–110. Replace the force and couple moment system acting on the overhang beam by a resultant force and couple moment at point A.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 26\left(\frac{5}{13}\right) - 30\sin 30° = -5\text{ kN} = 5\text{ kN} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -26\left(\frac{12}{13}\right) - 30\cos 30° = -49.98\text{ kN} = 49.98\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{5^2 + 49.98^2} = 50.23\text{ kN} = 50.2\text{ kN} \qquad \textbf{Ans.}$$

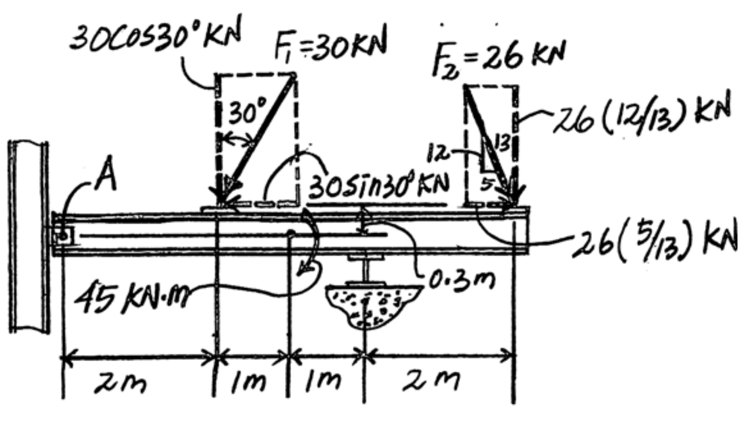
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{49.98}{5}\right] = 84.29° = 84.3° \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

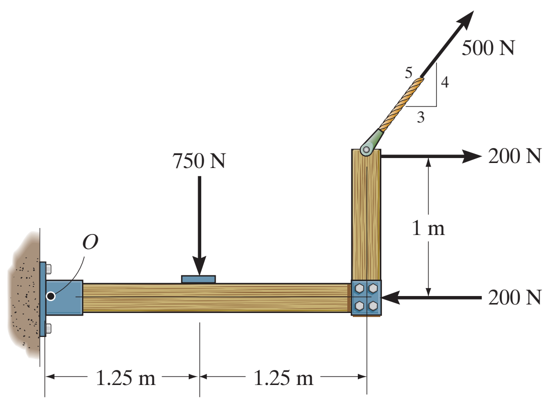
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad (M_R)_A = 30\sin 30°(0.3) - 30\cos 30°(2) - 26\left(\frac{5}{13}\right)(0.3) - 26\left(\frac{12}{13}\right)(6) - 45$$

$$= -239.46\text{ kN}\cdot\text{m} = 239\text{ kN}\cdot\text{m (clockwise)} \qquad \textbf{Ans.}$$



(a)

4–111. Replace the force system by a resultant force and couple moment at point O.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 200 - 200 + 500\left(\frac{3}{5}\right) = 300\text{ N} \rightarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -750 + 500\left(\frac{4}{5}\right) = -350\text{ N} = 350\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{300^2 + 350^2} = 461.0\text{ N} = 461\text{ N} \qquad \textbf{Ans.}$$

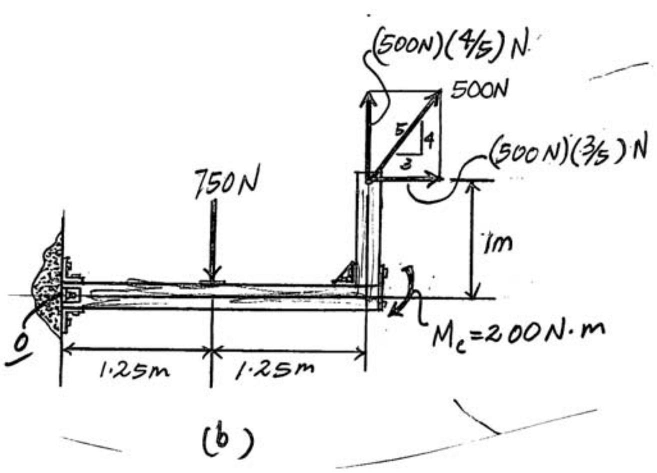
The angle θ of $\mathbf{F}_R$ is

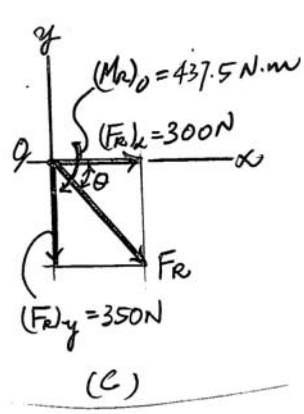
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{350}{300}\right] = 49.4° \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point O,

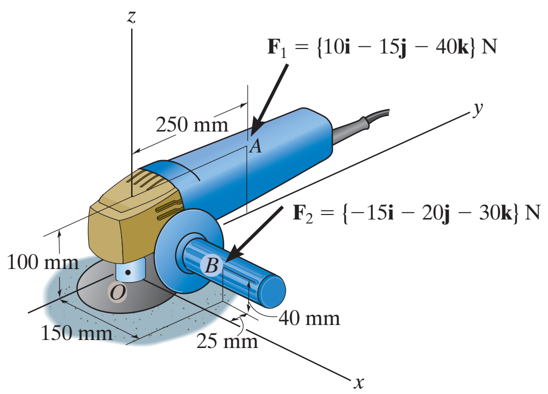
$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_O = -750(1.25) - 200(1) + 500\left(\frac{4}{5}\right)(2.50) - 500\left(\frac{3}{5}\right)(1)$$

$$= -438\text{ N}\cdot\text{m} = 438\text{ N}\cdot\text{m (clockwise)} \qquad \textbf{Ans.}$$





*4–112. Replace the two forces acting on the grinder by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



Equivalent Resultant Force: The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (10\mathbf{i} - 15\mathbf{j} - 40\mathbf{k}) + (-15\mathbf{i} - 20\mathbf{j} - 30\mathbf{k})$$
$$= [-5\mathbf{i} - 35\mathbf{j} - 70\mathbf{k}]\,\mathrm{N}\qquad \textbf{Ans.}$$

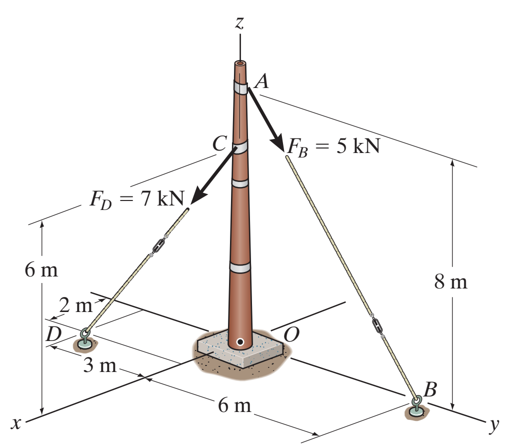
Equivalent Couple Moment: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$ are

$$\mathbf{r}_{OA} = (0-0)\mathbf{i} + (0.25-0)\mathbf{j} + (0.1-0)\mathbf{k} = [0.25\mathbf{j} + 0.1\mathbf{k}]\,\mathrm{m}$$
$$\mathbf{r}_{OB} = (0.15-0)\mathbf{i} + (0.25-0)\mathbf{j} + (0.04-0)\mathbf{k} = [0.15\mathbf{i} + 0.025\mathbf{j} + 0.04\mathbf{k}]\,\mathrm{m}$$

Thus, the resultant couple moment about point O is given by

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OA}\times\mathbf{F}_1 + \mathbf{r}_{OB}\times\mathbf{F}_2$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.25 & 0.1 \\ 10 & -15 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.025 & 0.04 \\ -15 & -20 & -30 \end{vmatrix}$$
$$= [-8.45\mathbf{i} + 4.90\mathbf{j} - 5.125\mathbf{k}]\,\mathrm{N}\cdot\mathrm{m}\qquad \textbf{Ans.}$$

•4–113. Replace the two forces acting on the post by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



Equivalent Resultant Force: The forces $\mathbf{F}_B$ and $\mathbf{F}_D$, Fig. *a*, expressed in Cartesian vector form can be written as

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 5\left[\frac{(0-0)\mathbf{i}+(6-0)\mathbf{j}+(0-8)\mathbf{k}}{\sqrt{(0-0)^2+(6-0)^2+(0-8)^2}}\right] = [3\mathbf{j}-4\mathbf{k}]\text{kN}$$

$$\mathbf{F}_D = F_D\mathbf{u}_{CD} = 7\left[\frac{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(2-0)^2+(-3-0)^2+(0-6)^2}}\right] = [2\mathbf{i}-3\mathbf{j}-6\mathbf{k}]\text{kN}$$

The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_D$$
$$= (3\mathbf{j}-4\mathbf{k}) + (2\mathbf{i}-3\mathbf{j}-6\mathbf{k})$$
$$= [2\mathbf{i}-10\mathbf{k}]\text{kN} \qquad \textbf{Ans.}$$

Equivalent Resultant Force: The position vectors $\mathbf{r}_{OB}$ and $\mathbf{r}_{OC}$ are

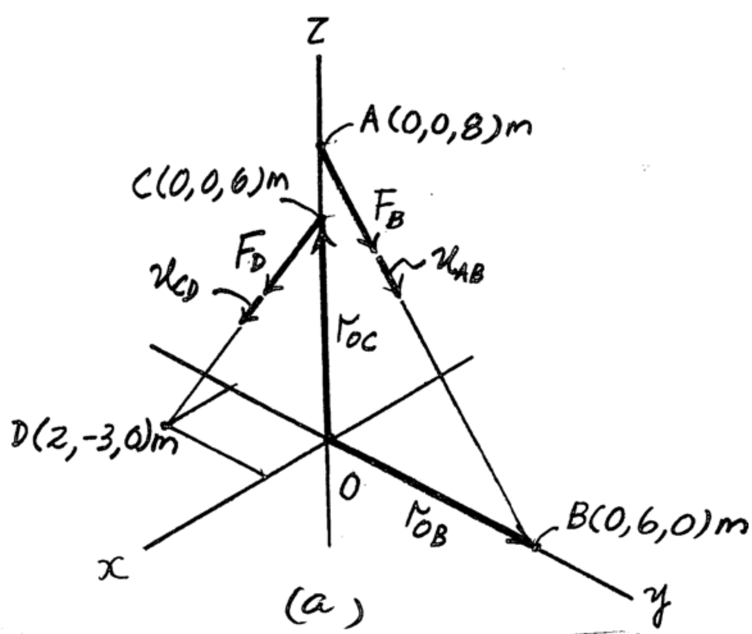
$\mathbf{r}_{OB} = \{6\mathbf{j}\}$ m $\qquad \mathbf{r}_{OC} = [6\mathbf{k}]$ m

Thus, the resultant couple moment about point O is given by

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OB}\times\mathbf{F}_B + \mathbf{r}_{OC}\times\mathbf{F}_D$$
$$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 6 & 0\\ 0 & 3 & -4\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 2 & -3 & -6\end{vmatrix}$$
$$= [-6\mathbf{i}+12\mathbf{j}]\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

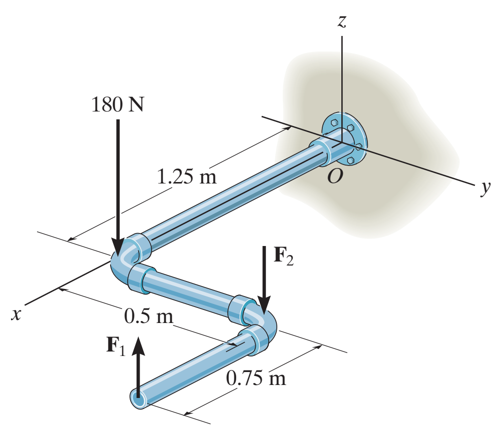
$$M_{R_A} = \Sigma M_A;\quad 10750d = -3500(3) - 5500(17) - 1750(25)$$

$$d = 13.7\text{ ft} \qquad \textbf{Ans.}$$



(a)

4–114. The three forces act on the pipe assembly. If $F_1 = 50$ N and $F_2 = 80$ N, replace this force system by an equivalent resultant force and couple moment acting at O. Express the results in Cartesian vector form.



$\mathbf{F}_R = \Sigma \mathbf{F}_z = \{-180\mathbf{k} + 50\mathbf{k} - 80\mathbf{k}\}$ N $= \{-210\mathbf{k}\}$ N **Ans**

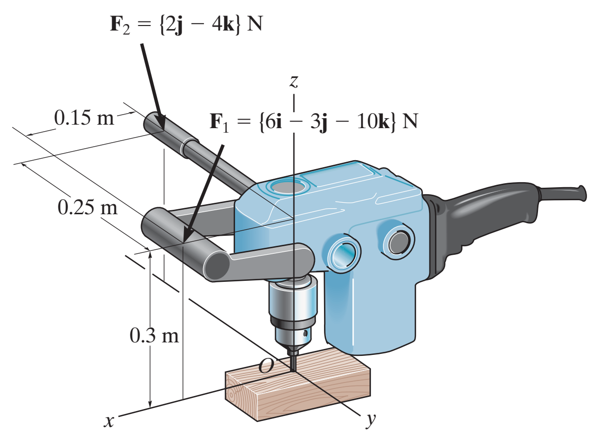
$\mathbf{M}_{RO} = \Sigma(\mathbf{r} \times \mathbf{F})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.25 & 0 & 0 \\ 0 & 0 & -180 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.25 & 0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0.5 & 0 \\ 0 & 0 & 50 \end{vmatrix}$$

$$= (225\mathbf{j}) + (-40\mathbf{i} + 100\mathbf{j}) + (25\mathbf{i} - 100\mathbf{j})$$

$= \{-15\mathbf{i} + 225\mathbf{j}\}$ N·m **Ans**

4–115. Handle forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to the electric drill. Replace this force system by an equivalent resultant force and couple moment acting at point O. Express the results in Cartesian vector form.



$\mathbf{F}_R = \Sigma\mathbf{F}$; $\quad \mathbf{F}_R = 6\mathbf{i} - 3\mathbf{j} - 10\mathbf{k} + 2\mathbf{j} - 4\mathbf{k}$

$= \{6\mathbf{i} - 1\mathbf{j} - 14\mathbf{k}\}$ N **Ans**

$\mathbf{M}_{RO} = \Sigma\mathbf{M}_O$;

$$\mathbf{M}_{RO} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix}$$

$= 0.9\mathbf{i} + 3.30\mathbf{j} - 0.450\mathbf{k} + 0.4\mathbf{i}$

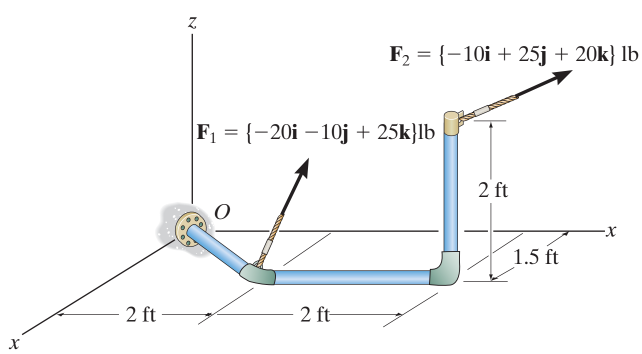
$= \{1.30\mathbf{i} + 3.30\mathbf{j} - 0.450\mathbf{k}\}$ N·m **Ans**

Note that $F_{Rz} = -14$ N pushes the drill bit down into the stock.

$(M_{RO})_x = 1.30$ N·m and $(M_{RO})_y = 3.30$ N·m cause the drill bit to bend.

$(M_{RO})_z = -0.450$ N·m causes the drill case and the spinning drill bit to rotate about the z-axis.

***4–116.** Replace the force system acting on the pipe assembly by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



Equivalent Resultant Force: The resultant force $\mathbf{F}_R$ can be determined from

$$\mathbf{F}_R = \Sigma\mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (-20\mathbf{i} - 10\mathbf{j} + 25\mathbf{k}) + (-10\mathbf{i} + 25\mathbf{j} + 20\mathbf{k})$$

$$= [-30\mathbf{i} + 15\mathbf{j} + 45\mathbf{k}]\text{ lb} \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$, Fig. a, are

$$\mathbf{r}_{OA} = (1.5 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [1.5\mathbf{i} + 2\mathbf{j}]\text{ ft}$$

$$\mathbf{r}_{OB} = (1.5 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = [1.5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}]\text{ ft}$$

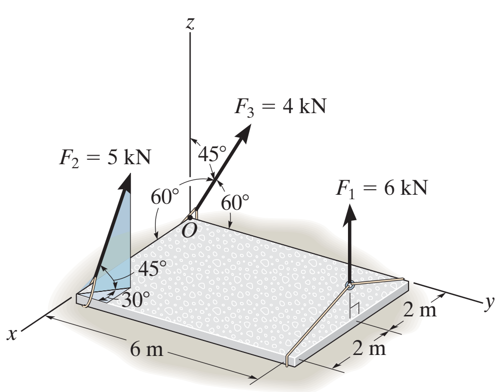
Thus, the resultant couple moment about point O is

$$\mathbf{M}_W = \Sigma\mathbf{M}_O; \qquad (\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OB} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix}$$

$$= [80\mathbf{i} - 87.5\mathbf{j} + 102.5\mathbf{k}]\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

•4–117. The slab is to be hoisted using the three slings shown. Replace the system of forces acting on slings by an equivalent force and couple moment at point O. The force $\mathbf{F}_1$ is vertical.



