CEN 100 | 100 | CEN 201

CEN100A – CEN201 (B+C) – FALL 03 – EX#1

Solutions – Dr. E. Cilaksr

EX1-1/5

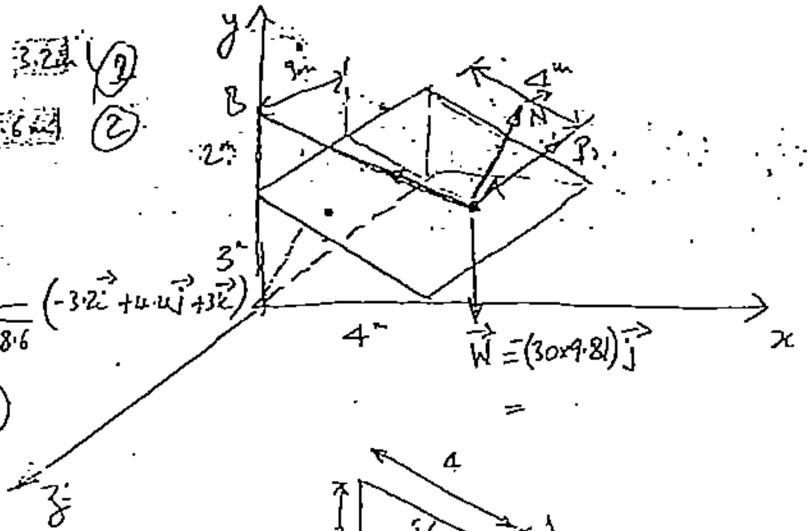
22/1/04

Pb 2.76 (Beer):

$$\frac{y_A}{3}=\frac{4-x_A}{4}=\frac{1}{5} \Rightarrow x_A = 4-\frac{4}{5}=3.2\,m \quad (2)$$

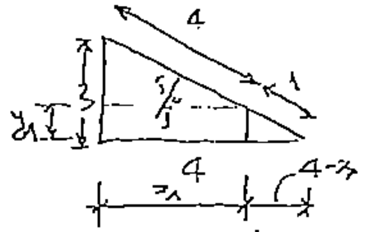
$$\Rightarrow y_A=\frac{3}{5}=0.6\,m \quad (2)$$

$$z_A=-3\,m$$



$$\vec{u}_{AB}=\frac{-3.2\vec{i}+(5-0.6)\vec{j}+3\vec{k}}{\sqrt{3.2^2+4.4^2+3^2}}\,(3)(1)=\frac{1}{\sqrt{38.6}}\left(-3.2\vec{i}+4.4\vec{j}+3\vec{k}\right)$$

$$\vec{u}_{AB}=-0.515\vec{i}+0.708\vec{j}+0.483\vec{k} \quad (1)$$



$$\vec{F}_{AB}=F_{AB}\vec{u}_{AB} \quad (2)$$

$$\vec{u}_N=\frac{3}{5}\vec{i}+\frac{4}{5}\vec{j}+0\vec{k} \quad (5)$$

$$\Rightarrow \vec{N}=N\vec{u}_N \quad (2)$$

$$\vec{W}=-294.3\vec{j} \quad (1)$$

$$\vec{P}=-P\vec{k} \quad (1)$$

$$\sum\vec{F}=\vec{0} \Rightarrow (4)\ \sum F_x=0=-0.515F_{AB}+\frac{3}{5}N=0 \quad (1) \Rightarrow \boxed{N=\frac{2.575}{3}F_{AB}}$$

$$(4)\ \sum F_y=0=0.708F_{AB}+\frac{4}{5}N-294.3=0 \quad (2)$$

$$(4)\ \sum F_z=0=0.483F_{AB}-P=0 \quad (3)$$

$$(2) \Rightarrow 0.708F_{AB}+\frac{4}{5}\times\frac{2.575}{3}\times F_{AB}-294.3=0 \Rightarrow \Rightarrow \boxed{F_{AB}=211.618\,N}\ (1)$$

$$\Rightarrow \boxed{N=181.624\,N}\ (1)$$

$$(3) \Rightarrow \boxed{P=101.922\,N}\ (1)$$

Ex 1-2

Pb 4.67 (modified): (2)

coordinates of point F with $\vec{CF}$ or $\vec{DF}$ (6) (6)

