Math 215: Linear Algebra I
Exam 1: 19 April, 2000
Time: 90 minutes

1) Solve by Gauss-Jordan elimination the system **(15 points)**

$$\begin{aligned} &x_1 + 3x_2 - 2x_3 + 2x_5 = 0 \\ &2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1 \\ &5x_3 + 10x_4 + 15x_6 = 5 \\ &2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6 \end{aligned}$$

2) For which values of λ does the following system of equations have **non-trivial** solutions: **(10 points)**

$$\begin{aligned} (\lambda - 3)x + y = 0 \\ x + (\lambda - 3)y = 0 \end{aligned}$$

3) Let A and B be two matrices such that AB and BA are both defined. Show that AB and BA are both square matrices. **(10 points)**

4) Evaluate $tr(B'A' + 2C')$ knowing that **(10 points)**

$$A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}, \quad C = \begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}$$

5) Show that if A is an $n \times n$ matrix that satisfies $A^2 - 3A + I_n = 0$ then A is invertible with A^{-1} satisfying $A^{-1} = 3I_n - A$. **(10 points)**

6) Let A and B be two square matrices such that $AB = 0$. If A is invertible, show that $B = 0$. **(5 points)**

7) Show that a square matrix with a row of zeros cannot have an inverse. **(10 points)**

8) Reduce the following matrix to a row echelon form (in 3 operations). **(10 points)**

$$A = \begin{pmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{pmatrix}$$

9) Prove that if A is any $m \times n$ matrix, then there is an invertible $m \times m$ matrix C such that CA is in reduced row echelon form **(10 points)**

10) Find the inverse of $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. **(10 points)**