1) **(15 points)** Find the linearization $L(x, y)$ of the function $f(x,y)$ at P_0. Then find an upper bound for the magnitude $|E|$ of the error in the approximation $f(x,y) \approx L(x,y)$ over the rectangle R.

$f(x,y) = \ln x + \ln y$ at $P_0(1,1)$.
$R: |x-1| \le 0.2,\ |y-1| \le 0.2$.

2) **(20 points)**

a) Find $\partial w/\partial v$ when $u=-1$, $v=2$ if $w = xy + \ln z$, $x = v^2/u$, $y = u+v$, $z = \cos u$.

b) Find the value of $\partial z/\partial x$ at the given point if that is given as a differentiable function of x and y.
$\sin(x+y)+\sin(y+z)+\sin(x+z)=0$, (π,π,π)

3) **(15 points)** You plan to calculate the volume inside of stretch of pipeline that is about 10 cm. in diameters and 100 meter long.

a) With which measurement should you be more careful – the length, or the diameter? Why?

b) Estimate the change (in cm^3) in the volume when the diameter increased by 0.5 cm.

4) **(20 points)**

a) The derivative of $f(x,y)$ at $P_0(1,2)$ in the direction of $\mathbf{i}+\mathbf{j}$ is $2\sqrt{2}$ and in the direction of $-2\mathbf{j}$ is -3. What is the derivative of f in the direction of $-\mathbf{i}-2\mathbf{j}$? Give reasons for your answer.

b) Find an equation for the plane that is tangent to the given surface at the given point.

5) **(30 points)** Find the extreme values of $f(x,y,z) = x^2yz+1$ on the intersection of the plane $z=1$ with the sphere $x^2+y^2+z^2=10$.

1. (15%) Find the limit of $f(x, y) = \dfrac{x^3 - xy^2}{x^2 + y^2}$ as $(x, y) \to (0, 0)$ or show that the limit does not exist.

2. (15%) Find the value of $\partial z / \partial x$ at the point (1, 1, 1) if the equation $xy + z^3 x - 2yz = 0$ defines z as a function of the two independent variables x and y and the partial derivative exists.

3. **(20%)** Give a reasonable square centered at (1, 1) over which the value of $f(x, y) = x^3 y^4$ will not vary by more than ± 0.1.

4. (25%) Find the absolute maxima and minima of the function on the given domain.

   $T(x, y) = x^2 + xy + y^2 - 6x$ on the rectangular plate $0 \le x \le 5$, $-3 \le y \le 3$.

5. **(25%)** The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 + 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant?