Thursday November 16, 2000

1) (10%) Find the length of the curve

$$\mathbf{r}(t) = \left(\sqrt{2}t\right)\mathbf{i} + \left(\sqrt{2}t\right)\mathbf{j} + \left(1 - t^2\right)\mathbf{k}$$

from $(0, 0, 1)$ to $\left(\sqrt{2}, \sqrt{2}, 0\right)$.

2) (20%) Find the limit or show that the limit doesn't exist.

a) $\lim_{(x,y)\to(0,0)} \ln \frac{y^2 + 5x}{2x^2 + x}$

b) $\lim_{(x,y)\to(0,0)} \frac{2x^2 y}{x^5 + y}$

3) (10%) Show that the parabola $y = ax^2$, $a \neq 0$, has its largest curvature at its vertex and has no minimum curvature. (*Note:* Since the curvature of a curve remains the same if the curve is translated or rotated, this result is true for any parabola.

4) (10%) Can anything be said about the acceleration of a particle that is moving at a constant speed? Give reasons for your answer.

5) (20%) a) Find the function's domain, b) Find the function's range, c) describe the function's level curves, d) find the boundary of the function's domain, e) determine if the domain is an open region, a closed region, or neither, and f) decide if the domain is bounded or unbounded and g) sketch the surface $f(x, y) = e^{-(x^2+y^2)}$.

6) (30%) Find T, N, B, κ, and τ for the space curves.

$$\mathbf{r}(t) = \left(e^t \cos t\right)\mathbf{i} + \left(e^t \sin t\right)\mathbf{j} + 2\mathbf{k}$$