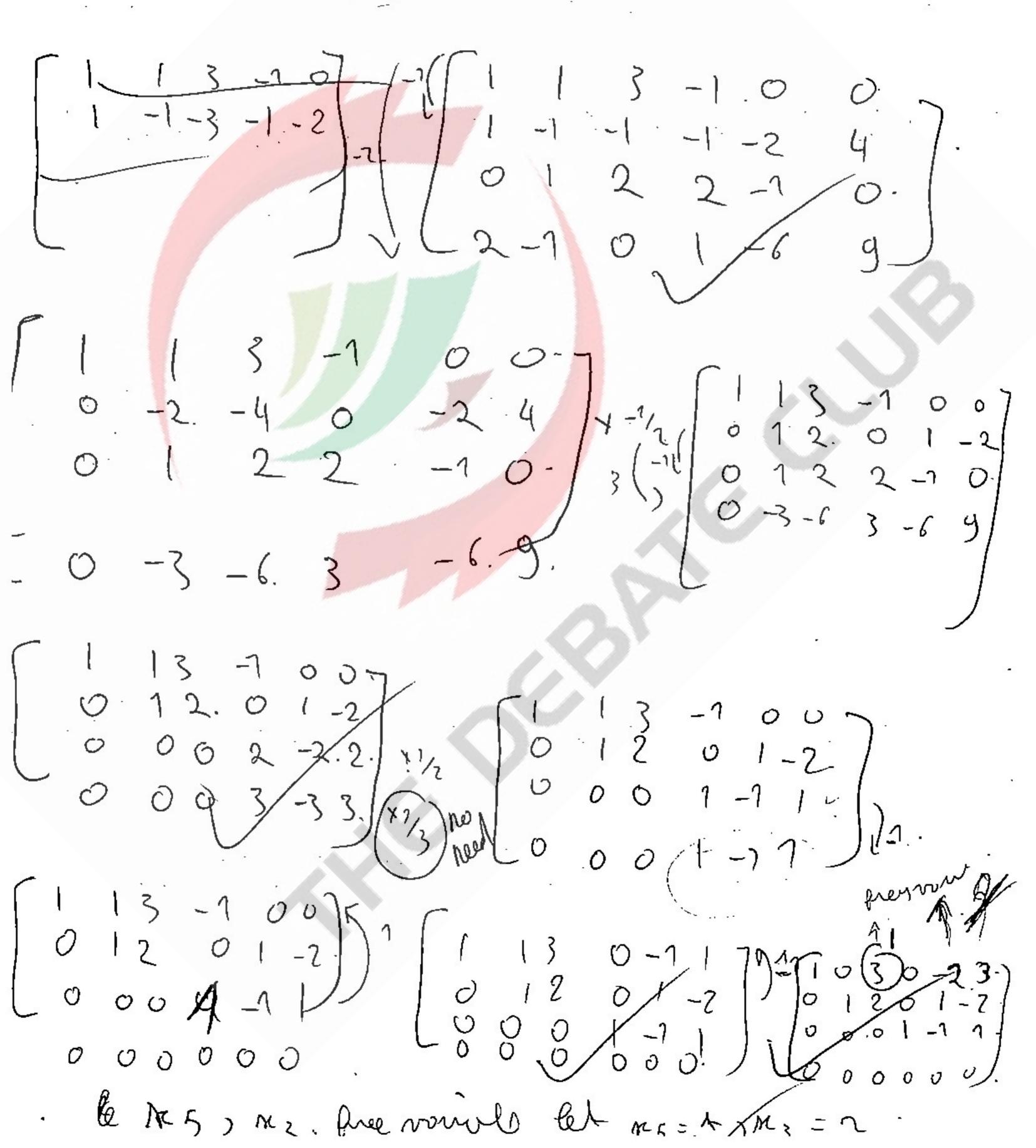
1) (16 points) Use Gauss-Jordan elimination to solve the following system of equations:

