

**Notre Dame University**

**Faculty of Engineering**

ENG 202 Computers and Engineering

Project Fall 2010

**Temperature distribution in a rectangular metal plate**

**Notes:**

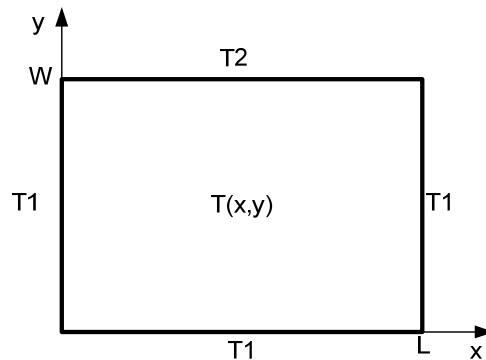
**Project deadline: January 28, 2011**

You can do the project alone or in group of maximum two students.

Project should be presented in a report format both printed and on a CD. Project should include a cover page that includes the students names, ID, major, title, instructor name, and due date. The report should include all the Matlab code and outputs of each section. The Matlab code should be well commented and written in only one M-file. Separation between sections should be clear in the M-file.

Project accounts for 10% of the final grade, taken from the 40% final exam grade. (i.e. Project = 10%, final exam: 30%)

Many applications require us to know the temperature distribution in an object. For example, this information is important for controlling the material properties, such as hardness, when cooling an object formed from molten metal. In a heat transfer course, the following description of the temperature distribution in a flat, rectangular metal plate is often derived. The temperature is held constant at  $T_1$  on three sides and at  $T_2$  on the fourth side as shown in the figure below.



The temperature  $T(x,y)$  as a function of the  $xy$  coordinates shown is given by

$$T(x,y) = (T_2 - T_1)w(x,y) + T_1$$

where

$$w(x,y) = \frac{2}{\pi} \sum_{n \text{ odd}} \frac{2}{n} \sin\left(\frac{n\pi x}{L}\right) \frac{\sinh\left(\frac{n\pi y}{L}\right)}{\sinh\left(\frac{n\pi W}{L}\right)}$$

Use the following data:  $T_1 = 70$  F,  $T_2 = 200$  F, and  $W = L = 2$  ft.

- The terms in the preceding series become smaller in magnitude as  $n$  increases. Write a MATLAB program to verify this fact for  $n=1,3,\dots,19$  for the center of the plate ( $x = y = 1$ ).
- Using  $x = y = 1$ , write a MATLAB program to determine how many terms are required in the series to produce a temperature calculation that is accurate to within 1 percent. (That is, for what value of  $n$  will the addition of the next term in the series produce a change in  $T$  of less than 1 percent.) Use your physical insight to determine whether this answer gives the correct temperature at the center of the plate.
- Modify the program from part a to compute the temperatures in the plate; use a spacing of 0.2 for both  $x$  and  $y$ . (That is, you should get an  $m \times n$  matrix containing the values of  $T(x,y)$ ).
- Using a spacing of 0.2 for both  $x$  and  $y$ , generate a surface mesh plot and a contour plot of the temperature distribution in the plate.
- Write a MATLAB program to animate the surface plot, to see how the precision of the temperature distribution changes when  $n$  increases from 1 to 19.