Force Vectors :

$$\mathbf{F}_1 = \{6.00\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_2 = 5(-\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k})$$
$$= \{-1.768\mathbf{i} + 3.062\mathbf{j} + 3.536\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_3 = 4(\cos 60°\mathbf{i} + \cos 60°\mathbf{j} + \cos 45°\mathbf{k})$$
$$= \{2.00\mathbf{i} + 2.00\mathbf{j} + 2.828\mathbf{k}\}\ \text{kN}$$

Equivalent Force and Couple Moment At Point O :

$$\mathbf{F}_R = \Sigma\mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= (-1.768 + 2.00)\,\mathbf{i} + (3.062 + 2.00)\,\mathbf{j} + (6.00 + 3.536 + 2.828)\,\mathbf{k}$$

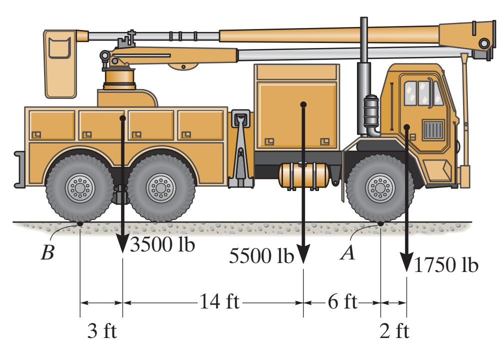
$$= \{0.232\mathbf{i} + 5.06\mathbf{j} + 12.4\mathbf{k}\}\ \text{kN} \qquad \textbf{Ans}$$

The position vectors are $\mathbf{r}_1 = \{2\mathbf{i} + 6\mathbf{j}\}$ m and $\mathbf{r}_2 = \{4\mathbf{i}\}$ m.

$$\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O; \quad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 6 & 0 \\ 0 & 0 & 6.00 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ -1.768 & 3.062 & 3.536 \end{vmatrix}$$

$$= \{36.0\mathbf{i} - 26.1\mathbf{j} + 12.2\mathbf{k}\}\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

4–118. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from B.

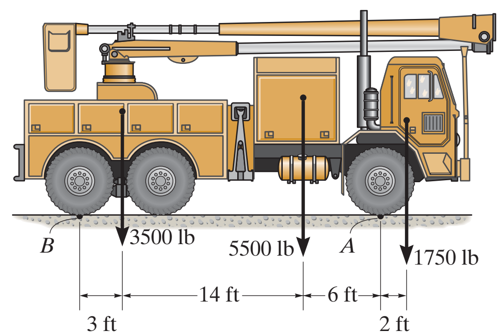


$$+\uparrow F_R = \Sigma F_y; \quad F_R = -1750 - 5500 - 3500$$
$$= -10750\ \text{lb} = 10.75\ \text{kip}\downarrow \qquad \textbf{Ans}$$

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad 10750d = -3500(3) - 5500(17) - 1750(25)$$

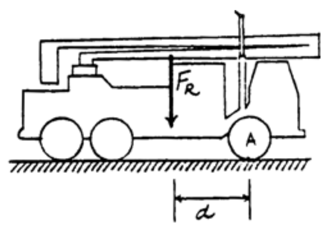
$$d = 13.7\ \text{ft} \qquad \textbf{Ans.}$$

4–119. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.



Equivalent Force :

$$+\uparrow F_R = \Sigma F_y ; \quad F_R = -1750 - 5500 - 3500$$
$$= -10750 \text{ lb} = 10.75 \text{ kip} \downarrow \qquad \textbf{Ans}$$

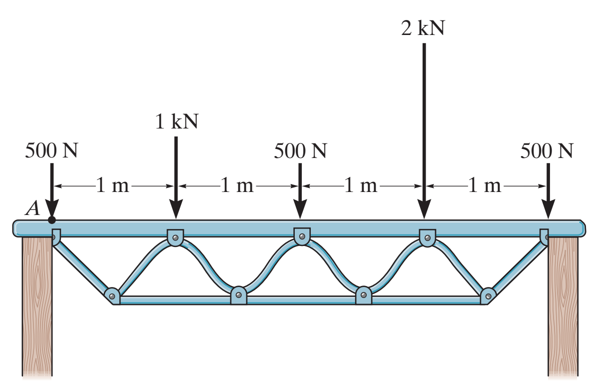


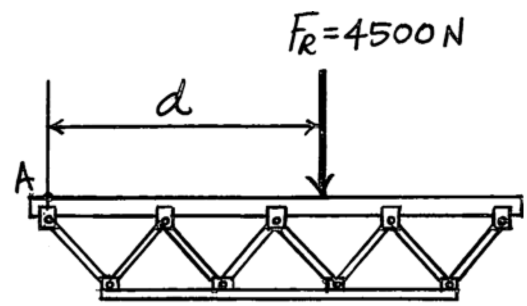
Location of Resultant Force From Point A :

$$\circlearrowleft + M_{R_A} = \Sigma M_A ; \quad 10750(d) = 3500(20) + 5500(6) - 1750(2)$$

$$d = 9.26 \text{ ft} \qquad \textbf{Ans}$$

*__4–120.__ The system of parallel forces acts on the top of the *Warren truss.* Determine the equivalent resultant force of the system and specify its location measured from point A.





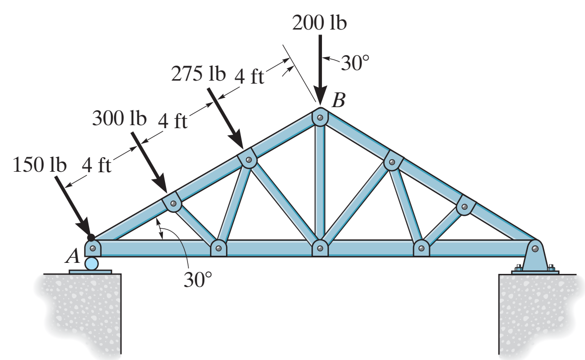
$$+\downarrow F_R = \Sigma F ; \quad F_R = 500 + 1000 + 500 + 2000 + 500$$

$$F_R = 4500 \text{ N} = 4.50 \text{ kN} \qquad \textbf{Ans}$$

$$\circlearrowleft + M_R = \Sigma M_A ; \quad 4500\,(d) = 1000(1) + 500\,(2) + 2000\,(3) + 500\,(4)$$

$$d = 2.22 \text{ m} \qquad \textbf{Ans}$$

•4–121. The system of four forces acts on the roof truss. Determine the equivalent resultant force and specify its location along AB, measured from point A.



$\nearrow + F_{Rx} = \Sigma F_x;\quad F_{Rx} = 200 \sin 30° = 100 \text{ lb}$

$\searrow + F_{Ry} = \Sigma F_y;\quad F_{Ry} = 150 + 300 + 275 + 200 \cos 30° = 898.2 \text{ lb}$

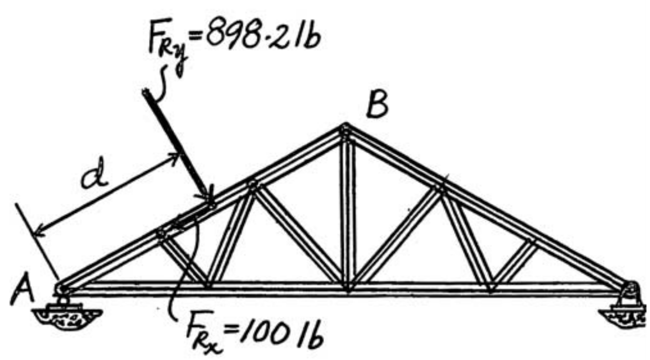
$F_R = \sqrt{(100)^2 + (898.2)^2} = 904 \text{ lb}$ **Ans**

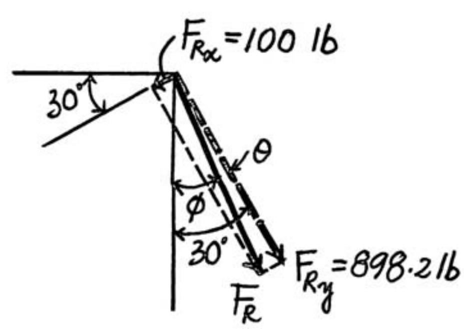
$\theta = \tan^{-1}\left(\frac{100}{898.2}\right) = 6.35°$

$\phi = 30° - 6.35° = 23.6°$ **Ans**

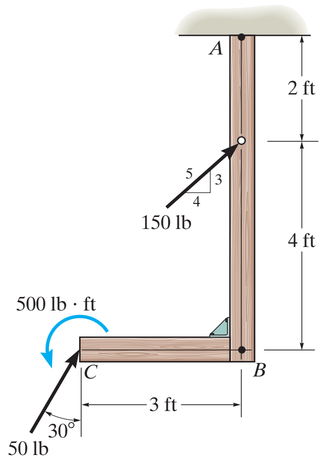
$\curvearrowleft + M_{RA} = \Sigma M_A;\quad 898.2\,(d) = 4\,(300) + 8\,(275) + 12 \cos 30°\,(200)$

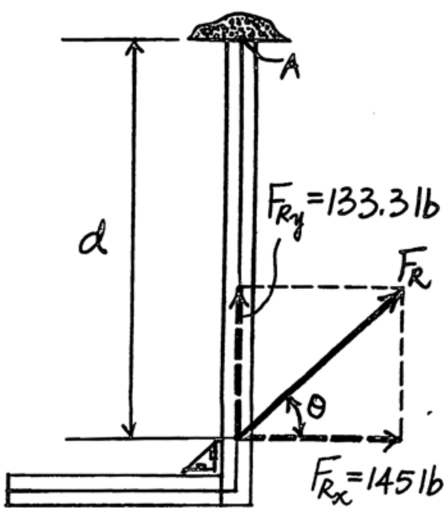
$d = 6.10 \text{ ft}$ **Ans**





4–122. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from A.





$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 150\left(\frac{4}{5}\right) + 50 \sin 30° = 145 \text{ lb}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 50 \cos 30° + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$

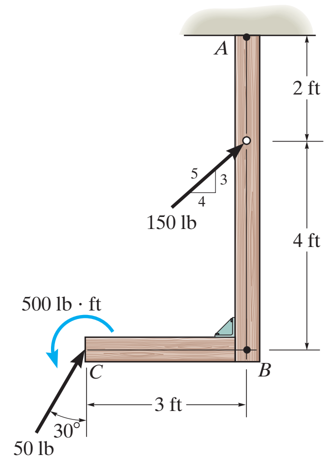
$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb}$ **Ans**

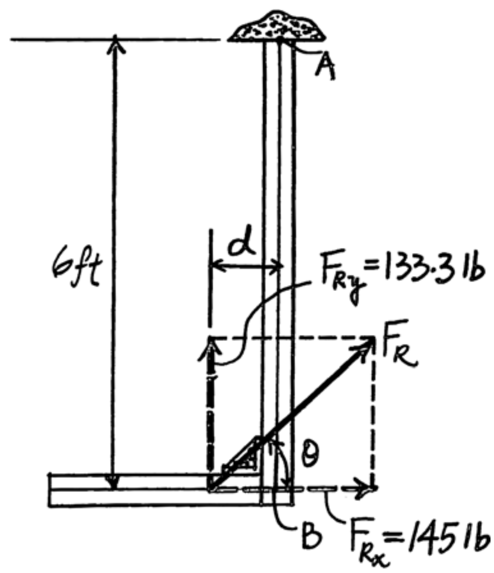
$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6° \measuredangle$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 145(d) = 150\left(\frac{4}{5}\right)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500$

$d = 5.24 \text{ ft}$ **Ans**

4–123. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from *B*.





$\xrightarrow{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = 150\left(\frac{4}{5}\right) + 50\sin 30^\circ = 145 \text{ lb}$

$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 50\cos 30^\circ + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$

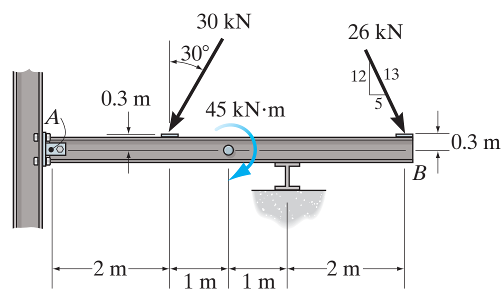
$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb}$ **Ans**

$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6^\circ$ ∠ **Ans**

$\curvearrowleft + M_{RA} = \Sigma M_A; \quad 145(6) - 133.3(d) = 150\left(\frac{4}{5}\right)(2) - 50\cos 30^\circ(3) + 50\sin 30^\circ(6) + 500$

$d = 0.824 \text{ ft}$ **Ans**

***4–124.** Replace the force and couple moment system acting on the overhang beam by a resultant force, and specify its location along *AB* measured from point *A*.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. *a*. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 26\left(\frac{5}{13}\right) - 30\sin 30° = -5\text{ kN} = 5\text{ kN} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -26\left(\frac{12}{13}\right) - 30\cos 30° = -49.98\text{ kN} = 49.98\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{5^2 + 49.98^2} = 50.23\text{ kN} = 50.2\text{ kN} \quad \textbf{Ans.}$$

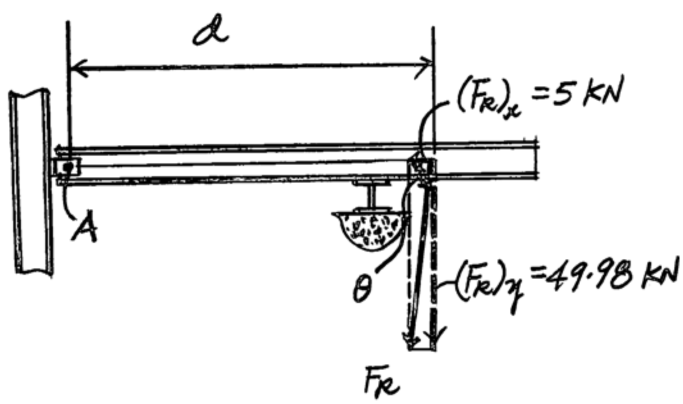
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{49.98}{5}\right] = 84.29° = 84.3° \qquad \textbf{Ans.}$$

Location of Resultant Force: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point *A*,

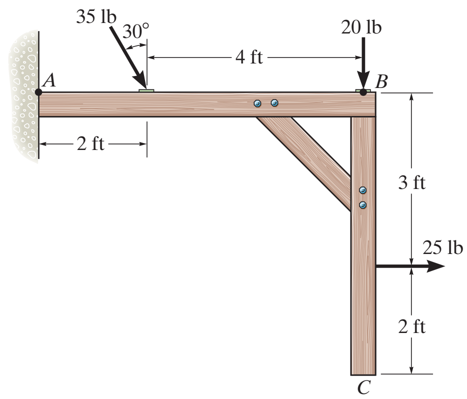
$$\circlearrowleft+(M_R)_A = \Sigma M_A;\ -49.98(d) = 30\sin 30°(0.3) - 30\cos 30°(2) - 26\left(\frac{5}{13}\right)(0.3) - 26\left(\frac{12}{13}\right)(6) - 45$$

$$d = 4.79\text{ m} \qquad \textbf{Ans.}$$



(b)

•4–125. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from point A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$

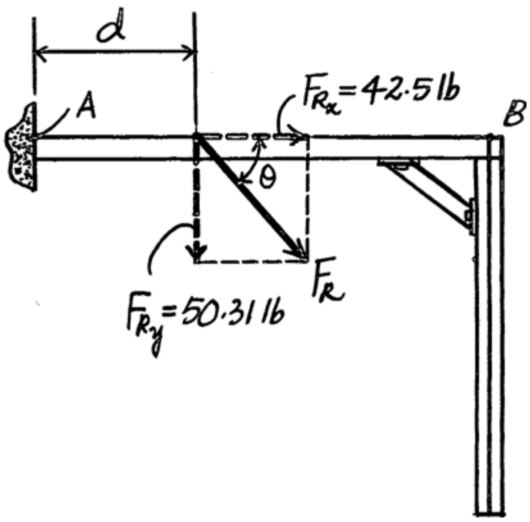
$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$ **Ans**

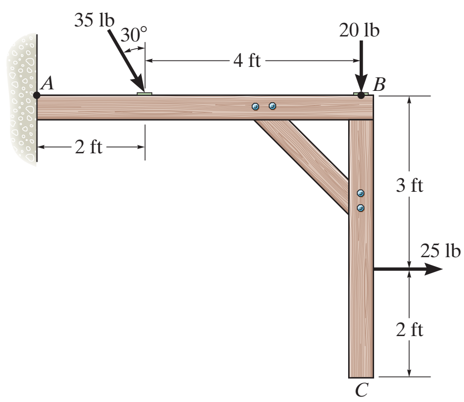
$\theta = \tan^{-1}\left(\dfrac{50.31}{42.5}\right) = 49.8°$ **Ans**

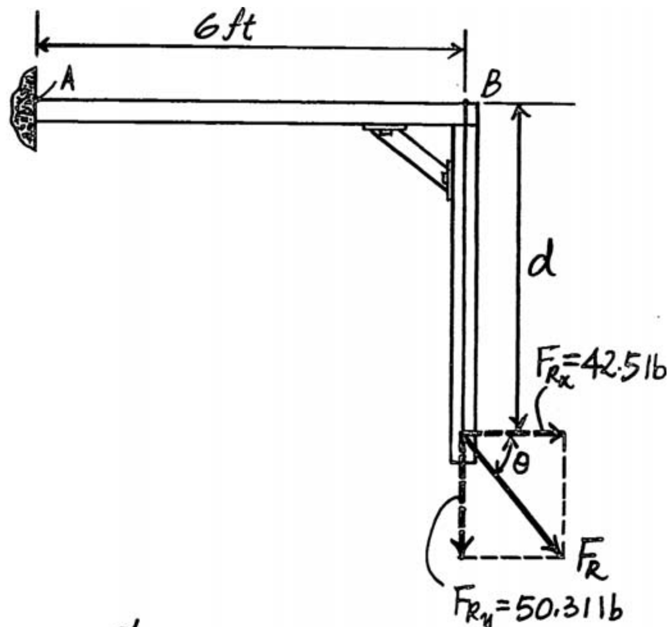
$\curvearrowleft +M_{RA} = \Sigma M_A;\quad 50.31\,(d) = 35 \cos 30°\,(2) + 20\,(6) - 25\,(3)$

$d = 2.10 \text{ ft}$ **Ans**



4–126. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from point B.