$$x_1 + x_2 + 3x_3 - x_4 = 0$$

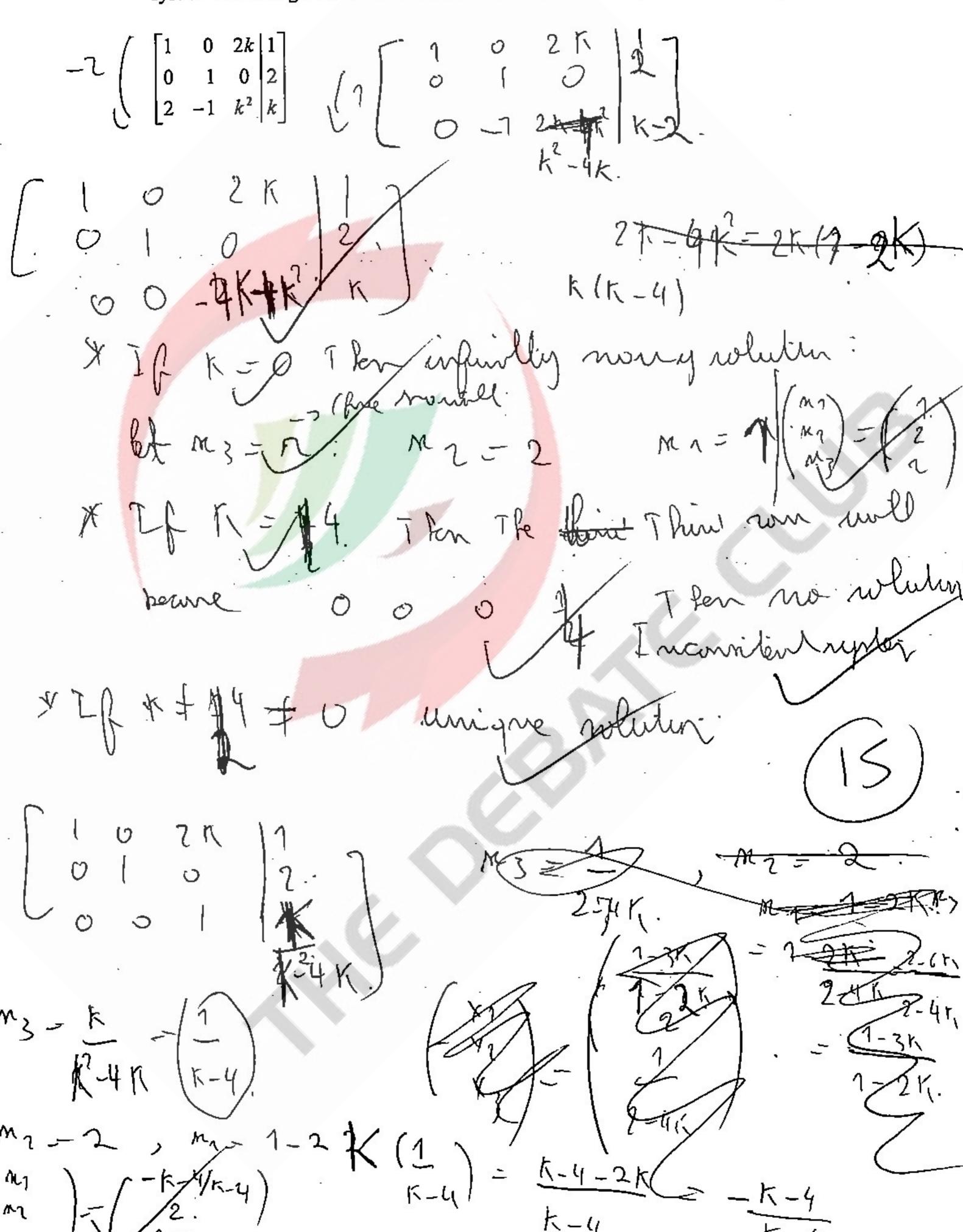
$$x_1 - x_2 - x_3 - x_4 - 2x_5 = 4$$

$$x_2 + 2x_3 + 2x_4 - x_5 = 0$$

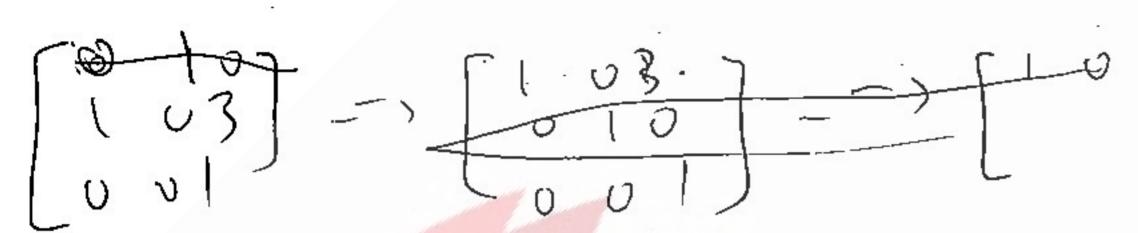
$$2x_1 - x_2 + x_4 - 6x_5 = 9$$



2) (15 points) Discuss, according to the values of k, the nature of the solutions of the system whose augmented matrix is below. Then, solve the system whenever possible.



