NOTRE DAME UNIVERSITY

Faculty of Engineering
Department of Civil and Environmental Engineering
Spring 2010

Instructor : ________________

Course Code : CEN 202 - Statics **Section:** _______

Exam No. 1 Closed Book, Closed Notes **Time:** 1½ Hours

Name : _____key____________key________
Last , First

ID No.: ________________

Problem	Points
1	
2	
3	

TOTAL: $\frac{\quad}{100}$

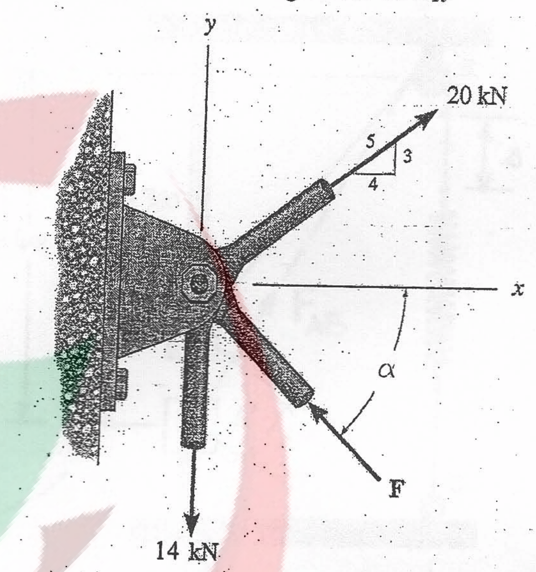
Grade = $\frac{\quad}{100}$

PROBLEM 1: **(35 points)**

The assembly is subjected to the three forces shown, where $0 \le \alpha \le 180°$.

a. If $\alpha = 30°$, what is the value of the force **F** that makes the resultant $\mathbf{F}_R$ as small as possible? Calculate then the magnitude of $\mathbf{F}_R$.

b. If $F = 16.1245$ kN, what is the value of the angle α that makes the resultant $\mathbf{F}_R$ as small as possible? Calculate then the magnitude of $\mathbf{F}_R$.



$F_{Rx} = 20\left(\frac{4}{5}\right) - F\cos\alpha = 16 - F\cos\alpha = 16 - \frac{F\sqrt{3}}{2}$ 6

$F_{Ry} = 20\left(\frac{3}{5}\right) - 14 + F\sin\alpha = \frac{F}{2} - 2$ 6

$F_R^2 = F_{Rx}^2 + F_{Ry}^2 = (16 - F\cos\alpha)^2 + (F\sin\alpha - 2)^2 = F^2 - 2(16\cos\alpha + 2\sin\alpha)F + 260$

$F_R^2 = F^2 - 4(8\cos\alpha + \sin\alpha)F + 260 = F^2 - 4\left(8\frac{\sqrt{3}}{2} + \frac{1}{2}\right)F + 260$ [1]

$= F^2 - 2(8\sqrt{3} + 1)F + 260 \Rightarrow F_R = \left[F^2 - 4(8\cos\alpha + \sin\alpha)F + 260\right]^{1/2}$ 3

a) $\frac{dF_R^2}{dF} = 0 \Rightarrow$ 6 $2F - 4(8\cos\alpha + \sin\alpha) = 0 \Rightarrow F = 2(8\cos\alpha + \sin\alpha)$ [2]

$F_R = \left[260 - 4(8\cos\alpha + \sin\alpha)^2\right]^{1/2}$ [3]

$\alpha = 30° \Rightarrow [2] \Rightarrow F = 14.8564$ kN 2 , $[3] \Rightarrow F_R = 6.2679$ kN 2

b) $[1] \Rightarrow \frac{dF_R^2}{d\alpha} = 0 \Rightarrow$ 6 $0 - 4(-8\sin\alpha + \cos\alpha)F + 0 = 0 \Rightarrow \tan\alpha = \frac{1}{8}$

$\Rightarrow \alpha = \tan^{-1}(1/8) + k\pi = 0.1244 \text{ rad} + k\pi$

$\alpha = 7.1250° + k(180°)$

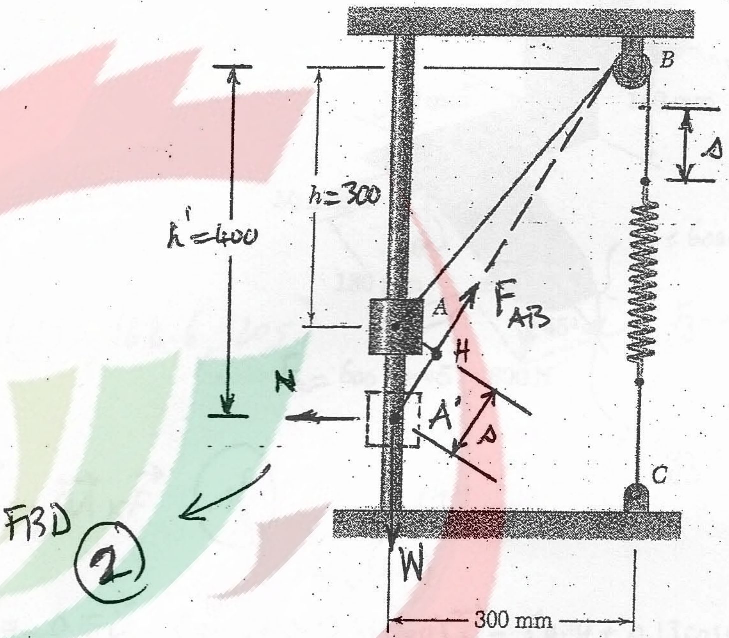
1 $\alpha = 7.1250° \Rightarrow F_R = 0.0000$ kN 1