$\xrightarrow{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$

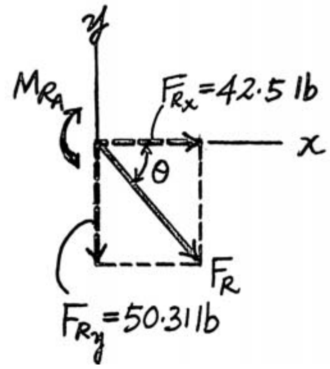
$+\downarrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$ **Ans**

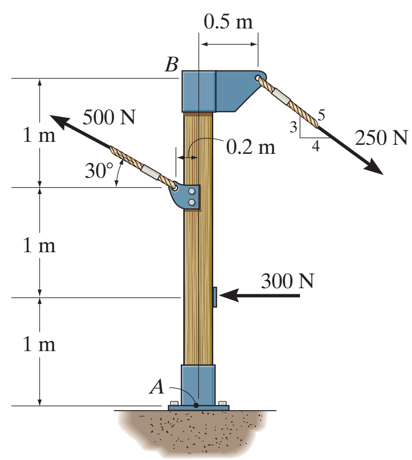
$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A; \quad 50.31(6) - 42.5(d) = 35 \cos 30° (2) + 20(6) - 25(3)$

$d = 4.62 \text{ ft}$ **Ans**



4–127. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point A.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = 500\sin 30° - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

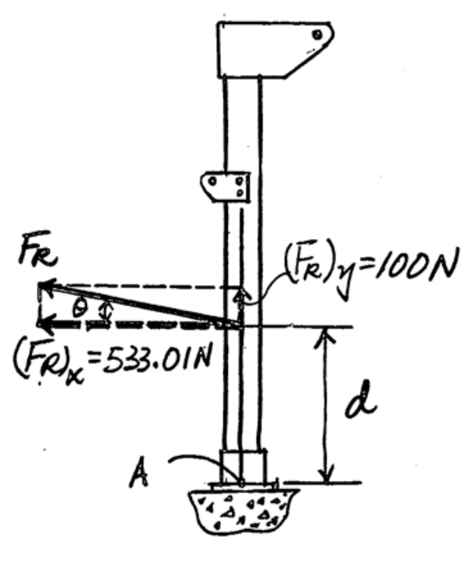
Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

$$\curvearrowleft +(M_R)_A = \Sigma M_A; \quad 533.01(d) = 500\cos 30°(2) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$d = 0.8274\text{ mm} = 827\text{ mm} \qquad \textbf{Ans.}$$

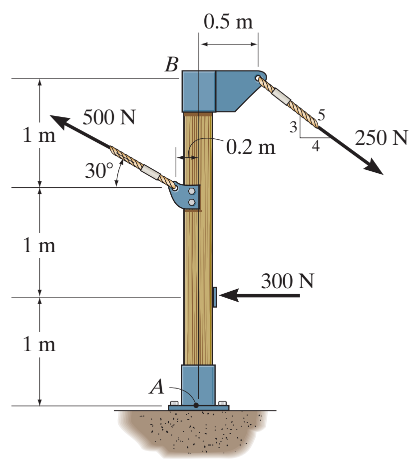
use Fig. (a)
of
Prob. 4-109

(a)



(b)

***4–128.** Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 500\sin 30° - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

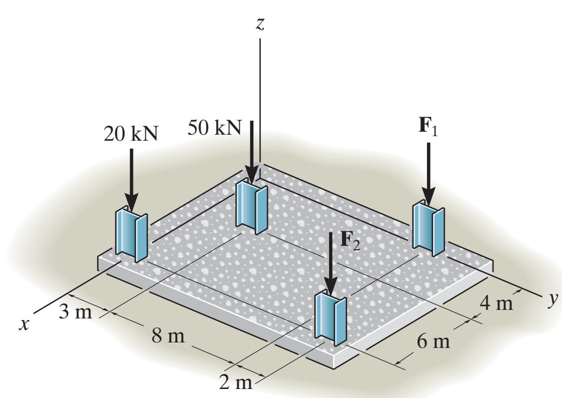
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

$$\circlearrowleft +(M_R)_b = \Sigma M_b; \quad -533.01(d) = -500\cos 30°(1) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 300(2)$$

$$d = 2.17\text{ m} \qquad \textbf{Ans.}$$

•4–129. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.



$$+\uparrow F_R = \Sigma F_z; \quad F_R = -20 - 50 - 30 - 40 = -140\text{ kN} = 140\text{ kN}\downarrow \qquad \textbf{Ans}$$

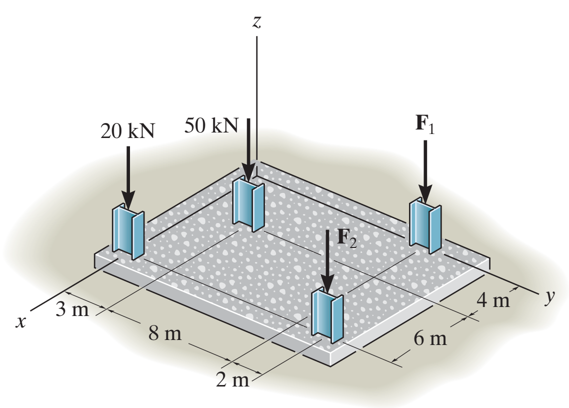
$$(M_R)_x = \Sigma M_x; \quad -140y = -50(3) - 30(11) - 40(13)$$

$$y = 7.14\text{ m} \qquad \textbf{Ans}$$

$$(M_R)_y = \Sigma M_y; \quad 140x = 50(4) + 20(10) + 40(10)$$

$$x = 5.71\text{ m} \qquad \textbf{Ans}$$

4–130. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



$+\downarrow F_R = \Sigma F_z;$ $\quad F_R = 20 + 50 + 20 + 50 = 140$ kN **Ans**

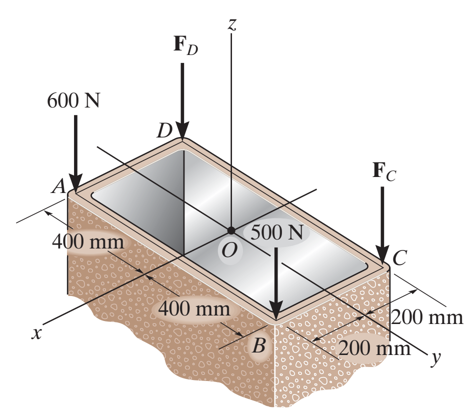
$M_{R\,y} = \Sigma M_y;$ $\quad 140(x) = (50)(4) + 20(10) + 50(10)$

$x = 6.43$ m **Ans**

$M_{R\,x} = \Sigma M_x;$ $\quad -140(y) = -(50)(3) - 20(11) - 50(13)$

$y = 7.29$ m **Ans**

4–131. The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



Since the resultant force passes through point O, the resultant moment components about x and y axes are both zero.

$\Sigma M_x = 0;$ $\quad F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0$

$F_C - F_D = 100 \quad (1)$

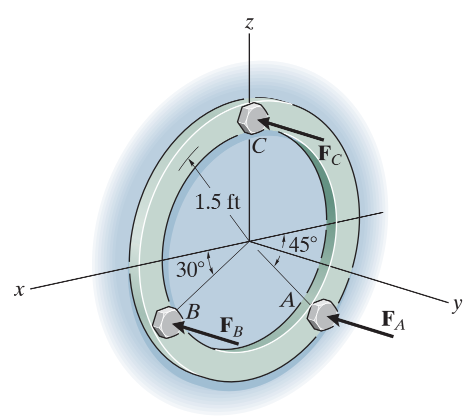
$\Sigma M_y = 0;$ $\quad 500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0$

$F_C + F_D = 1100 \quad (2)$

Solving Eqs. (1) and (2) yields:

$F_C = 600$ N $\quad F_D = 500$ N **Ans**

***4–132.** Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location (x, z) on the plate. $F_A = 200$ lb, $F_B = 100$ lb, and $F_C = 400$ lb.



Equivalent Force :

$$F_R = \Sigma F_y; \qquad -F_R = -400 - 200 - 100$$

$$F_R = 700 \text{ lb} \qquad \textbf{Ans}$$

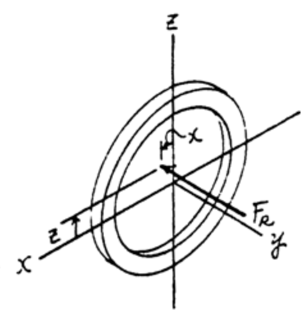
Location of Resultant Force :

$$M_{R_x} = \Sigma M_x; \qquad 700(z) = 400(1.5) - 200(1.5\sin 45°) - 100(1.5\sin 30°)$$

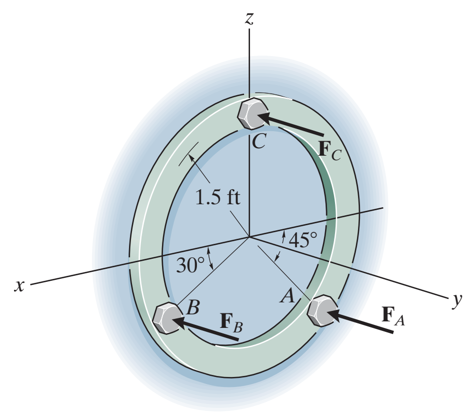
$$z = 0.447 \text{ ft} \qquad \textbf{Ans}$$

$$M_{R_z} = \Sigma M_z; \qquad -700(x) = 200(1.5\cos 45°) - 100(1.5\cos 30°)$$

$$x = -0.117 \text{ ft} \qquad \textbf{Ans}$$



•4–133. The three parallel bolting forces act on the circular plate. If the force at A has a magnitude of $F_A = 200$ lb, determine the magnitudes of $\mathbf{F}_B$ and $\mathbf{F}_C$ so that the resultant force $\mathbf{F}_R$ of the system has a line of action that coincides with the y axis. *Hint:* This requires $\Sigma M_x = 0$ and $\Sigma M_z = 0$.



Since F_R coincides with y axis, $M_{R_x} = M_{R_z} = 0$.

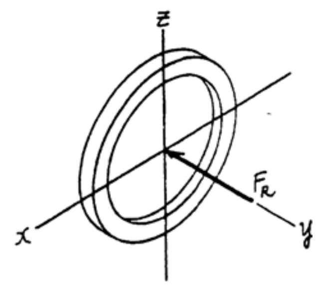
$M_{R_z} = \Sigma M_z$; $0 = 200(1.5\cos 45°) - F_B(1.5\cos 30°)$

$F_B = 163.30 \text{ lb} = 163 \text{ lb}$ **Ans**

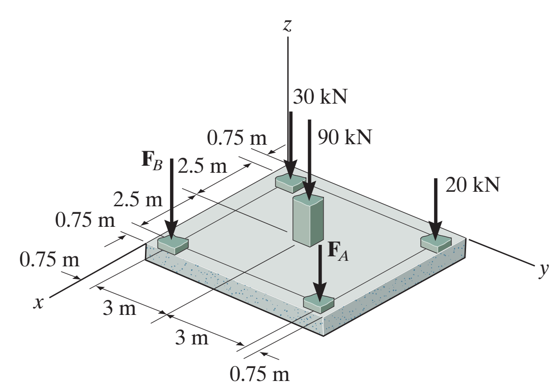
Using the result $F_B = 163.30$ lb,

$M_{R_x} = \Sigma M_x$; $0 = F_C(1.5) - 200(1.5\sin 45°) - 163.30(1.5\sin 30°)$

$F_C = 223$ lb **Ans**



4–134. If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.



Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$, F . b,

$+\uparrow F_R = \Sigma F_z$; $-F_R = -30 - 20 - 90 - 35 - 40$

$F_R = 215$ kN **Ans.**

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

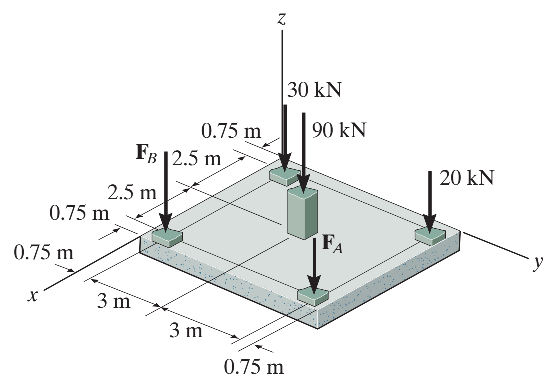
$(M_R)_x = \Sigma M_x$; $-215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75)$

$y = 3.68$ m **Ans.**

$(M_R)_y = \Sigma M_y$; $215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)$

$x = 3.54$ m **Ans.**

4–135. If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$,

$+\uparrow F_R = \Sigma F_z;$ $\quad -F_R = -30 - 20 - 90 - F_A - F_B$

$$F_R = 140 + F_A + F_B \qquad (1)$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$(M_R)_x = \Sigma M_x;$ $\quad -F_R(3.75) = -F_B(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - F_A(6.75)$

$$F_R = 0.2F_B + 1.8F_A + 132 \qquad (2)$$

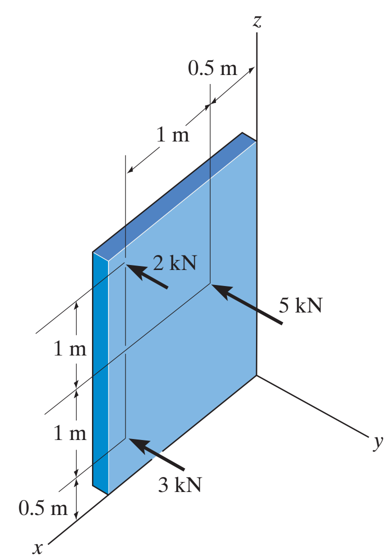
$(M_R)_y = \Sigma M_y;$ $\quad F_R(3.25) = 30(0.75) + 20(0.75) + 90(3.25) + F_A(5.75) + F_B(5.75)$

$$F_R = 1.769F_A + 1.769F_B + 101.54 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_A = 30\text{ kN}$ $\quad F_B = 20\text{ kN}$ $\quad F_R = 190\text{ kN}$ **Ans.**

***4–136.** Replace the parallel force system acting on the plate by a resultant force and specify its location on the x–z plane.



Resultant Force: Summing the forces acting on the plate,

$(F_R)_y = \Sigma F_y;$ $\quad F_R = -5\text{ kN} - 2\text{ kN} - 3\text{ kN}$

$$= -10\text{ kN} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{F}_R$ acts along the negative y axis.

Resultant Moment: Using the right - hand rule, and equating the moment of $\mathbf{F}_R$ to the sum of the moments of the force system about the x and z axes,

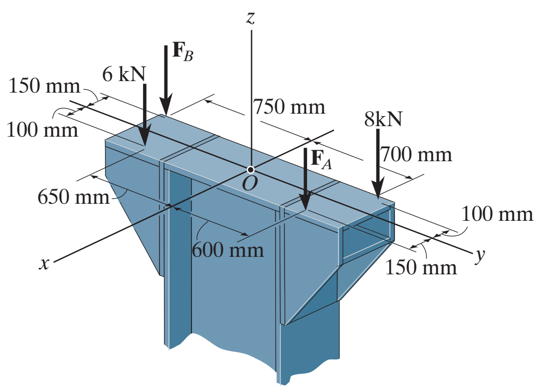
$(M_R)_x = \Sigma M_x;$ $\quad (10\text{ kN})(z) = (3\text{ kN})(0.5\text{ m}) + (5\text{ kN})(1.5\text{ m}) + 2\text{ kN}(2.5\text{ m})$

$$z = 1.40\text{ m} \qquad \textbf{Ans.}$$

$(M_R)_z = \Sigma M_z;$ $\quad -(10\text{ kN})(x) = -(5\text{ kN})(0.5\text{ m}) - (2\text{ kN})(1.5\text{ m}) - (3\text{ kN})(1.5\text{ m})$

$$x = 1.00\text{ m} \qquad \textbf{Ans.}$$

•4–137. If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system acting on the corbels by a resultant force, and specify its location on the x–y plane.



Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the z axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -6 - 5 - 7 - 8$$

$$F_R = 26 \text{ kN} \qquad \textbf{Ans.}$$

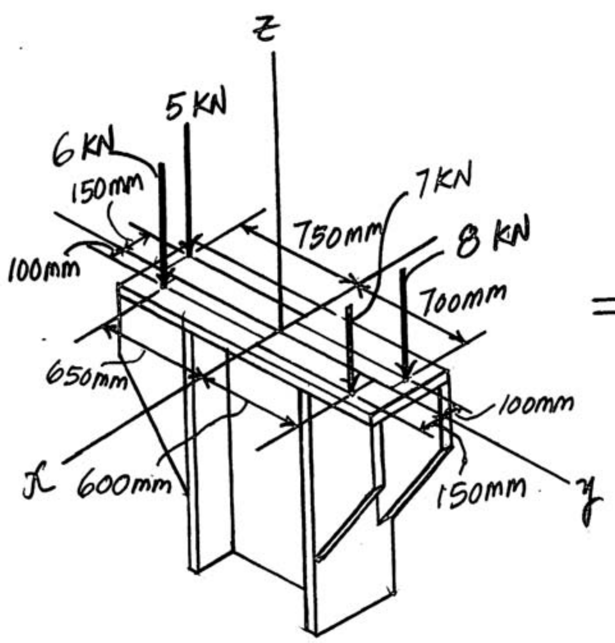
Point of Application: By equating the moment of the forces shown in Fig. *a* and $\mathbf{F}_R$, Fig. *b*, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -26(y) = 6(650) + 5(750) - 7(600) - 8(700)$$

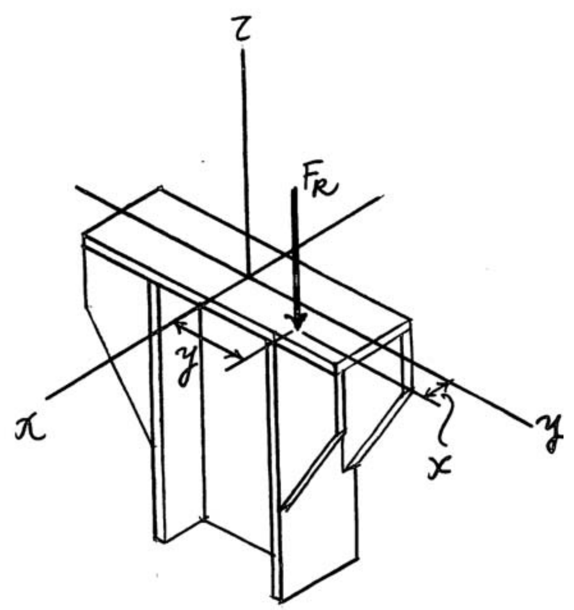
$$y = 82.7 \text{ mm} \qquad \textbf{Ans.}$$

$$(M_R)_y = \Sigma M_y; \qquad 26(x) = 6(100) + 7(150) - 5(150) - 8(100)$$

$$x = 3.85 \text{ mm} \qquad \textbf{Ans.}$$

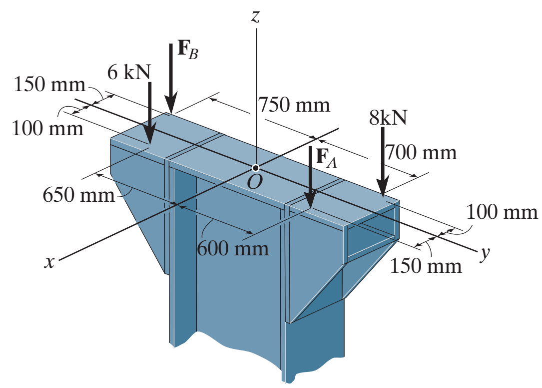


(a)



(b)

4–138. Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O of the column.



Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the z axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$+\uparrow F_R = \Sigma F_z;$ $\quad -F_R = -F_A - F_B - 8 - 6$

$$F_R = F_A + F_B + 14 \qquad (1)$$

Point of Application: Since $\mathbf{F}_R$ is required to pass through point O, the moment of $\mathbf{F}_R$ about the x and y axes are equal to zero. Thus,

$(M_R)_x = \Sigma M_x;$ $\quad 0 = F_B(750) + 6(650) - F_A(600) - 8(700)$

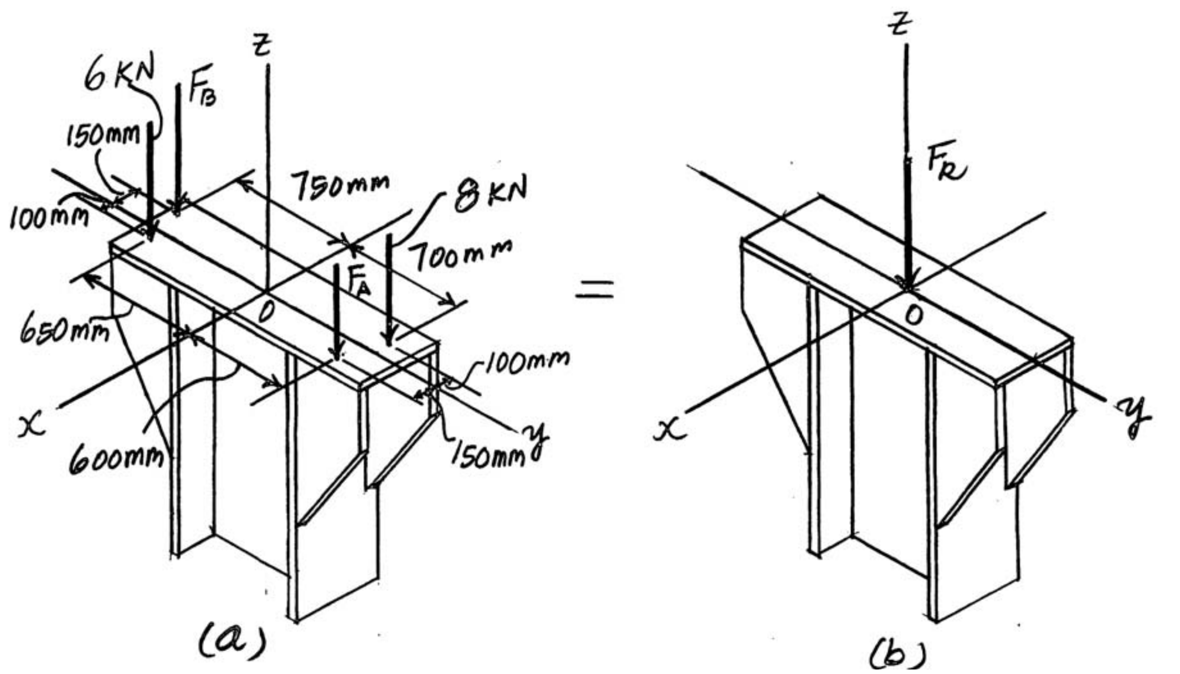
$$750F_B - 600F_A - 1700 = 0 \qquad (2)$$

$(M_R)_y = \Sigma M_y;$ $\quad 0 = F_A(150) + 6(100) - F_B(150) - 8(100)$

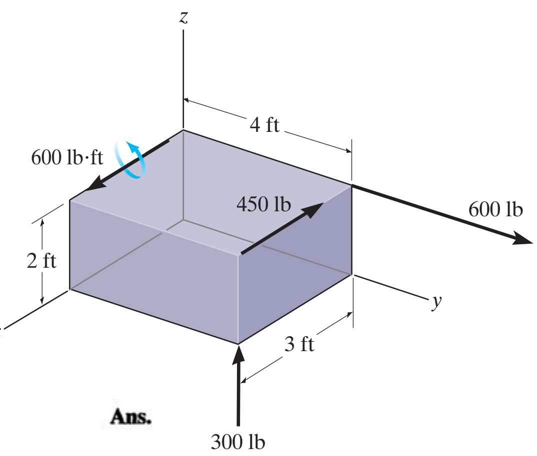
$$150F_A - 150F_B + 200 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_A = 18.0\text{ kN}$ $\quad F_B = 16.7\text{ kN}$ $\quad F_R = 48.7\text{ kN}$ **Ans.**



4–139. Replace the force and couple moment system acting on the rectangular block by a wrench. Specify the magnitude of the force and couple moment of the wrench and where its line of action intersects the *x–y* plane.



Equivalent Resultant Force: The resultant forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ expressed in Cartesian vector form can be written as $\mathbf{F}_1 = [600\mathbf{j}]$ lb, $\mathbf{F}_2 = [-450\mathbf{i}]$ lb, and $\mathbf{F}_3 = [300\mathbf{k}]$ lb. The force of the wrench can be determined from

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= 600\mathbf{j} - 450\mathbf{i} + 300\mathbf{k} = [-450\mathbf{i} + 600\mathbf{j} + 300\mathbf{k}]\text{ lb}$$

Thus, the magnitude of the wrench force is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2} = \sqrt{(-450)^2 + 600^2 + 300^2} = 807.77\text{ lb} = 808\text{ lb}\qquad\textbf{Ans.}$$

Equivalent Couple Moment: Here, we will assume that the axis of the wrench passes through point P, Figs. *a* and *b*. Since $\mathbf{M}_W$ is collinear with $\mathbf{F}_R$,

$$\mathbf{M}_W = M_W\mathbf{u}_{F_R} = M_W\left[\frac{-450\mathbf{i} + 600\mathbf{j} + 300\mathbf{k}}{\sqrt{(-450)^2 + 600^2 + 300^2}}\right]$$
$$= -0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k}$$

The position vectors $\mathbf{r}_{PA}$, $\mathbf{r}_{PB}$, and $\mathbf{r}_{PC}$ are

$$\mathbf{r}_{PA} = (0 - x)\mathbf{i} + (4 - y)\mathbf{j} + (2 - 0)\mathbf{k} = -x\mathbf{i} + (4 - y)\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{r}_{PB} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + (0 - 0)\mathbf{k} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j}$$
$$\mathbf{r}_{PC} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + (2 - 0)\mathbf{k} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + 2\mathbf{k}$$

The couple moment $\mathbf{M}$ expressed in Cartesian vector form is written as $\mathbf{M} = [600\mathbf{i}]$ lb · ft. Summing the moments of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ about point P and including $\mathbf{M}$,

$$\mathbf{M}_W = \Sigma\mathbf{M}_P;\qquad \mathbf{M}_W = \mathbf{r}_{PA}\times\mathbf{F}_1 + \mathbf{r}_{PC}\times\mathbf{F}_2 + \mathbf{r}_{PB}\times\mathbf{F}_3 + \mathbf{M}$$

$$-0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -x & (4-y) & 2\\ 0 & 600 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ (3-x) & (4-y) & 2\\ -450 & 0 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ (3-x) & (4-y) & 0\\ 0 & 0 & 300\end{vmatrix} + 600\mathbf{i}$$

$$-0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k} = (600 - 300y)\mathbf{i} + (300x - 1800)\mathbf{j} + (1800 - 600x - 450y)\mathbf{k}$$

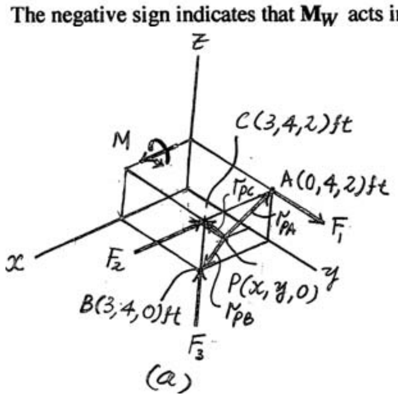
Equating the **i**, **j**, and **k** components,

$$-0.5571M_w = 600 - 300y \qquad (1)$$
$$0.7428M_w = 300x - 1800 \qquad (2)$$
$$0.3714M_w = 1800 - 600x - 450y \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

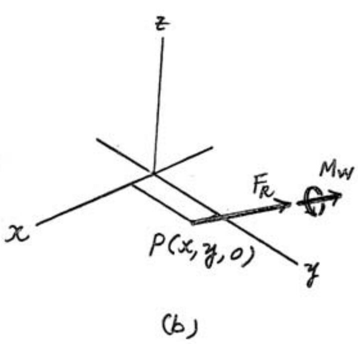
$x = 3.52$ ft $\qquad y = 0.138$ ft $\qquad M_W = -1003$ lb · ft $\qquad$ **Ans.**

The negative sign indicates that $\mathbf{M}_W$ acts in the opposite sense to that of $\mathbf{F}_R$.



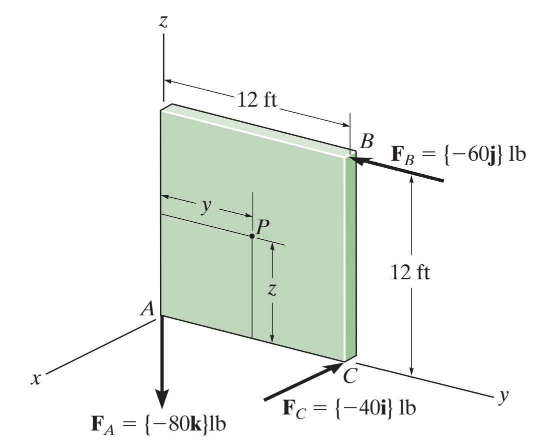
(a)

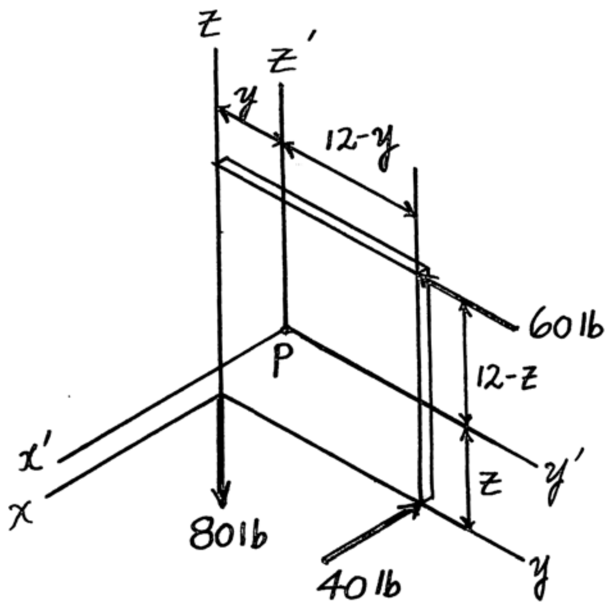
=



(b)

***4–140.** Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.





Resultant Force Vector :

$$\mathbf{F}_R = \{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}\}\ \text{lb}$$

$$F_R = \sqrt{(-40)^2 + (-60)^2 + (-80)^2} = 107.70\ \text{lb} = 108\ \text{lb} \qquad \textbf{Ans}$$

$$\mathbf{u}_{F_R} = \frac{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}}{107.70} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$$

Resultant Moment: The line of action of $\mathbf{M}_R$ of the wrench is parallel to the line of action of $\mathbf{F}_R$. Assume that both $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Therefore, $\mathbf{u}_{M_R} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$.

$$(M_R)_{x'} = \Sigma M_{x'}; \qquad -0.3714M_R = 60(12 - z) + 80y \qquad [1]$$

$$(M_R)_{y'} = \Sigma M_{y'}; \qquad -0.5571M_R = 40z \qquad [2]$$

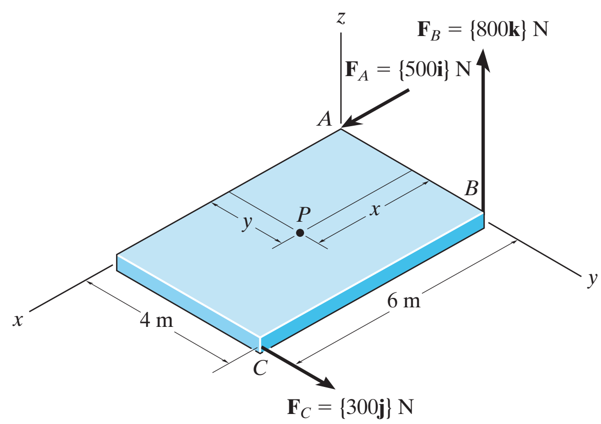
$$(M_R)_{z'} = \Sigma M_{z'}; \qquad -0.7428M_R = 40(12 - y) \qquad [3]$$

Solving Eqs.[1], [2], and [3] yields :

$$M_R = -624\ \text{lb} \cdot \text{ft} \qquad z = 8.69\ \text{ft} \qquad y = 0.414\ \text{ft} \qquad \textbf{Ans}$$

The negative sign indicates that the line of action for $\mathbf{M}_R$ is directed in the opposite sense to that of $\mathbf{F}_R$.