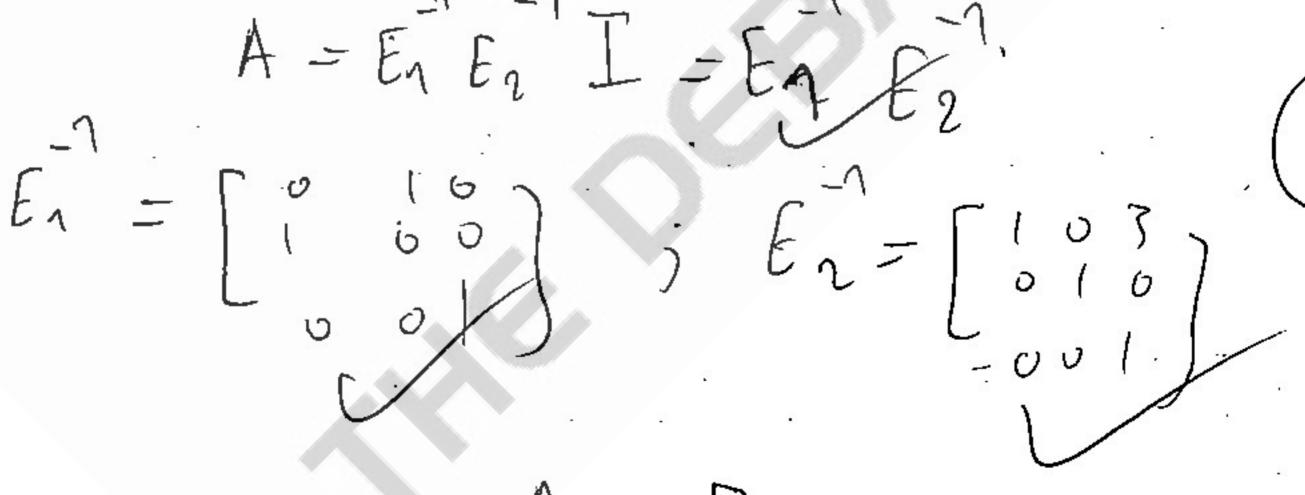
- 3) (15 points) Consider the matrix  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ .
  - a) (7 points) Find elementary matrices  $E_1$  and  $E_2$  such that  $E_2E_1A=I$ .



$$E_{1} = \begin{bmatrix} 0 & 10 \\ 1 & 00 \end{bmatrix}; E_{2} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \end{bmatrix}$$

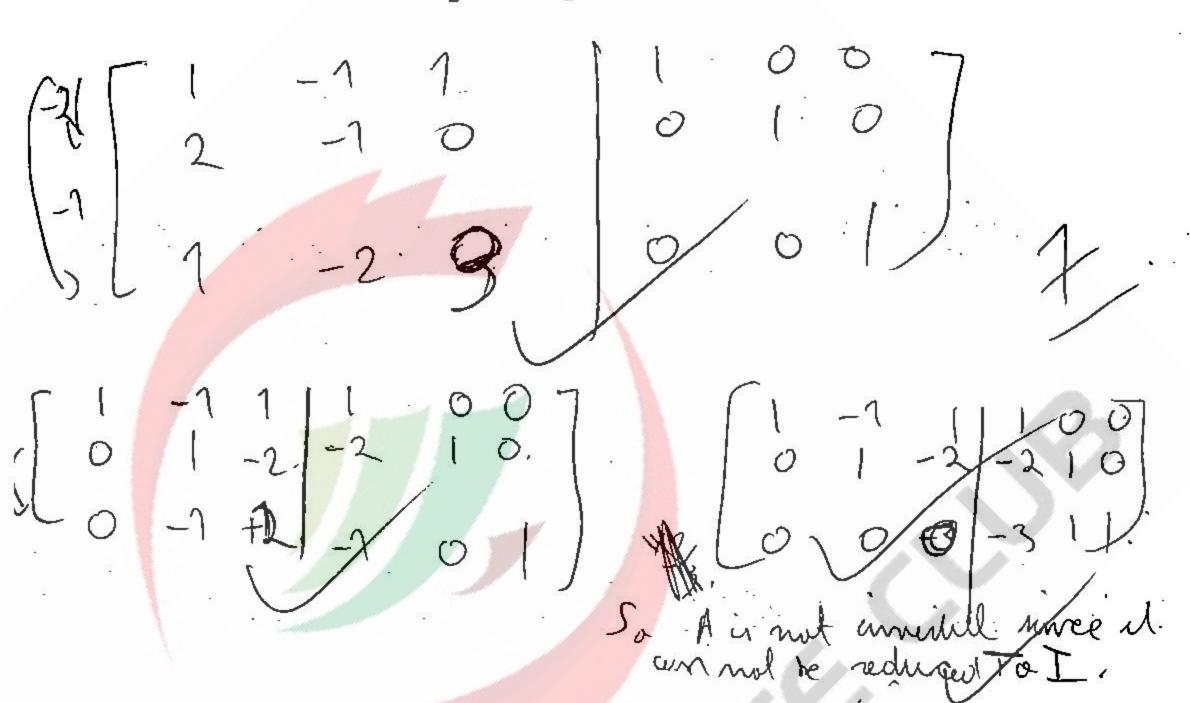
b) (8 points) Use part (a) to express the matrix A as a product of two elementary

