MAT 215 – Linear Algebra I
Exam # 1

1. (a) Use Gauss-Jordan elimination to find the solution set (S) of the linear system

$$\begin{aligned} x+y+z+w&=4 \\ x-y+2z+3w&=5 \\ 2x+y+z-w&=4 \end{aligned}$$

(b) Express the solution set (S) as the sum of a particular solution of the system and the general solution of the associated homogeneous system.

2. If $A=\begin{bmatrix}1 & 4\\ 4 & 1\end{bmatrix}$. Find a column vector $\mathbf{x}\neq\mathbf{0}$ such that $A\mathbf{x}=5\mathbf{x}$.

3. Let $A=\begin{bmatrix}1 & 1 & 2\\ 2 & 3 & 2\\ 1 & 1 & 3\end{bmatrix}$.

(a) Find the inverse of A. What is the rank of A? Is A row equivalent to I_3? Why?

(b) Find the solution of the system $A\mathbf{x}=\mathbf{b}$ where $\mathbf{b}=[2 \quad 8 \quad 1]^T$.

4. Show that if A is an $m\times m$ skew-symmetric matrix, then A^n is symmetric if n is even and skew-symmetric if n is odd for any integer $n\geq 2$.

5. Determine, according to the values of k, whether the linear system below has a unique solution, no solution or infinitely many solutions.

$$\begin{aligned} x+y+kz&=1 \\ x+ky+z&=1 \\ kx+ky+z&=1 \end{aligned}$$

6 Show that if A is an $n\times n$ invertible matrix such that $A^2=A$, then $A^{-1}=I_n$.

7. Find the reduced row echelon form of $A=\begin{bmatrix}1 & 1 & 1 & 0\\ 1 & -1 & 1 & 1\\ 0 & 1 & 2 & 1\end{bmatrix}$.

8. List all 2×2 elementary matrices.

9. Let A be any $n\times p$ matrix. Prove that there exists an invertible matrix C, such that CA is in a reduced row echelon form. (**Hint:** think of elementary $n\times n$ matrices)