1 $\alpha = 187.1250° \Rightarrow F_R = 32.2490$ kN 1

PROBLEM 2: **(35 points)**

Collar ***A*** shown in the figure can slide on a frictionless vertical rod and is attached as shown to a spring. The constant of the spring is 600 N/m, and the spring is unstretched when $h = 300$ mm. Knowing that the system is in equilibrium when $h = 400$ mm:

a. Determine the weight of the collar.
b. Determine the tension in cable ***AB***.
c. Determine the normal reaction exerted by the vertical rod on the collar ***A***.



AB when the spring is unstretched $= \sqrt{0.3^2+0.3^2} = 0.3\sqrt{2} = 0.4243$ m (3)

Collar moves from A to A' such that: $A'B = \sqrt{0.3^2+0.4^2} = 0.5000$ m (3)

Since the cable is inextensible ⇒ the stretch Δ in the spring is:

$\Delta = A'B - AB = 0.5 - 0.4243 \text{ m} = 0.0757 \text{ m}$ (6)

⇒ $F_{AB} = k\Delta = 600(0.0757) = 45.4416$ N (5)

(6) (2)

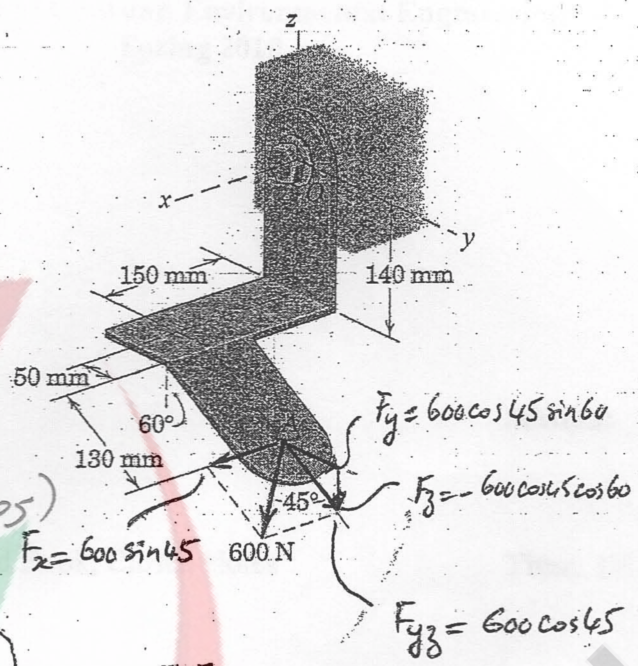
FBD of collar at A' = $+\uparrow \sum F_y = 0 \Rightarrow F_{AB}\left(\frac{4}{5}\right) - W = 0 \Rightarrow W = 36.3532\text{N}$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -N + F_{AB}\left(\frac{3}{5}\right) = 0 \Rightarrow 27.2649 \text{ N} = N$

(6) (2)

PROBLEM 3: **(30 points)**

Determine the vector expression for the moment $\mathbf{M}_O$ of the 600-N force about point O.



A(150, 162.6, -205)

$\vec{M}_o = \vec{OA} \times \vec{F}$ (1 1/2) [1]

5 x (1 1/2)

$\vec{OA} = 0.15\vec{i} + (0.05 + 0.13\sin 60)\vec{j} - (0.14 + 0.13\cos 60)\vec{k}$

$\vec{OA} = 0.15\vec{i} + 0.1626\vec{j} - 0.2050\vec{k}$ (1 1/2)

4 x (1 1/2)

$\vec{F} = F_x\vec{i} + \vec{F}_{yz} = 600\sin 45\vec{i} + 600\cos 45(\sin 60\vec{j} - \cos 60\vec{k})$

$= 600\sin 45\vec{i} + 600\cos 45\sin 60\vec{j} - 600\cos 45\cos 60\vec{k} =$

$= 424.2641\vec{i} + 367.4235\vec{j} - 212.1320\vec{k}$ (1 1/2)

$$[1] \Rightarrow \quad \vec{M}_o = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.15 & 0.1626 & -0.2050 \\ 424.2641 & 367.4235 & -212.1320 \end{vmatrix} \quad (1\tfrac{1}{2})$$

6 x (1 1/2)

$$= \big(0.1626(-212.1320) - (-0.2050)(367.4235)\big)\vec{i} - \big(0.15(-212.1320) - (-0.2050)(424.2641)\big)\vec{j} + \big(0.15(367.4235) - 0.1626(424.2641)\big)\vec{k}$$

$= 40.8327\vec{i} - 55.1543\vec{j} - 13.8647\vec{k}$ (1 1/2)