$$A = E_1 E_2 = E_1 E_2$$



4) (14 points)

a) (7 points) Let  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ . Find  $A^{-1}$  or prove that A is not invertible.



 $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}.$ b) (7 points) Find the matrix X such that:

$$A = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

$$A = \frac{1}{5-6} \begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 8 & -10 & -1 \\ 21 & -25 & -3 \end{bmatrix}$$

5) (6 points) Prove the following theorem: If  $A\vec{x} = \vec{b}$  is a system of linear equations, then

The publish can be natured if we show That An? Is
has infurity mony solution. let nn, me de goluturis for An = b (with X) = X2.

16 Am2-Am1= H(m2-Mn)= b'-b= 3 let no 7 his rolution; on K = scalar. (42)

A (1/1 + Km2) = 4 mn + KAmo - b + Kxo - 5 Then mi + Kmo is a whiting for the mys. Am = b'

Then The mystern has infinelly many solution

6) (16 points) Let A be an $n \times n$ matrix. Prove each of the following statements:
a) (4 points) If A is a diagonal matrix all of whose entries are d, then $Ir(A) = nd$ we have a The only we have a large of the only we have a large of the only we have the same and the s
Translation of the mount manufacture is sugar
19 d
The frame in entries since non of them is
2 and each entry is of The Trial = It
b) (4 points) $A - A^T$ is skew-symmetric.  Skew symmetric.
(A-A) - A - (A-A)-
i An A-A in stan symbleg.
c) (4 points) If A is invertible and $k \neq 0$ is a scalar, then $kA''$ is invertible and
c) (4 points) If A is invertible and $k \neq 0$ is a scalar, then $kA^{n}$ is invertible and $(kA^{n})^{-1} = \frac{1}{k}A^{-n}$ . $(kA^{n})^{-1} = \frac{1}{k}A^{-n}$ $(kA^{n})^{-1} = \frac{1}{k}A^{-n}$
Amoentation (A) (A) (A) - I ) The
A (A) = (A VA) - d Jungers of.
A is investible The The Tist insertable.
A is unseabled The $t$ A is unevertical.  ( $t$ A) $t$ A $t$
d) (4 points) Let I be the $n \times n$ identity matrix. Then $IA = A$ .
I f A is n x/m munce (tAn) = 1 An.
PLR 7A
bij - Elimvanj = Jioranin
bij - E Iimvanj = Jiovanj + Tijvanji ivanj
Segnel - Iii.xo.
So B=IA in equivalent to A.  This and motion  become me one my en motion
beaux me hure mye n'4m and more andries
Peaux me hure on ye or your and sur more andries
(16)

7) (18 points) Let A and B be two $n \times n$ matrices. Prove each of the following states	nents.
a) (4 points) If A and B are both invertible, then AB is invertible and $(AB)^{-1} = B$ AB + BA  A T A T - A A T - T  b) (4 points) If A is invertible and B is row-equivalent to A, then B is invertible.	estitue 7
Brown equiplem to A Than ErEr Cr & B = A	
The flowertony motives is one inventure matures. Si	
(6 points) Let $A\bar{x} = \bar{0}$ be a homogeneous system of linear equations and let $B\bar{x} = \bar{0}$ invertible. Show that if $A\bar{x} = \bar{0}$ has only the trivial solution, then $(BA)\bar{x} = \bar{0}$ only the trivial solution.	, 00
If Am = 5' less only The turnal solution + Bon Air invention	B of En Ex
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motures is investige	of Demeth
Then (DA) is = 0 will have only	mulices is
The turned solutions.	virgertide
d) (4 points) A square matrix $M$ is said to be orthogonal if $MM^T = I$ . Show the and $B$ are both orthogonal, then $AB$ is also orthogonal.	at if A
A is orthogonal Than A A = I	
BU W BB-I	4
ABX (AB) = ABXBA - AIA - AP  orthogon.	T = I
	18