•4–141. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



$$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990\text{ N} \qquad \textbf{Ans}$$

$$\mathbf{u}_{FR} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$$

$$M_{R_{x'}} = \Sigma M_{x'}; \qquad M_{R_{x'}} = 800(4-y)$$

$$M_{R_{y'}} = \Sigma M_{y'}; \qquad M_{R_{y'}} = 800x$$

$$M_{R_{z'}} = \Sigma M_{z'}; \qquad M_{R_{z'}} = 500y + 300(6-x)$$

Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{FR}$,

$$M_R(0.5051) = 800(4-y)$$

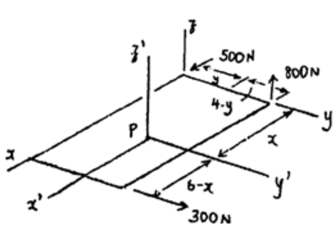
$$M_R(0.3030) = 800x$$

$$M_R(0.8081) = 500y + 300(6-x)$$

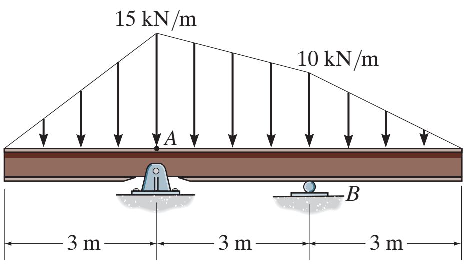
$$M_R = 3.07\text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$x = 1.16\text{ m} \qquad \textbf{Ans}$$

$$y = 2.06\text{ m} \qquad \textbf{Ans}$$



4–142. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Loading: The distributed loading can be divided into four parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also indicated in Fig. *a*.

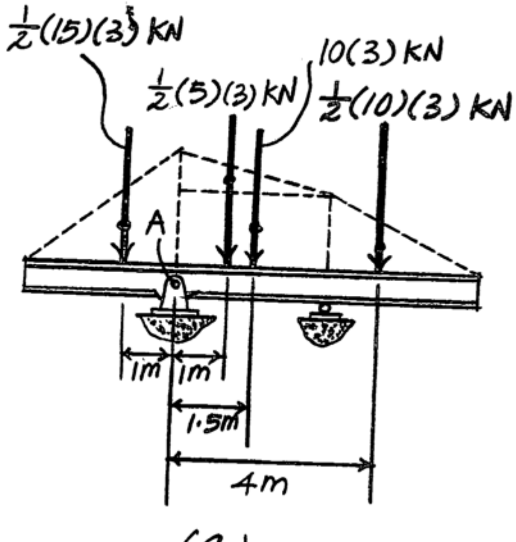
Resultants: Equating the sum of the forces along the *y* axis of Figs. *a* and *b*,

$$+\downarrow F_R = \Sigma F_y; \qquad F_R = \frac{1}{2}(15)(3) + \frac{1}{2}(5)(3) + 10(3) + \frac{1}{2}(10)(3) = 75\text{ kN} \downarrow \qquad \textbf{Ans.}$$

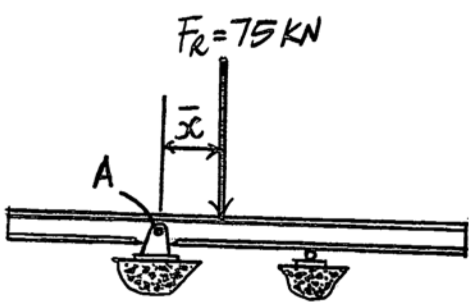
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point *A*,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad -75(\bar{x}) = \frac{1}{2}(15)(3)(1) - \frac{1}{2}(5)(3)(1) - 10(3)(1.5) - \frac{1}{2}(10)(3)(4)$$

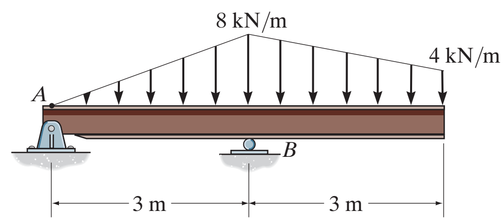
$$\bar{x} = 1.20\text{ m} \qquad \textbf{Ans.}$$



(a)



4–143. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Loading: The distributed loading can be divided into three parts as shown in Fig. *a*.

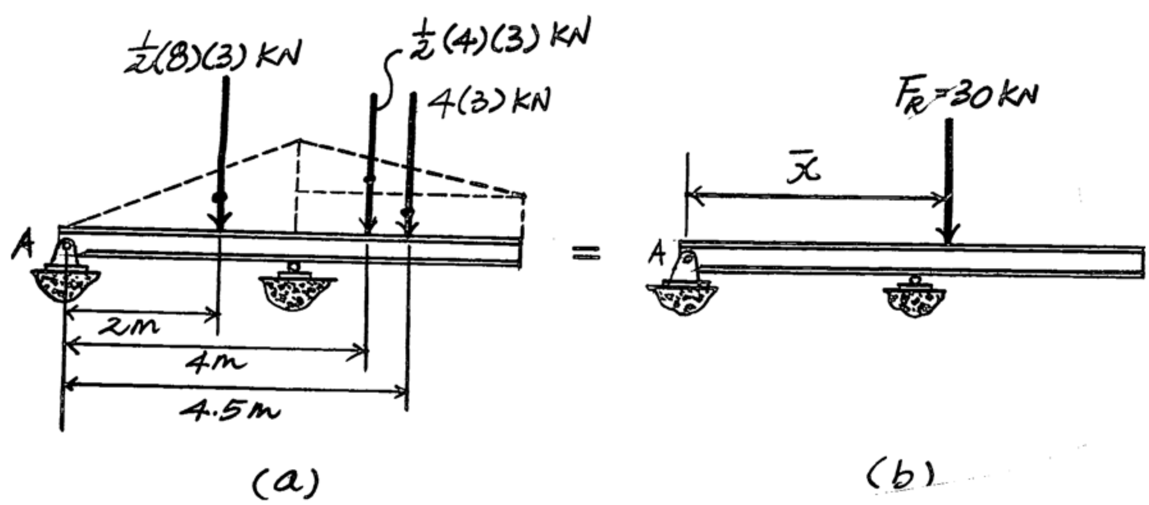
Resultants: Equating the sum of the forces along the *y* axis of Figs. *a* and *b*,

$+\downarrow F_R = \Sigma F_y;\qquad F_R = \frac{1}{2}(8)(3) + \frac{1}{2}(4)(3) + 4(3) = 30\text{ kN}\downarrow$ **Ans.**

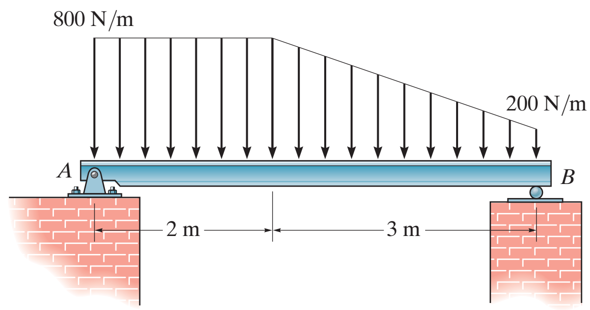
If we equate the moments of F_R, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point A,

$\circlearrowleft + (M_R)_A = \Sigma M_A;\quad -30(\bar{x}) = -\frac{1}{2}(8)(3)(2) - \frac{1}{2}(4)(3)(4) - 4(3)(4.5)$

$(\bar{x}) = 3.4\text{ m}$ **Ans.**



***4–144.** Replace the distributed loading by an equivalent resultant force and specify its location, measured from point A.

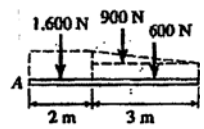


$+\downarrow F_R = \Sigma F;\quad F_R = 1600 + 900 + 600 = 3100\text{ N}$

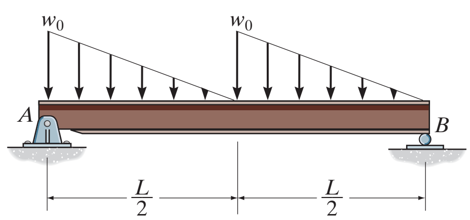
$F_R = 3.10\text{ kN}\downarrow$ **Ans**

$\curvearrowleft + M_{RA} = \Sigma M_A;\quad x(3100) = 1600(1) + 900(3) + 600(3.5)$

$x = 2.06\text{ m}$ **Ans**



•4–145. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Loading: The distributed loading can be divided into two parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also shown in Fig. *a*.

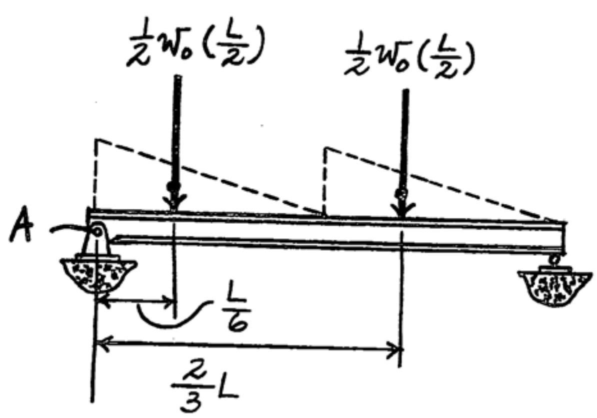
Resultants: Equating the sum of the forces along the *y* axis of Figs. *a* and *b*,

$$+\downarrow F_R = \Sigma F;\qquad F_R = \frac{1}{2}w_0\left(\frac{L}{2}\right) + \frac{1}{2}w_0\left(\frac{L}{2}\right) = \frac{1}{2}w_0 L\downarrow \qquad \textbf{Ans.}$$

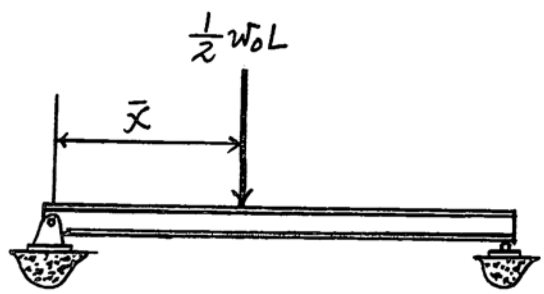
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point A,

$$\curvearrowleft + (M_R)_A = \Sigma M_A;\quad -\frac{1}{2}w_0 L(\bar{x}) = -\frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{2}{3}L\right)$$

$$\bar{x} = \frac{5}{12}L \qquad \textbf{Ans.}$$

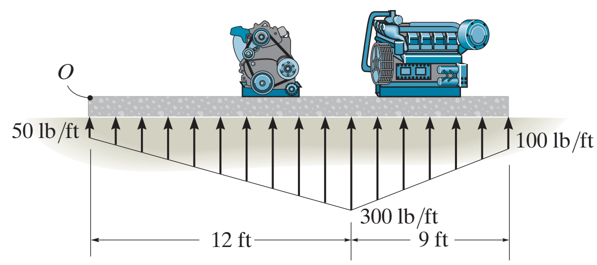


(a)



(b)

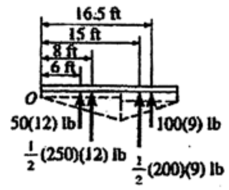
4–146. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



$+\uparrow F_R = \Sigma F_y;\quad F_R = 50(12) + \frac{1}{2}(250)(12)$

$+ \frac{1}{2}(200)(9) + 100(9)$

$= 3900\text{ lb} = 3.90\text{ kip}\uparrow$ **Ans**

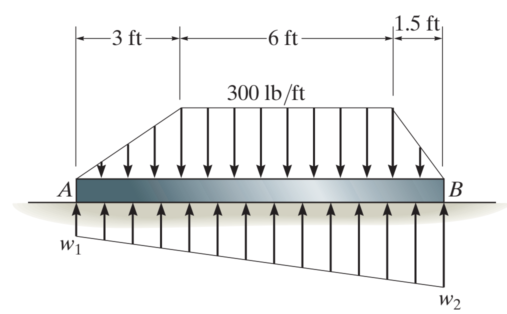


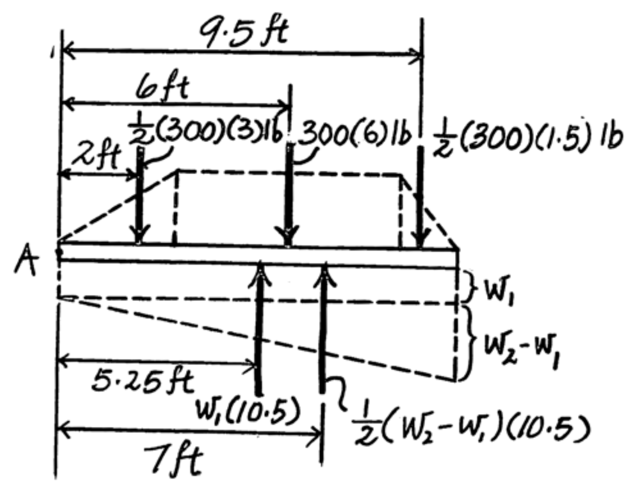
$\curvearrowleft + M_{R_O} = \Sigma M_O;\quad 3900(d) = 50(12)(6) + \frac{1}{2}(250)(12)(8)$

$+ \frac{1}{2}(200)(9)(15) + 100(9)(16.5)$

$d = 11.3\text{ ft}$ **Ans**

4–147. Determine the intensities w_1 and w_2 of the distributed loading acting on the bottom of the slab so that this loading has an equivalent resultant force that is equal but opposite to the resultant of the distributed loading acting on the top of the plate.





$\uparrow + F_R = \Sigma F;\quad 0 = w_1(10.5) + \frac{1}{2}(w_2 - w_1)(10.5) - \frac{1}{2}(300)(3) - 300(6) - \frac{1}{2}(300)(1.5)$

$w_1 + w_2 = 471.429 \qquad (1)$

$\curvearrowleft + M_{RA} = \Sigma M_A;\quad 0 = w_1(10.5)(5.25) + \frac{1}{2}(w_2 - w_1)(10.5)(7) - \frac{1}{2}(300)(3)(2)$

$- 300(6)(6) - \frac{1}{2}(300)(1.5)(9.5)$

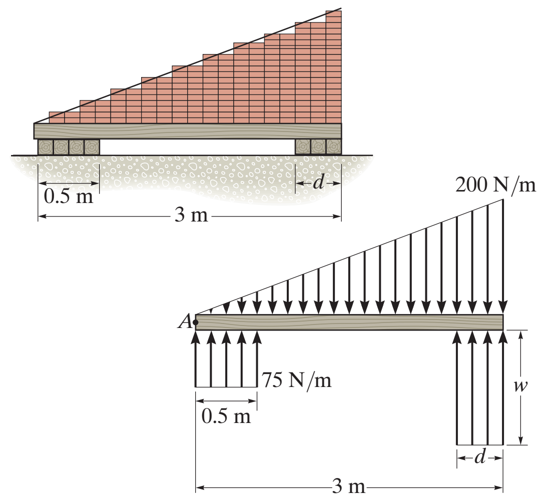
$w_1 + 2w_2 = 753.061 \qquad (2)$

Solving Eqs. (1) and (2),

$w_1 = 190\text{ lb/ft}$ **Ans**

$w_2 = 282\text{ lb/ft}$ **Ans**

***4–148.** The bricks on top of the beam and the supports at the bottom create the distributed loading shown in the second figure. Determine the required intensity w and dimension d of the right support so that the resultant force and couple moment about point A of the system are both zero.



Require $F_R = 0$.

$$+\uparrow F_R = \Sigma F_y; \quad 0 = wd + 37.5 - 300$$

$$wd = 262.5 \qquad [1]$$

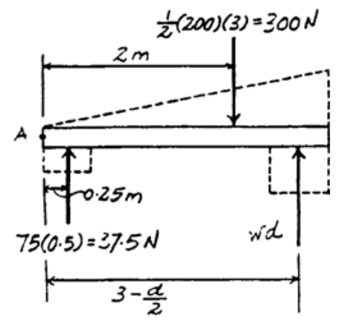
Require $M_{R_A} = 0$.

