MAT 215 – Linear Algebra I
Spring 2001 - Exam # 2
Duration: 1 hour

1) Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

a) Find the cofactor matrix of A.
b) Find $\det(A)$.
c) Find the inverse of A.

(30 points)

2) Determine the following.

a) If $S = \{(x, y) / x^2 + y^2 \leq 1\}$, is S closed under addition of $\mathbb{R}^2$? Why?
b) If $S = \{(x, y) / y \geq 0\}$, is S closed under scalar multiplication of $\mathbb{R}^2$? Why?
c) If $S = \{\alpha(1,1,1) + \beta(1,0,0) / \alpha, \beta \in \mathbb{R}\}$, is S a subspace of $\mathbb{R}^3$? Why?

(15 points)

3) Let $S = \{v_1 = (1,-1,2), v_2 = (-1,0,3), v_3 = (0,-1,5), v_4 = (3,-2,2)\}$.

a) Determine whether $\mathbb{R}^3$ is spanned by S.
b) Find if possible a subset of S that can serve as a basis for $\mathbb{R}^3$.

(20 points)

4) Let $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 3 & 3 \end{bmatrix}$.

a) Find a spanning set for the solution space of $AX = 0$.
b) Find a basis for the Null Space of A.
c) Find the dimension of the Null Space of A.

(20 points)

5) Let $B = \{u, v, w\}$ denote a basis in a 3-dimensional vector space V. Show that the set $B' = \{u + v, v + w, w + u\}$ is a basis for V.

(15 points)