1) (12 points) For each of the following multiple
the correct answer. If more than one letter is ci
receive no credit for that problem.

Question A

$$\int_0^{\sqrt{3}} \int_y^2 \frac{1}{4} y e^{x^3} \, dx \, dy =$$

a) $\frac{1}{12}(e^4 - 1)$

b) $\frac{1}{16}(e^4 - 1)$

THE DEBATE CLUB

Question B

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{2}{(x^2+y^2+1)^2} \, dy\,dx =$$

a) $\frac{2\pi}{5}$

b) $\frac{9\pi}{10}$

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2) **(15 points)** Find the flux of the field $\mathbf{F} = x^2\,\mathbf{i} + y^2\,\mathbf{j}$ around the closed curve that consists of the semicircle $\mathbf{r}_1(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j}$, $0 \le t \le \pi$, followed by the line segment $\mathbf{r}_2(t) = t\mathbf{i}$, $-a \le t \le a$.

$\vec{F} = x^2 \vec{i} + y^2 \vec{j}$

$\vec{r}_{1(t)} = (a\cos t)\vec{i} + a\sin t\,\vec{j}$

$\frac{d\vec{r}_1}{dt} = (-a\sin t)\vec{i} + a\cos t\,\vec{j}$

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3) (15 points) Let D be the region bounded from below by the xy-plane, from above by the cone $z = \sqrt{3x^2 + 3y^2}$ and laterally by the cylinder $x^2 + (y-1)^2 = 1$.

a) (4 points) Sketch the region D

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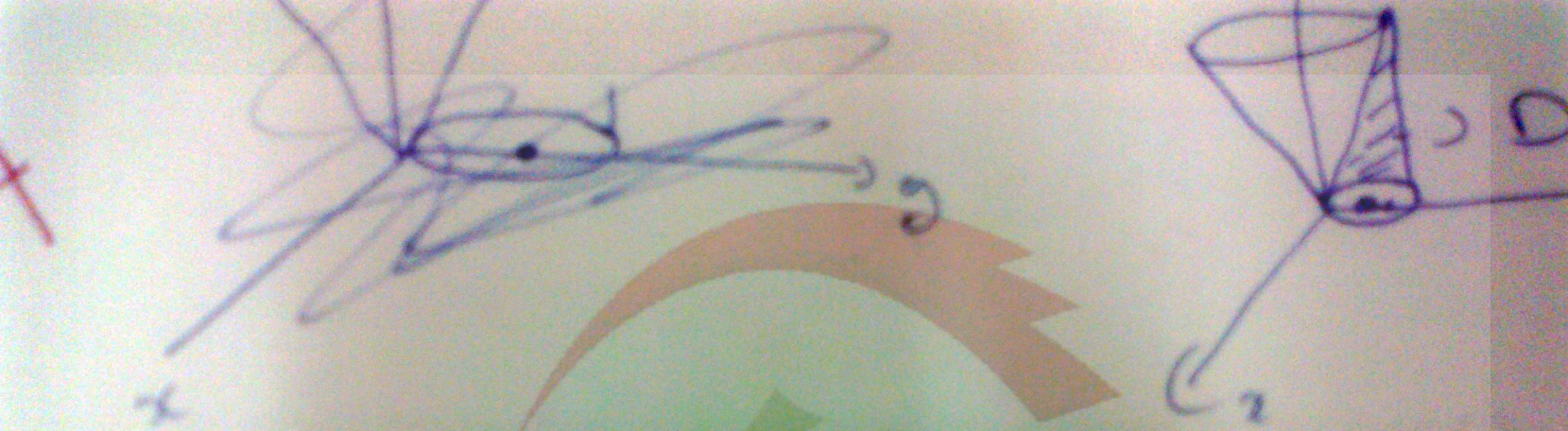

b) (11 points) Set up triple integral, in cylindrical coordinates, for the volume of D according to the order of integration $dzdrd\theta$

$$\int_0^{\pi} \int_0^{2\cos\theta} \int_0^{\sqrt{3x^2+3y^2}=\sqrt{3}r} r \, dz \, dr \, d\theta$$

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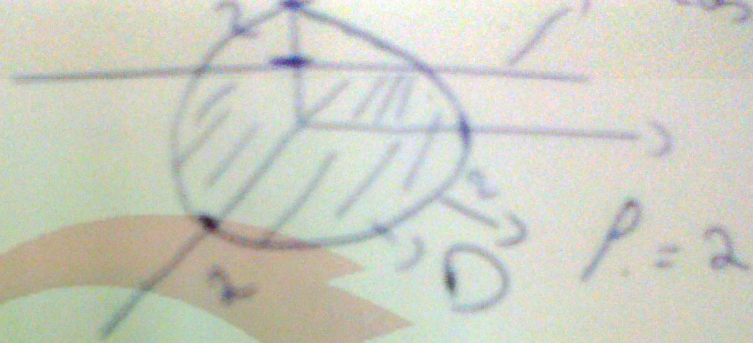
4) (20 points) Let D be the region bounded from above by the plane $z = 1$ a
below by the sphere $x^2 + y^2 + z^2 = 4$.

a) (4 points) Sketch the region D

THE DEBATE CLUB

...spherical coordinates, for th



b) (16 points) Set up triple integral, in spherical coordinates, for the volume of D according to the order of integration $d\rho d\phi d\theta$

$$\int_0^{2\pi} \int_{\pi/3}^{\pi/2} \int_0^{\frac{1}{\cos\phi}} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

THE DEBATE CLUB

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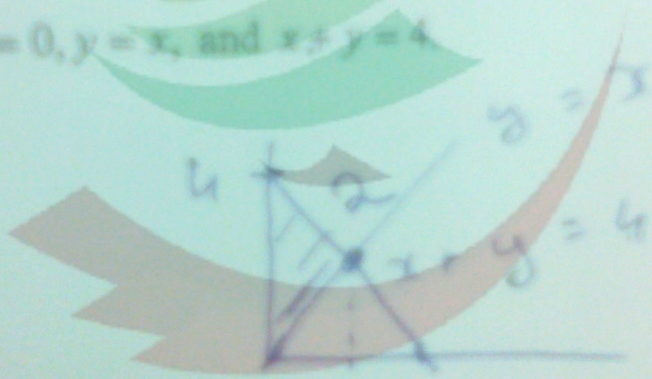
5) (20 points) Use the transformation $x = u + 4v$ and $y = u$ to evaluate the double integral $\iint_R y^3\sqrt{x-y}\,dydx$, where R is the triangular region in the xy-plane bounded by the lines $y = 0, y = x$, and $x = 4$.



$$\int_0^2 \int_0^x$$

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6) **(18 points)** Use the Lagrange Multipliers Method to find the points on the intersection of the sphere $x^2 + y^2 + z^2 = 4z$ and the plane $z = 1$ that are closest and farthest from the point $(0,1,0)$.

$= x^2 + (y-1)^2 + z^2$

$= x^2 + y^2 + z^2 - 4z$

$= 1$

$= \lambda 2y + 4\lambda$

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