MAT 215 (Linear Algebra I), Final Exam, Fall98-99, Duration: 2 hours

1. (15%) For which values of a will the following system have no solutions? Exactly one solution? Infinitely many solutions?

$$\begin{aligned} x + 2y - \quad & 3z = 4 \\ 3x - y + \quad & 5z = 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

2. (15%) Let A and B be $n \times n$ matrices, and m be any positive integer. Use determinants to answer the following questions:

(a) If AB is invertible, then A and B are both invertible.
(b) If A^m is invertible, then A is invertible.
(c) If $A^m = 0$, then A cannot be invertible.
(d) If n is odd and $AB + BA = 0$, then A or B is not invertible.
(e) Give an example of a 2×2 nonzero matrix C whose square $C^2 = 0$.

3. (15%) (a) Determine whether or not the three vectors $u = (2, -1, 0)$, $v = (1, -1, 1)$, and $w = (0, 2, 3)$ form a basis for the vector space R^3.

(b) Prove that the set $S_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix} \right\}$ is not a spanning set for the vector space M_{22}.

(c) Extend S_1 to a spanning set S_2 of M_{22}.

(d) Is S_2 a basis for M_{22}? Why or why not?

4. (20%) Consider the matrix $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A.

(b) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

(c) Use (a) and (b) to evaluate A^{11}.