E is point of application of $\vec F$

$M_{b-b'}$ of F $\qquad$ b-b' = CD

$$\vec{u}_{b-b'} = \frac{-600\vec{i} + 200\vec{j} + 300\vec{k}}{\sqrt{600^2+200^2+300^2}} = -\frac{6}{7}\vec{i} + \frac{2}{7}\vec{j} + \frac{3}{7}\vec{k} \quad (4)\ (1)\ (1)\ (6)$$

$$\underbrace{M_{b-b'} = \vec{u}_{b-b'} \cdot (\vec{r}_{CF} \times \vec{F})}_{3} = \underbrace{\begin{vmatrix} -\frac{6}{7} & \frac{2}{7} & \frac{3}{7} \\ \underbrace{\left(500+150\times\frac{3}{5}-200\times\frac{4}{5}\right)}_{430} & \underbrace{\left(200+150\times\frac{4}{5}+200\times\frac{3}{5}\right)}_{440} & 0 \\ 0 & 0 & -60 \end{vmatrix}}_{12} \quad (4)\ (15)$$

$\vec{CF} = 430\vec{i} + 440\vec{j}$

$$= -\frac{6}{7}\left(440\times(-60) - 0\times 0\right) - \frac{2}{7}\times\left(430\times(-60)\right) + \frac{3}{7}\times 0$$

$$= \frac{+210000}{7} = 30000 \text{ N·m} = 30 \text{ N·m} \quad (2)$$

$$M_{b-b'} = \vec{u}_{b-b'} \cdot (\vec{r}_{AF} \times \vec{F}) = \begin{vmatrix} -\frac{6}{7} & \frac{2}{7} & \frac{3}{7} \\ \underbrace{\left(600+500+150\times\frac{3}{5}-200\times\frac{4}{5}\right)}_{1030} & \underbrace{\left(150\times\frac{4}{5}+200\times\frac{3}{5}\right)}_{240} & -300 \\ 0 & 0 & -60 \end{vmatrix}$$

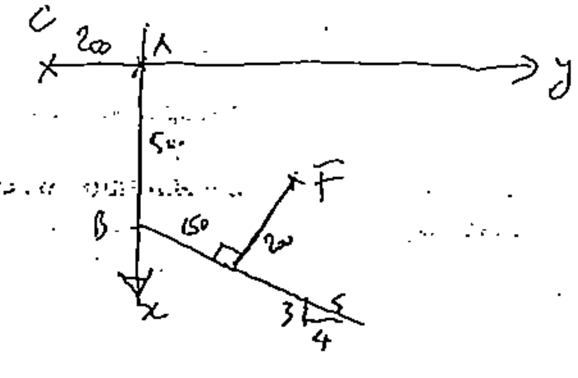
$$= -60\times\left(-\frac{6}{7}\times 240 - 1030\times\frac{2}{7}\right) = \frac{210000}{7} = 30.000 \text{ N·m}$$

Cartesian vector:

$$\vec{M}_{b-b'} = M_{b-b'} \cdot \vec{u}_{b-b'} = 30\times\left(-\frac{6}{7}\vec{i} + \frac{2}{7}\vec{j} + \frac{3}{7}\vec{k}\right)$$

$$= -25.714\vec{i} + 8.571\vec{j} + 12.857\vec{k}$$

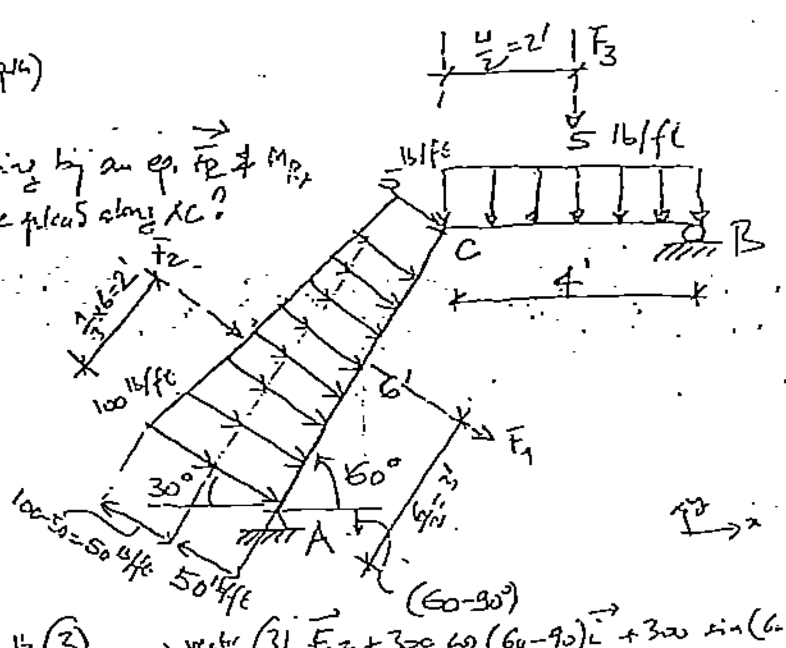
$$\vec{AF} = \left(500 + 150\times\frac{3}{5} - 200\times\frac{4}{5}\right)\vec{i} + \left(150\times\frac{4}{5} + 200\times\frac{3}{5}\right)\vec{j} + 0\vec{k} = 430\vec{i} + 240\vec{j} + 0\vec{k}$$



Pb 4.153: (9%)

Replace the loading by an eq. $\vec{F_R}$ & M_{R_A}

where should F_R be placed along AC?



$F_1 = 50\times 6 = 300^{lb}$ (3) → vector (3) $\vec{F_1} = +300\cos(60-90)\vec{i} + 300\sin(60-90)\vec{j} = 300\left(\frac{\sqrt{3}}{2}\vec{i} - \right.$

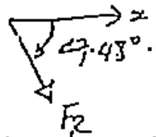
$F_2 = (100-50)\times\frac{6}{2} = 150^{lb}$ (3) → " (3) $\vec{F_2} = 150\left(\frac{\sqrt{3}}{2}\vec{i} - \frac{1}{2}\vec{j}\right)$

$F_3 = 5\times 4 = 200^{lb}$ (2) → , (2) $\vec{F_3} = -200\vec{j}$

$F_{Rx} = F_1\cos(60-90) + F_2\cos(60-90) + F_3\cos(-90) = 389.711^{lb}$ (2) $\Rightarrow \cancel{F_R = 576}.628$ lb.

$F_{Ry} = F_1\sin(60-90) + F_2\sin(60-90) + F_3\sin(-90) = -425^{lb}$ (2)

$tg\,\theta = \frac{F_{Ry}}{F_{Rx}} = -\frac{425}{389.711} = -1.09055 \Rightarrow \theta = -47.48°$ (3)



$M_{R_A} = -F_1\times\frac{6}{2} - F_2\times\frac{6}{3} - F_3\times\left(6\cos 60 + \frac{4}{2}\right) = -2200$ lb·ft $= 2200$ lb·ft ↻

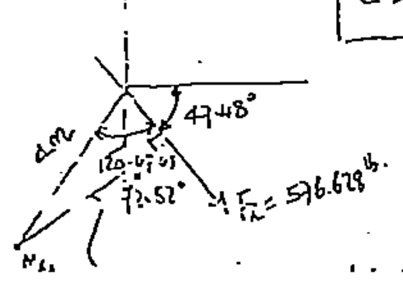
location of $\vec{F_R}$ along AC: $\vec{d} = d_x\vec{i} + d_y\vec{j} = d\cos 60\,\vec{i} + d\sin 60\,\vec{j}$

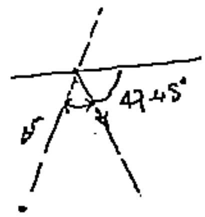
$$\vec{M_{R_A}} = \vec{d}\times\vec{F_R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ d_x & d_y & 0 \\ F_{Rx} & F_{Ry} & 0 \end{vmatrix} = \left(d_x F_{Ry} - d_y F_{Rx}\right)\vec{k} = M_{R_A}\vec{k}$$

$$d_x F_{Ry} - d_y F_{Rx} = M_{R_A}$$

$$d\cos 60\, F_{Ry} - d\sin 60\, F_{Rx} = M_{R_A} \Rightarrow d = \frac{M_{R_A}}{F_{Ry}\cos 60 - F_{Rx}\sin 60} = \frac{-2200}{-425\times\cos 60 - 389.711\times\sin 60}$$

$$\boxed{d = 4\,ft}$$





Pb 4.153 (9th):

(b) $M_{R_C} = F_1 \times \frac{6}{2} + F_2 \times \frac{2}{3} \times 6 - F_3 \times \frac{4}{2} = 300 \times 3 + 150 \times 4 - 200 \times 2 = 1100$ lb·ft
$= 900 + 600 - 400 = 1100$

location of $\vec{F_R}$ along AC measured from C is $\vec{d} = d_x \vec{i} + d_y \vec{j} = d\cos 60 \vec{i} + d \sin 60 \vec{j}$ (3)