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad 0 = 37.5(0.25) + wd\left(3 - \frac{d}{2}\right) - 300(2)$$

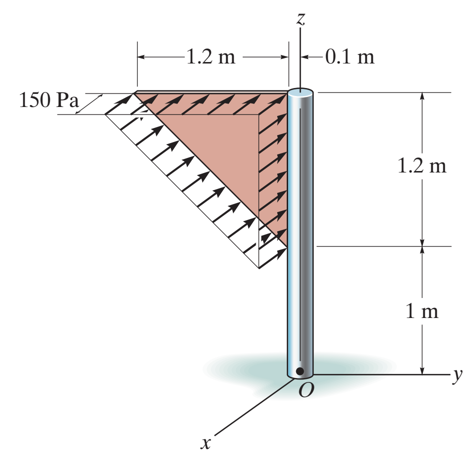
$$3wd - \frac{wd^2}{2} = 590.625 \qquad [2]$$

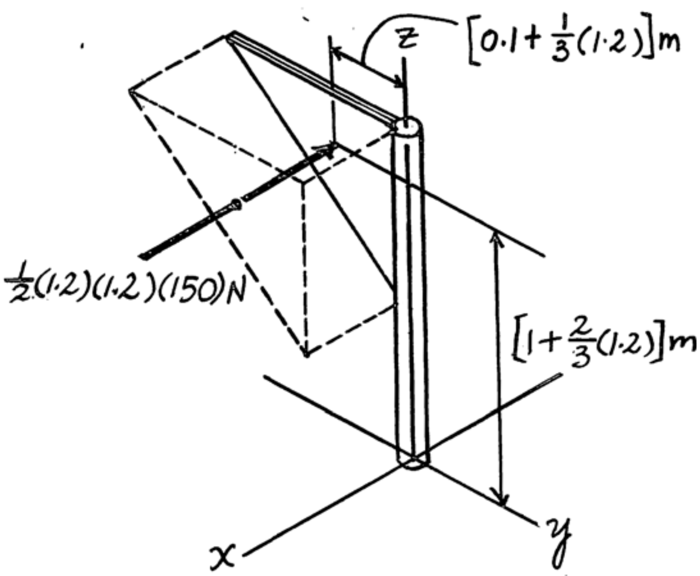
Solving Eqs. [1] and [2] yields

$d = 1.50$ m $\qquad w = 175$ N/m $\qquad$ **Ans**



•4–149. The wind pressure acting on a triangular sign is uniform. Replace this loading by an equivalent resultant force and couple moment at point O.





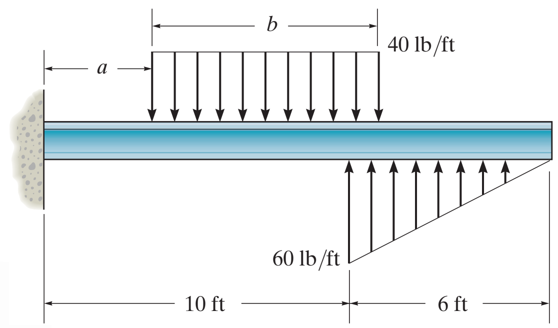
$$F_R = \frac{1}{2}(1.2)(1.2)(150)$$

$\mathbf{F}_R = \{-108\,\mathbf{i}\}$ N **Ans**

$$\mathbf{M}_{RO} = -\left(1+\frac{2}{3}(1.2)\right)(108)\,\mathbf{j} - \left(0.1+\frac{1}{3}(1.2)\right)(108)\,\mathbf{k}$$

$\mathbf{M}_{RO} = \{-194\,\mathbf{j} - 54\,\mathbf{k}\}$ N·m **Ans**

4–150. The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.



Require $F_R = 0$.

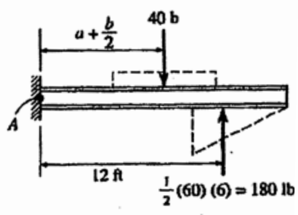
$+\uparrow F_R = \Sigma F_y;\quad 0 = 180 - 40b$

$b = 4.50$ ft **Ans**

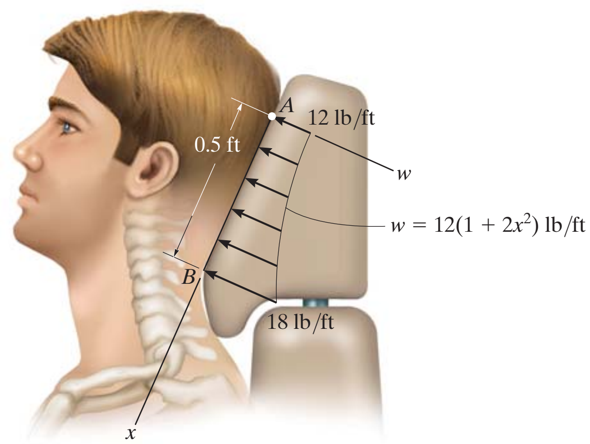
Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

$\curvearrowleft + M_{R_A} = \Sigma M_A;\quad 0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$

$a = 9.75$ ft **Ans**



4–151. Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point A.

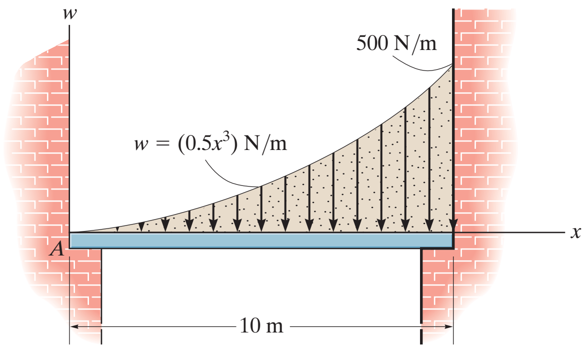


$$F_R = \int w(x)\,dx = \int_0^{0.5} 12\left(1 + 2x^2\right)dx = 12\left[x + \frac{2}{3}x^3\right]_0^{0.5} = 7\text{ lb} \quad \textbf{Ans}$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^{0.5} x(12)\left(1 + 2x^2\right)dx}{7} = \frac{12\left[\frac{x^2}{2} + (2)\frac{x^4}{4}\right]_0^{0.5}}{7}$$

$\bar{x} = 0.268$ ft **Ans**

***4–152.** Wind has blown sand over a platform such that the intensity of the load can be approximated by the function $w = (0.5x^3)$ N/m. Simplify this distributed loading to an equivalent resultant force and specify its magnitude and location measured from A.



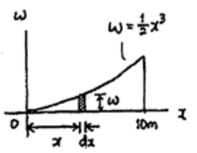
$dA = w\,dx$

$F_R = \int dA = \int_0^{10} \frac{1}{2}x^3\,dx$

$= \left[\frac{1}{8}x^4\right]_0^{10}$

$= 1250$ N

$F_R = 1.25$ kN **Ans**



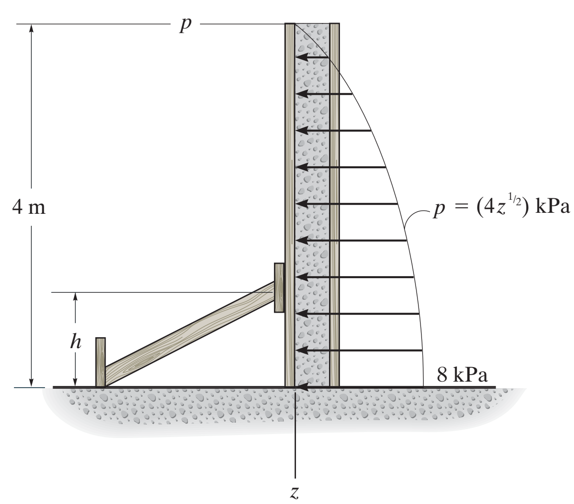
$\int \tilde{x}\,dA = \int_0^{10} \frac{1}{2}x^4\,dx$

$= \left[\frac{1}{10}x^5\right]_0^{10}$

$= 10\,000$ N·m

$\bar{x} = \frac{10\,000}{1250} = 8.00$ m **Ans**

•4–153. Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.

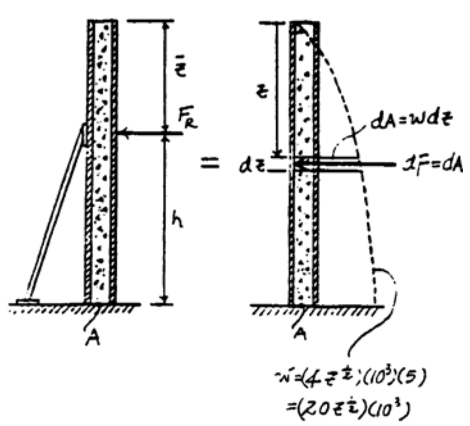


Equivalent Resultant Force :

$\xrightarrow{+} F_R = \Sigma F_x; \quad -F_R = -\int_A dA = -\int_0^z w\,dz$

$F_R = \int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)\left(10^3\right) dz$

$= 106.67\left(10^3\right)$ N $= 107$ kN $\leftarrow$ **Ans**



Location of Equivalent Resultant Force :

$\bar{z} = \frac{\int_A \tilde{z}\,dA}{\int_A dA} = \frac{\int_0^z zw\,dz}{\int_0^z w\,dz}$

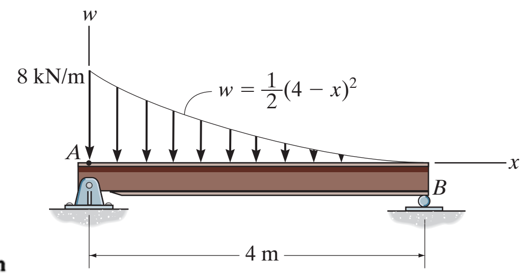
$= \frac{\int_0^{4\text{m}} z\left[\left(20z^{\frac{1}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)(10^3)\,dz}$

$= \frac{\int_0^{4\text{m}} \left[\left(20z^{\frac{3}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)(10^3)\,dz}$

$= 2.40$ m

Thus, $\quad h = 4 - \bar{z} = 4 - 2.40 = 1.60$ m **Ans**

4–154. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Resultant: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. *a*. Thus,

$$dF_R = w\,dx = \frac{1}{2}(4-x)^2\,dx = \left(\frac{x^2}{2} - 4x + 8\right)dx$$

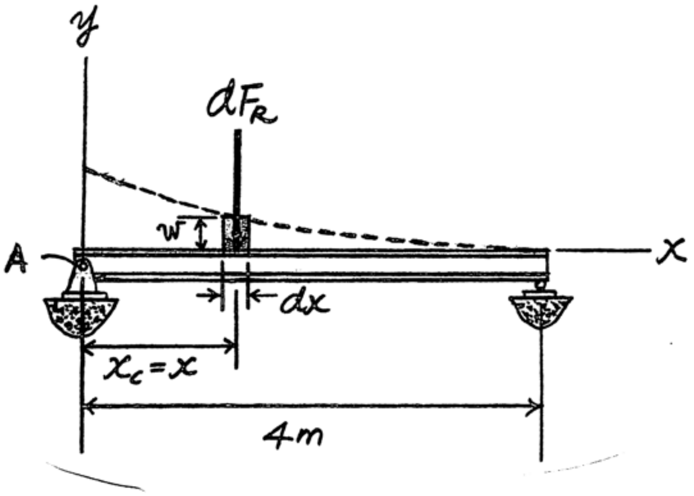
Integrating $d\mathbf{F}_R$ over the entire length of the beam gives the resultant force $\mathbf{F}_R$.

$$+\downarrow \qquad F_R = \int_L dF_R = \int_0^{4\text{ m}} \left(\frac{x^2}{2} - 4x + 8\right)dx = \left(\frac{x^3}{6} - 2x^2 + 8x\right)\Bigg|_0^{4\text{ m}}$$

$$= 10.667\text{ kN} = 10.7\text{ kN} \downarrow \qquad \textbf{Ans.}$$

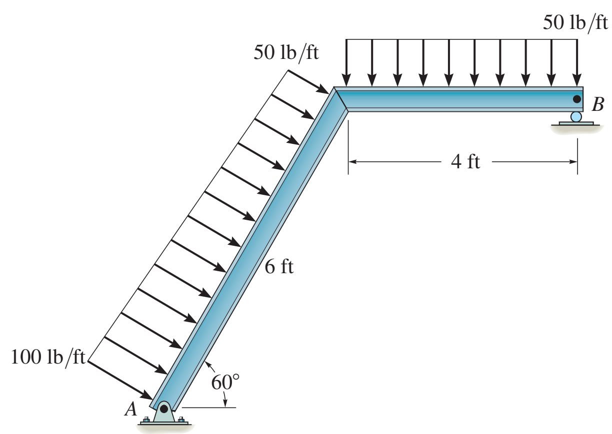
Location. The location of $d\mathbf{F}_R$ on the beam is $x_c = x$, measured from point A. Thus, the location $\bar{x}$ of $\mathbf{F}_R$ measured from point A is

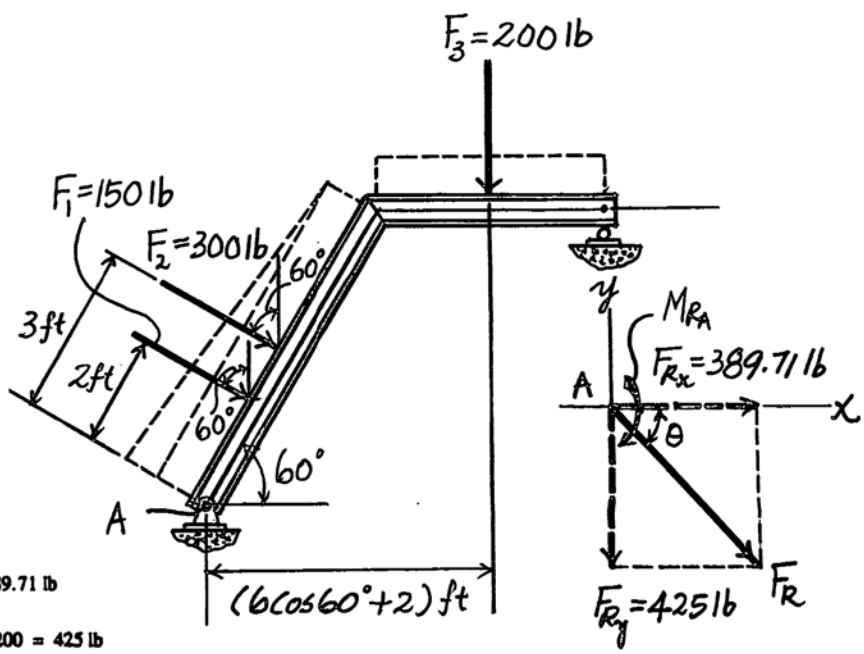
$$\bar{x} = \frac{\int_L x_c\,dF_R}{\int_L dF_R} = \frac{\int_0^{4\text{ m}} x\left(\frac{x^2}{2} - 4x + 8\right)dx}{10.667} = \frac{\left(\frac{x^4}{8} - \frac{4x^3}{3} + 4x^2\right)\Bigg|_0^{4\text{ m}}}{10.667} = 1\text{ m} \qquad \textbf{Ans.}$$



(a)

4–155. Replace the loading by an equivalent resultant force and couple moment at point A.





$F_1 = \frac{1}{2}(6)(50) = 150$ lb

$F_2 = (6)(50) = 300$ lb

$F_3 = (4)(50) = 200$ lb

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 150 \sin 60° + 300 \sin 60° = 389.71$ lb

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 150 \cos 60° + 300 \cos 60° + 200 = 425$ lb

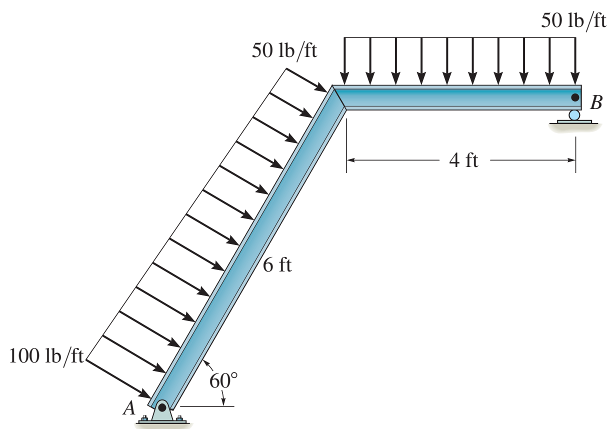
$F_R = \sqrt{(389.71)^2 + (425)^2} = 577$ lb **Ans**

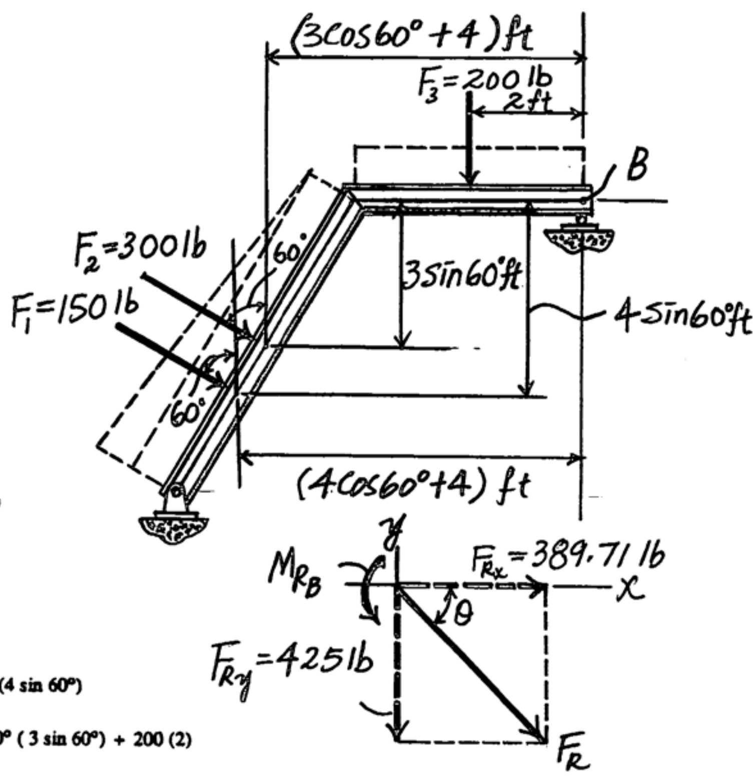
$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5°$ **Ans**

$\curvearrowleft + M_{RA} = \Sigma M_A;\quad M_{RA} = 150(2) + 300(3) + 200(6 \cos 60° + 2)$

$= 2200$ lb · ft $= 2.20$ kip · ft $\curvearrowright$ **Ans**

***4–156.** Replace the loading by an equivalent resultant force and couple moment acting at point *B*.





$F_1 = \frac{1}{2}(6)(50) = 150\text{ lb}$

$F_2 = (6)(50) = 300\text{ lb}$

$F_2 = (4)(50) = 200\text{ lb}$

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 150\sin 60° + 300\sin 60° = 389.71\text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 150\cos 60° + 300\cos 60° + 200 = 425\text{ lb}$

$F_R = \sqrt{(389.71)^2 + (425)^2} = 577\text{ lb}$ **Ans**

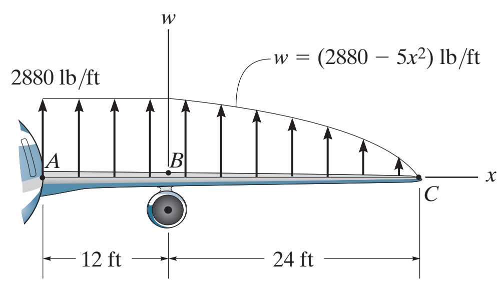
$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5°$ **Ans**

$\curvearrowleft + M_{RB} = \Sigma M_B;\quad M_{RB} = 150\cos 60°(4\cos 60° + 4) + 150\sin 60°(4\sin 60°)$

$+ 300\cos 60°(3\cos 60° + 4) + 300\sin 60°(3\sin 60°) + 200(2)$

$M_{RB} = 2800\text{ lb}\cdot\text{ft} = 2.80\text{ kip}\cdot\text{ft}$ **Ans**

•4–157. The lifting force along the wing of a jet aircraft consists of a uniform distribution along AB, and a semiparabolic distribution along BC with origin at B. Replace this loading by a single resultant force and specify its location measured from point A.



