Tuesday April 16, 2002 Test I

There are 6 problems total. Answer them all.

1) **(15%)** Find the length of the indicated portion of the curve:

$$\mathbf{r}(t) = \left(e^t \cos t\right)\mathbf{i} + \left(e^t \sin t\right)\mathbf{j} + e^t\mathbf{k}, \quad -\ln 4 \le t \le 0$$

2) **(30%)** Find **T**, **N**, **B**, κ, τ, and write the acceleration **a** in the form $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ for the space curve $\mathbf{r}(t) = \sin t\,\mathbf{i} + \sqrt{2}\cos t\,\mathbf{j} + \sin t\,\mathbf{k}$.

3) **(20%)** A particle traveling in a straight line is located at the point $(1, -1, 2)$ and has a speed 2 at time $t = 0$. The particle moves toward the point (3, 0, 3) with constant acceleration $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. Find its position vector $\mathbf{r}(t)$ at time t.

4) **(10%)** Sketch the surface (*S*) whose spherical equation is $\rho = \sqrt{2}\sec\phi$

5) **(10%)** Sketch the surface $z^2 + 4y^2 - 4x^2 = 4$.

6) **(15%)** Find the limit of f as $(x, y) \to (0,0)$ or show that the limit does not exist:

a) $f(x, y) = \dfrac{x^3 - y^3}{x^3 + y^3}$

b) $f(x, y) = \tan^{-1}\left(\dfrac{|x| + |y|}{x^2 + y^2}\right)$