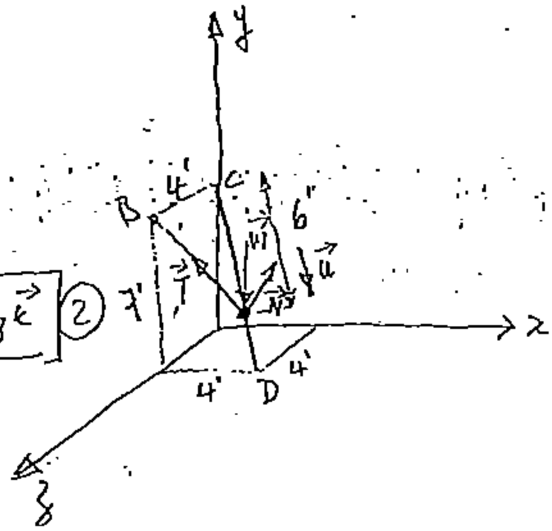
$$\vec{M_{R_C}} = \vec{d} \times \vec{F_R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ d\cos 60 & d\sin 60 & 0 \\ 389.711 & -425 & 0 \end{vmatrix} = d(-425 \times \cos 60 - 389.711 \times \sin 60)\vec{k} = M_{R_C} \vec{k}$$
$$= -550 d \vec{k} = 1100 \vec{k} \quad (3)$$

$$\Rightarrow \quad \boxed{d = -2 \text{ ft}}$$

Pb 2.86 (modified)

If $T = 32^{lb} \Rightarrow N \,\&\, W = ??$

Unit vector along smooth rod:



$$\vec{u}_{CD} = \frac{+4\vec{i} - 7\vec{j} + 4\vec{k}}{\sqrt{4^2+7^2+4^2}} = \frac{1}{9}\left(+4\vec{i} - 7\vec{j} + 4\vec{k}\right) \quad (2)\ (1)\ (1)$$

The Normal Reaction to the rod $\boxed{\vec{N} = N_x\vec{i} + N_y\vec{j} + N_z\vec{k}}$ (2)

$\vec{N} \perp \vec{u}_{CD} \Rightarrow \vec{N}\cdot\vec{u} = 0$ (2)

$$\Rightarrow \boxed{4N_x - 7N_y + 4N_z = 0} \quad (1)\ (4)$$

$$\vec{CA} = 6\vec{u}_{CD} \quad (2)$$

$$(x_A - 0)\vec{i} + (y_A - 7)\vec{j} + (z_A - 0)\vec{k} = 6\vec{u}_{CD} = \frac{6}{9}(4\vec{i} - 7\vec{j} + 4\vec{k}) \quad (2)$$

$$\Rightarrow x_A = \frac{8}{3} \ ; \ y_A = \frac{7}{3} \ ; \ z_A = \frac{8}{3} \quad (3)$$

$$\vec{u}_{AB} = \left(0 - \frac{8}{3}\right)\vec{i} + \left(7 - \frac{7}{3}\right)\vec{j} + \left(4 - \frac{8}{3}\right)\vec{k} \Big/ \sqrt{\left(\tfrac{8}{3}\right)^2 + \left(\tfrac{14}{3}\right)^2 + \left(\tfrac{4}{3}\right)^2} = \frac{\frac{2}{3}(-4\vec{i} + 7\vec{j} + 2\vec{k})}{\frac{2}{3}\sqrt{4^2+7^2+2^2}} \quad (2)\ (1)$$

$$\vec{u}_{AB} = \frac{1}{\sqrt{69}}\left(-4\vec{i} + 7\vec{j} + 2\vec{k}\right) \quad (1)$$

$$\Rightarrow \vec{T} = T\vec{u}_{AB} = \frac{-128}{\sqrt{69}}\vec{i} + \frac{224}{\sqrt{69}}\vec{j} + \frac{64}{\sqrt{69}}\vec{k} \quad (1)$$

$$\vec{W} = -W\vec{j} \quad (1)$$

$$\sum \vec{F} = \vec{0} = \vec{N} + \vec{T} + \vec{W} = \vec{0} \Rightarrow$$

$$(2)\ \sum F_x = 0 = N_x - \frac{128}{\sqrt{69}} = 0 \Rightarrow \boxed{N_x = \frac{128}{\sqrt{69}} = 15.409^{lb}} \quad (1)\ (2)$$

$$(2)\ \sum F_y = 0 = N_y + \frac{224}{\sqrt{69}} - W = 0 \quad (3)$$

$$(2)\ \sum F_z = 0 = N_z + \frac{64}{\sqrt{69}} = 0 \quad (4) \Rightarrow \boxed{N_z = \frac{-64}{\sqrt{69}} = -7.705^{lb}} \quad (1)$$

$$(1) \Rightarrow 4N_x - 7N_y + 4N_z = 0 = 4\times\frac{128}{\sqrt{69}} - 7N_y + 4\left(\frac{-64}{\sqrt{69}}\right) = 0 \Rightarrow \boxed{N_y = \frac{256}{7\sqrt{69}} = 4.40} \quad (1)$$

$$(3) \Rightarrow \boxed{W = \frac{256}{7\sqrt{69}} + \frac{224}{\sqrt{69}} = \frac{1824}{7\sqrt{69}} = 31.369^{lb}} \quad (1)$$