Equivalent Resultant Force :

$+\uparrow F_R = \Sigma F_y;\quad F_R = 34560 + \int_0^x w\,dx$

$$F_R = 34560 + \int_0^{24\text{ ft}} \left(2880 - 5x^2\right) dx$$

$$= 80640 \text{ lb} = 80.6 \text{ kip} \uparrow \qquad \textbf{Ans}$$

Location of Equivalent Resultant Force :

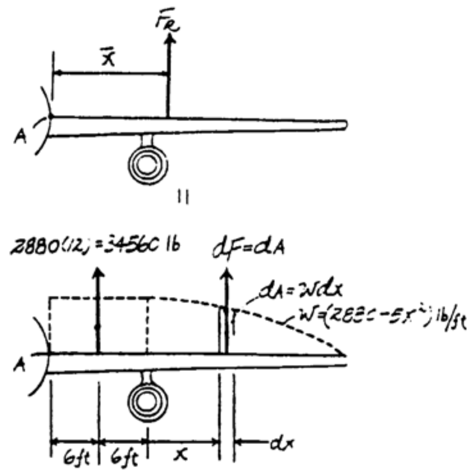
$\curvearrowleft + M_{R_A} = \Sigma M_A;$

$$80640\bar{x} = 34560(6) + \int_0^x (x+12)\,w\,dx$$

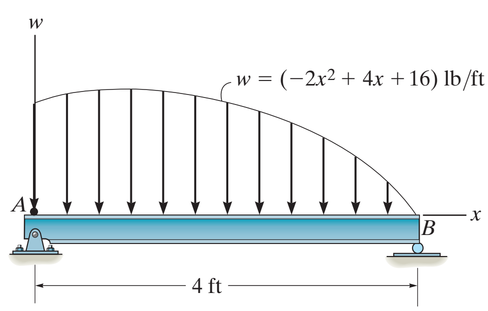
$$80640\bar{x} = 207360 + \int_0^{24\text{ ft}} (x+12)\left(2880 - 5x^2\right) dx$$

$$80640\bar{x} = 207360 + \int_0^{24\text{ ft}} \left(-5x^3 - 60x^2 + 2880x + 34560\right) dx$$

$$\bar{x} = 14.6 \text{ ft} \qquad \textbf{Ans}$$

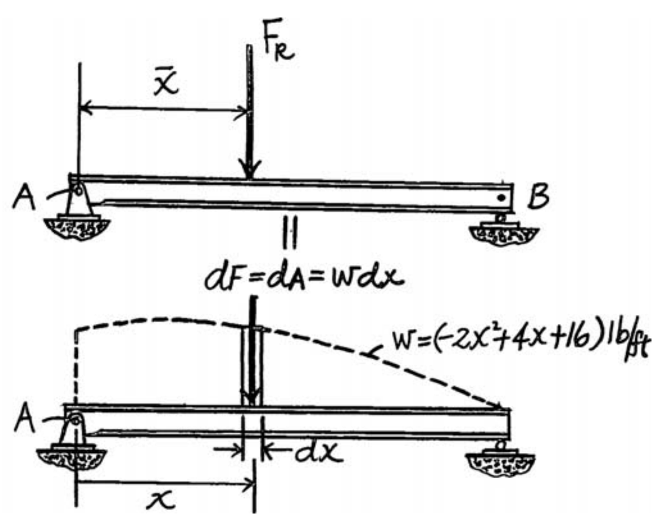


4–158. The distributed load acts on the beam as shown. Determine the magnitude of the equivalent resultant force and specify where it acts, measured from point A.

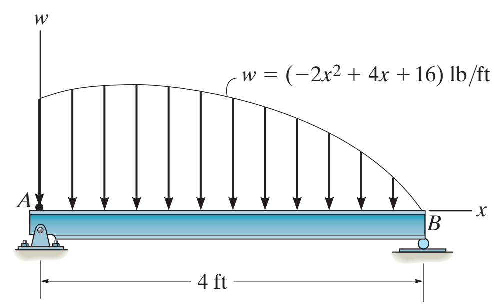


$$F_R = \int w(x)\,dx = \int_0^4 \left(-2x^2 + 4x + 16\right) dx = 53.333 = 53.3 \text{ lb} \qquad \textbf{Ans}$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^4 x\left(-2x^2 + 4x + 16\right) dx}{53.333} = 1.60 \text{ ft} \qquad \textbf{Ans}$$



4–159. The distributed load acts on the beam as shown. Determine the maximum intensity w_{max}. What is the magnitude of the equivalent resultant force? Specify where it acts, measured from point B.



$$\frac{dw}{dx} = -4x + 4 = 0$$

$$x = 1$$

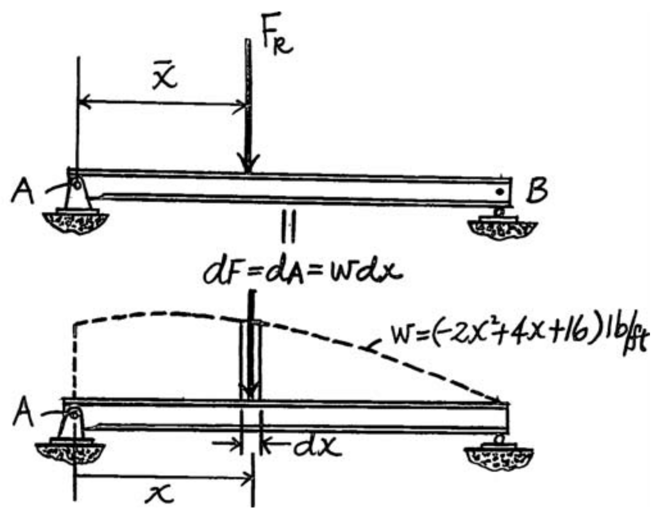
$$w_{max} = -2(1)^2 + 4(1) + 16 = 18 \text{ lb/ft} \quad \textbf{Ans}$$

$$F_R = \int w(x)\,dx = \int_0^4 (-2x^2 + 4x + 16)\,dx = 53.333 = 53.3 \text{ lb} \quad \textbf{Ans}$$

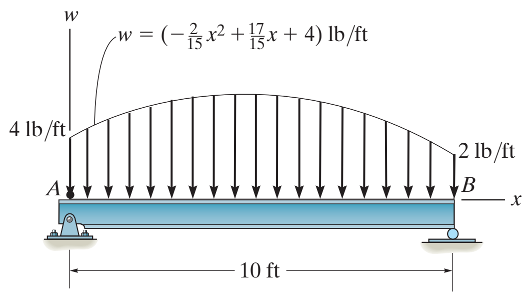
$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^4 x(-2x^2 + 4x + 16)\,dx}{53.333} = 1.60 \text{ ft}$$

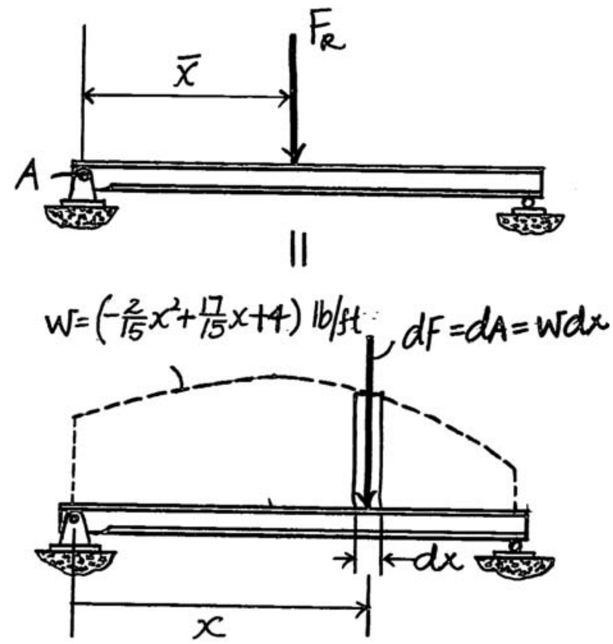
So that from B,

$$x' = 4 - 1.60 = 2.40 \text{ ft} \quad \textbf{Ans}$$



***4–160.** The distributed load acts on the beam as shown. Determine the magnitude of the equivalent resultant force and specify its location, measured from point A.



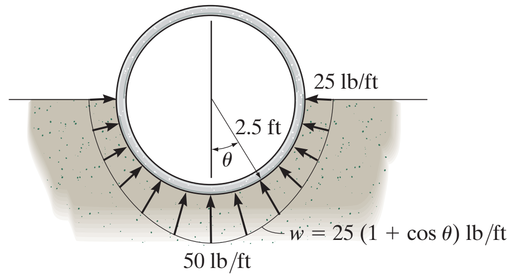


$$F_R = \int w(x)\,dx = \int_0^{10}\left(-\frac{2}{15}x^2 + \frac{17}{15}x + 4\right)dx = 52.22 = 52.2\text{ lb} \qquad \textbf{Ans}$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^{10} x\left(-\frac{2}{15}x^2 + \frac{17}{15}x + 4\right)dx}{52.22} = \frac{244.44}{52.22}$$

$$\bar{x} = 4.68\text{ ft} \qquad \textbf{Ans}$$

•4–161. If the distribution of the ground reaction on the pipe per foot of length can be approximated as shown, determine the magnitude of the resultant force due to this loading.



Resultant Components: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. *a*.

$$d\mathbf{F}_R = wr\,d\theta = 25(1+\cos\theta)(2.5\,d\theta) = 62.5(1+\cos\theta)\,d\theta$$

The horizontal and vertical components of $d\mathbf{F}_R$ are given by

$$\overset{+}{\leftarrow} \qquad (dF_R)_x = dF_R \sin\theta = 62.5(1+\cos\theta)\sin\theta\,d\theta = 62.5\left(\sin\theta + \frac{\sin 2\theta}{2}\right)d\theta$$

$$+\uparrow \qquad (dF_R)_y = dF_R \cos\theta = 62.5(1+\cos\theta)\cos\theta\,d\theta = 62.5\left(\cos\theta + \frac{\cos 2\theta + 1}{2}\right)d\theta$$

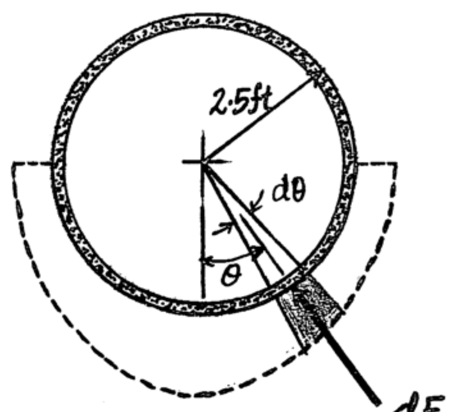
Integrating $(dF_R)_x$ and $(dF_R)_y$ from $\theta = -\frac{\pi}{2}$ rad to $\theta = \frac{-\pi}{2}$ rad gives the horizontal and vertical components of the resultant for $\mathbf{F}_R$.

$$\overset{+}{\leftarrow}(F_R)_x = \int_{-\pi/2}^{\pi/2} 62.5\left(\sin\theta + \frac{\sin 2\theta}{2}\right)d\theta = 62.5\left(-\cos\theta - \frac{\cos 2\theta}{4}\right)\Bigg|_{-\pi/2}^{\pi/2} = 0$$

$$+\uparrow(F_R)_y = \int_{-\pi/2}^{\pi/2} 62.5\left(\cos\theta + \frac{\cos 2\theta + 1}{2}\right)d\theta = 62.5\left(\sin\theta + \frac{\sin 2\theta}{4} + \frac{1}{2}\theta\right)\Bigg|_{-\pi/2}^{\pi/2} = 62.5\left(2 + \frac{\pi}{2}\right) = 223.17\text{ lb}\uparrow$$

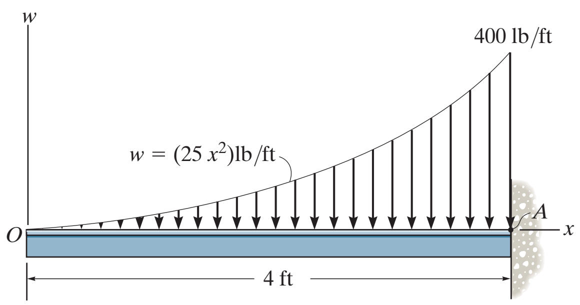
Thus,

$$F_R = (F_R)_y = 223.17\text{ lb} = 223\text{ lb}\uparrow \qquad \textbf{Ans.}$$



(a)

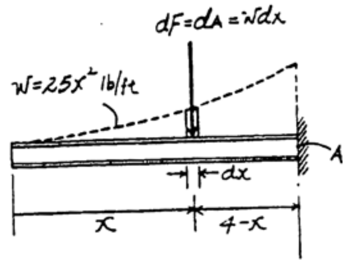
4–162. The beam is subjected to the parabolic loading. Determine an equivalent force and couple system at point A.



$+\uparrow F_R = \Sigma F_y;\quad F_R = -\int_A dA = -\int_0^x w\,dx$

$F_R = -\int_0^{4\text{ ft}} (25x^2)\,dx$

$= -533.33 \text{ lb} = 533 \text{ lb} \downarrow$ **Ans**



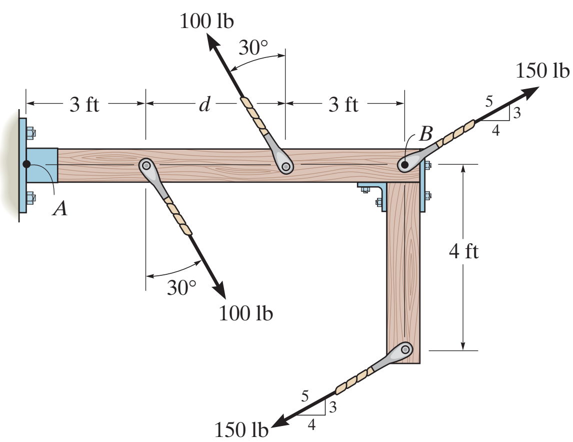
$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad M_{R_A} = \int_0^x (4-x)\,w\,dx$

$= \int_0^{4\text{ ft}} (4-x)(25x^2)\,dx$

$= \int_0^{4\text{ ft}} (-25x^3 + 100x^2)\,dx$

$= 533 \text{ lb}\cdot\text{ft}$ (*Counterclockwise*) **Ans**

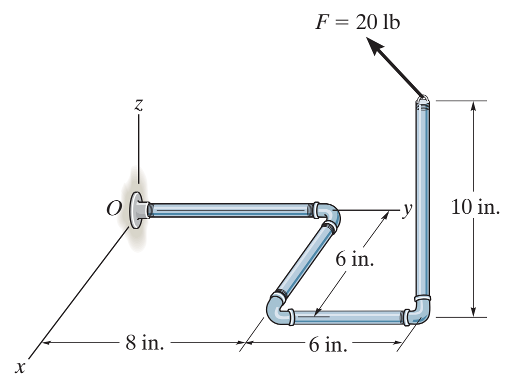
4–163. Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the 100-lb couple forces.



$\circlearrowleft + M_R = 0 = \Sigma M;\quad 0 = 100\cos 30°(d) - \frac{4}{5}(150)(4)$

$d = 5.54 \text{ ft}$ **Ans**

***4–164.** Determine the coordinate direction angles α, β, γ of **F**, which is applied to the end of the pipe assembly, so that the moment of **F** about O is zero.



Require $\mathbf{M}_O = \mathbf{0}$. This happens when force **F** is directed along line OA either from point O to A or from point A to O. The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_{AO}$ are

$$\mathbf{u}_{OA} = \frac{(6-0)\mathbf{i}+(14-0)\mathbf{j}+(10-0)\mathbf{k}}{\sqrt{(6-0)^2+(14-0)^2+(10-0)^2}}$$
$$= 0.3293\mathbf{i}+0.7683\mathbf{j}+0.5488\mathbf{k}$$

Thus,

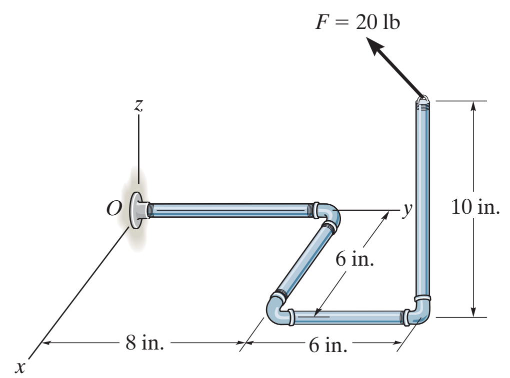
$$\alpha = \cos^{-1}0.3293 = 70.8° \qquad \textbf{Ans}$$
$$\beta = \cos^{-1}0.7683 = 39.8° \qquad \textbf{Ans}$$
$$\gamma = \cos^{-1}0.5488 = 56.7° \qquad \textbf{Ans}$$

$$\mathbf{u}_{AO} = \frac{(0-6)\mathbf{i}+(0-14)\mathbf{j}+(0-10)\mathbf{k}}{\sqrt{(0-6)^2+(0-14)^2+(0-10)^2}}$$
$$= -0.3293\mathbf{i}-0.7683\mathbf{j}-0.5488\mathbf{k}$$

Thus,

$$\alpha = \cos^{-1}(-0.3293) = 109° \qquad \textbf{Ans}$$
$$\beta = \cos^{-1}(-0.7683) = 140° \qquad \textbf{Ans}$$
$$\gamma = \cos^{-1}(-0.5488) = 123° \qquad \textbf{Ans}$$

•4–165. Determine the moment of the force **F** about point O. The force has coordinate direction angles of $\alpha = 60°$, $\beta = 120°$, $\gamma = 45°$. Express the result as a Cartesian vector.



Position Vector And Force Vectors :

$$\mathbf{r}_{OA} = \{(6-0)\mathbf{i}+(14-0)\mathbf{j}+(10-0)\mathbf{k}\}\ \text{in.}$$
$$= \{6\mathbf{i}+14\mathbf{j}+10\mathbf{k}\}\ \text{in.}$$

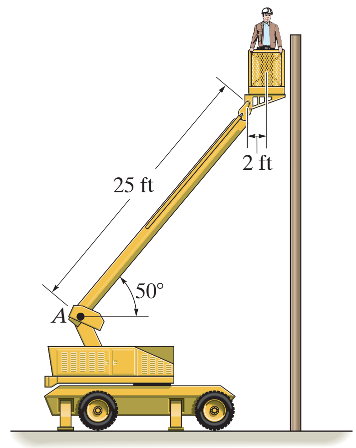
$$\mathbf{F} = 20(\cos 60°\mathbf{i}+\cos 120°\mathbf{j}+\cos 45°\mathbf{k})\ \text{lb}$$
$$= \{10.0\mathbf{i}-10.0\mathbf{j}+14.142\mathbf{k}\}\ \text{lb}$$

Moment of Force* F *About Point O : Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 14 & 10 \\ 10.0 & -10.0 & 14.142 \end{vmatrix}$$

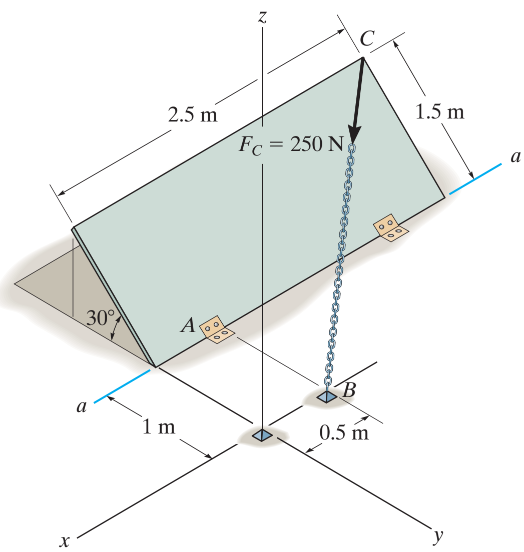
$$= \{298\mathbf{i}+15.1\mathbf{j}-200\mathbf{k}\}\ \text{lb}\cdot\text{in} \qquad \textbf{Ans}$$

4–166. The snorkel boom lift is extended into the position shown. If the worker weighs 160 lb, determine the moment of this force about the connection at A.



$$M_A = 160(2 + 25\cos 50°) = 2891 \text{ lb}\cdot\text{ft} = 2.89 \text{ kip}\cdot\text{ft} \quad \textbf{Ans}$$

4–167. Determine the moment of the force $\mathbf{F}_C$ about the door hinge at A. Express the result as a Cartesian vector.



Position Vector And Force Vector :

$$\mathbf{r}_{AB} = \{[-0.5 - (-0.5)]\mathbf{i} + [0 - (-1)]\mathbf{j} + (0 - 0)\mathbf{k}\} \text{ m} = \{1\mathbf{j}\} \text{ m}$$

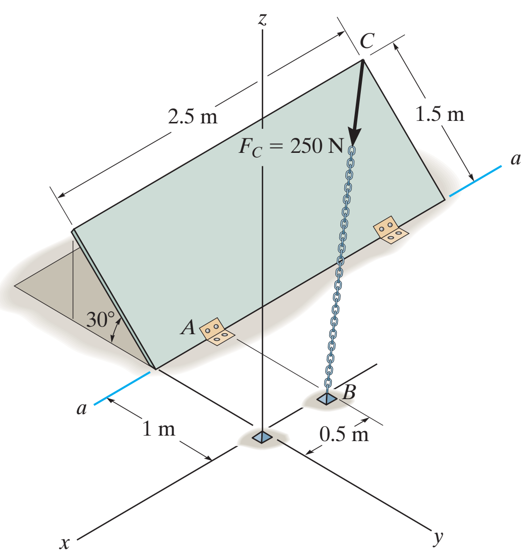
$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i} + \{0 - [-(1 + 1.5\cos 30°)]\}\mathbf{j} + (0 - 1.5\sin 30°)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2 + \{0 - [-(1+1.5\cos 30°)]\}^2 + (0 - 1.5\sin 30°)^2}}\right) \text{N}$$
$$= \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\} \text{ N}$$

Moment of Force $\mathbf{F}_C$ About Point A : Applying Eq. 4–7, we have

$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

$$= \{-59.7\mathbf{i} - 159\mathbf{k}\} \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

***4–168.** Determine the magnitude of the moment of the force $\mathbf{F}_C$ about the hinged axis aa of the door.



$$\mathbf{r}_{AB} = \{[-0.5 - (-0.5)]\mathbf{i} + [0 - (-1)]\mathbf{j} + (0 - 0)\mathbf{k}\} \text{ m} = \{1\mathbf{j}\} \text{ m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i} + \{0 - [-(1 + 1.5\cos 30°)]\}\mathbf{j} + (0 - 1.5\sin 30°)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2 + \{0 - [-(1+1.5\cos 30°)]\}^2 + (0 - 1.5\sin 30°)^2}}\right) \text{N}$$
$$= \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\} \text{ N}$$

Moment of Force $\mathbf{F}_C$ About a-aAxis : The unit vector along the $a-a$ axis is $\mathbf{i}$. Applying Eq. 4–11, we have

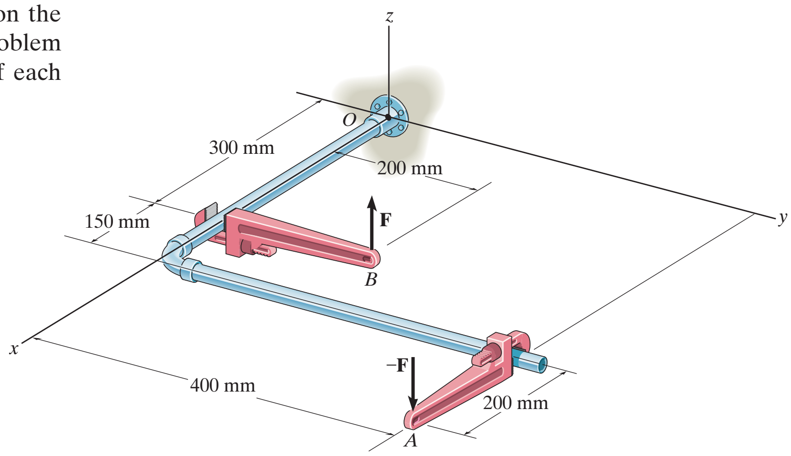
$$M_{a-a} = \mathbf{i}\cdot(\mathbf{r}_{AB} \times \mathbf{F}_C)$$
$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

$$= 1[1(-59.75) - (183.15)(0)] - 0 + 0$$
$$= -59.7 \text{ N}\cdot\text{m}$$

The negative sign indicates that $\mathbf{M}_{a-a}$ is directed toward negative x axis.

$$M_{a-a} = 59.7 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

•4–169. Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem (a) using Eq. 4–13 and (b) summing the moment of each force about point O. Take $\mathbf{F} = \{25\mathbf{k}\}$ N.



a) $\mathbf{M}_C = \mathbf{r}_{AB} \times (25\,\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}$$

$\mathbf{M}_C = \{-5\,\mathbf{i} + 8.75\,\mathbf{j}\}$ N·m   **Ans**

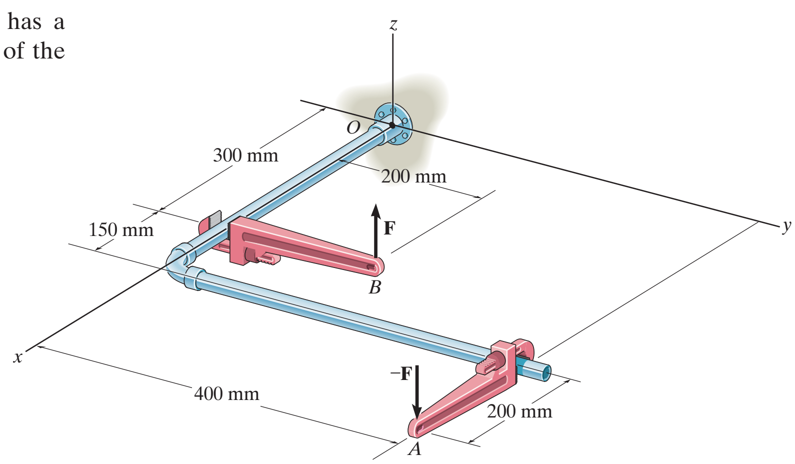
(b) $\mathbf{M}_C = \mathbf{r}_{OB} \times (25\,\mathbf{k}) + \mathbf{r}_{OA} \times (-25\,\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.4 & 0 \\ 0 & 0 & -25 \end{vmatrix}$$

$\mathbf{M}_C = (5 - 10)\,\mathbf{i} + (-7.5 + 16.25)\,\mathbf{j}$

$\mathbf{M}_C = \{-5\,\mathbf{i} + 8.75\,\mathbf{j}\}$ N·m   **Ans**

4–170. If the couple moment acting on the pipe has a magnitude of 400 N · m, determine the magnitude F of the vertical force applied to each wrench.



$\mathbf{M}_C = \mathbf{r}_{AB} \times (F\,\mathbf{k})$

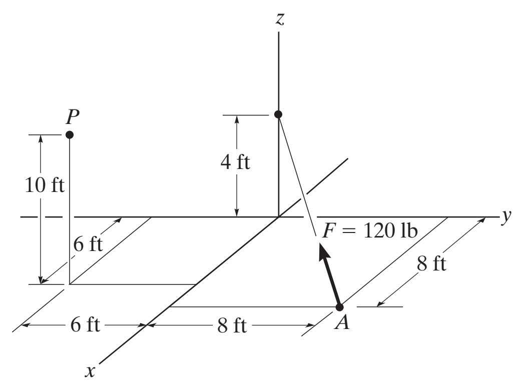
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$\mathbf{M}_C = \{-0.2F\,\mathbf{i} + 0.35F\,\mathbf{j}\}$ N·m

$M_C = \sqrt{(-0.2F)^2 + (0.35F)^2} = 400$

$$F = \frac{400}{\sqrt{(-0.2)^2 + (0.35)^2}} = 992 \text{ N}$$

4–171. Replace the force at A by an equivalent resultant force and couple moment at point P. Express the results in Cartesian vector form.

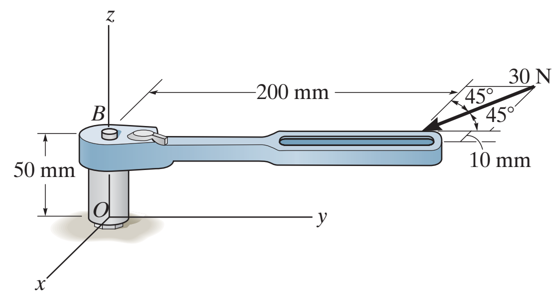


$$\mathbf{F}_R = 120\left(\frac{-8\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}}{\sqrt{(-8)^2 + (-8)^2 + 4^2}}\right) = \{-80\mathbf{i} - 80\mathbf{j} + 40\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{M}_{RP} = \Sigma\mathbf{M}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 14 & -10 \\ -80 & -80 & 40 \end{vmatrix}$$

$$= \{-240\mathbf{i} + 720\mathbf{j} + 960\mathbf{k}\}\text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

*__4–172.__ The horizontal 30-N force acts on the handle of the wrench. Determine the moment of this force about point O. Specify the coordinate direction angles α, β, γ of the moment axis.



Position Vector And Force Vectors :

$$\mathbf{r}_{OA} = \{(-0.01-0)\mathbf{i} + (0.2-0)\mathbf{j} + (0.05-0)\mathbf{k}\}\text{ m}$$
$$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})\text{ N}$$
$$= \{21.213\mathbf{i} - 21.213\mathbf{j}\}\text{ N}$$

Moment of Force F About Point O : Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$$= \{1.061\mathbf{i} + 1.061\mathbf{j} - 4.031\mathbf{k}\}\text{ N}\cdot\text{m}$$
$$= \{1.06\mathbf{i} + 1.06\mathbf{j} - 4.03\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \sqrt{1.061^2 + 1.061^2 + (-4.031)^2} = 4.301\text{ N}\cdot\text{m}$$

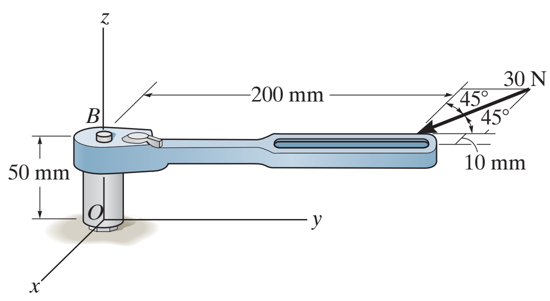
The coordinate direction angles for $\mathbf{M}_O$ are

$$\alpha = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-4.031}{4.301}\right) = 160° \qquad \textbf{Ans}$$

•**4–173.** The horizontal 30-N force acts on the handle of the wrench. What is the magnitude of the moment of this force about the z axis?



Position Vector And Force Vectors :

$$\mathbf{r}_{BA} = \{-0.01\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}$$
$$\mathbf{r}_{OA} = \{(-0.01-0)\mathbf{i} + (0.2-0)\mathbf{j} + (0.05-0)\mathbf{k}\}\ \text{m}$$
$$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\ \text{m}$$

$$\mathbf{F} = 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})\ \text{N}$$
$$= \{21.213\mathbf{i} - 21.213\mathbf{j}\}\ \text{N}$$

Moment of Force* F *About z Axis : The unit vector along the z axis is $\mathbf{k}$. Applying Eq. 4 – 11, we have

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{BA} \times \mathbf{F})$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$$= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]$$
$$= -4.03\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

Or

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{BA} \times \mathbf{F})$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$$= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]$$
$$= -4.03\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

The negative sign indicates that $\mathbf{M}_z$ is directed along the negative z axis.