Exam II (100)

1. Use the method of Least-Squares to fit an exponential function of **Y vs. X** to the following set of data. Write the appropriate equations in terms of cell references **without performing calculations.** (Note: i represents the point number) Use the excel sheet and the space below to solve.

I18 fx

	A	B	C	D	E	F	G
1	i	x	y	ln y	$x_i \ln y_i$		
2	1	0	0	=ln(C2)	=B2*D2		
3	2	1	1				
4	3	2	4				
5	4	3	9				
6	5	4	16				
7	6	5	25				
8	7	6	36				
9	8	7	49				
10	9	8	64				
11	10	9	81				
12	11	10	100				
13	12	12	144				
14		=SUM(B2:B13)	=SUM(C2:C13)	=SUM(D2:D13)	=SUM(E2:E13)		
15							
16							
17							

exponential funct.: $y = ae^{bx}$

$\ln y = \ln a + bx$ Let $c = \ln a$

$b\sum x_i + c(12) = \sum \ln y_i$

$b\sum x_i^2 + c\sum x_i = \sum x_i(\ln y_i)$

(Answer of b & c will appear in F1 & F2)

In A16: =F1*B14+F2*12-D14

In A17: =F1*B14^2+F2*B14-E14

In A18: =A16^2+A17^2

Go to Solver:

Target Cell A18

Set value 0

By changing F1:F2

Solve ↵

We get answer of b in F1 and C in F2

$a = e^c$

In Cell G1: =exp(F2) In F2 we get the value of a

Now have our exponential funct. $y = ae^{bx}$

2. The relationship between pressure, volume, and temperature for many real gases can be determined by Van der Waals's equation of state which is written as:

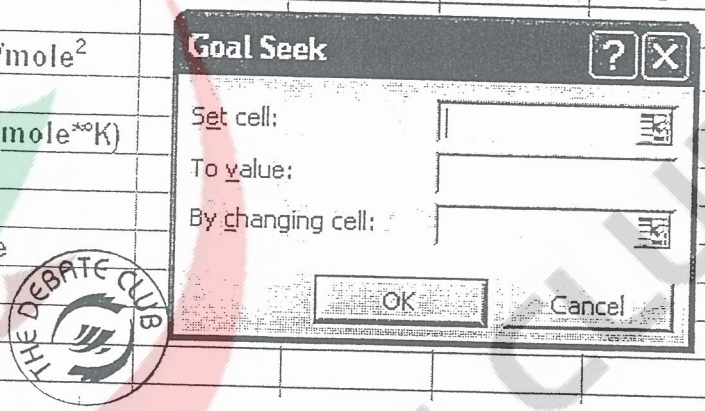
$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where P is the absolute pressure, V is the volume per mole, T is the absolute temperature, R is the ideal gas constant (0.082054 liter atm/mole °K) and a and b are constants that are unique to each particular gas.

Below is an excel sheet that was set up to solve Van der Waals's equation for the volume per mole (V) of an organic compound at 10 atm pressure and 400 °K. The Van der Waals's constant for this particular gas are a = 40.0 liter^2 atm/mole^2 and b = 0.2 liter/mole. Answer the following questions related to the excel worksheet below:

A13 f_x

	A	B	C
1			
2	a =	40	liter^2*atm/mole^2
3	b =	0.2	liter.mole
4	R =	0.08205	liter*atm/(mole**K)
5	T =	400	°K
6	P =	10	atm
7	V =	0.250818232	liters/mole
8	F(V) =	-3.8262E-06	
9			
10			



a) What is the excel formula that was entered into cell B8?

=(B6 + B2/B7^2)*(B7 - B3) - (B4 * B5)

b) What cell reference was entered into the 'Set cell' box in Goal Seek to solve the above problem? B8

c) What value was entered into the 'To Value' box in Goal Seek to solve the above problem? 0

d) What cell reference was entered into the 'By changing cell' box in Goal Seek to solve the above problem? B7

3. A steel company manufactures four different types of steel alloys, called A1, A2, A3 and A4. Each Alloy contains small amounts of chromium (Cr), molybdenum (Mo), titanium (Ti), and nickel (Ni). The required composition of each alloy is given below:

Alloy	Cr	Mo	Ti	Ni
A1	1.6 %	0.7 %	1.2 %	0.3 %
A2	0.6	0.3	1.0	0.8
A3	0.3	0.7	1.1	1.5
A4	1.4	0.9	0.7	2.2

The alloying materials are available in the following amounts:

Material	Availability
Cr	1200 kg/day
Mo	800
Ti	1000
Ni	1500

a) Formulate a system of equations that will determine the daily production rate for each alloy (Note: All amounts of each material is used up to create the alloys):

$$\frac{1.6}{100} \times A1 + \frac{0.6}{100} \times A2 + \frac{0.3}{100} \times A3 + \frac{1.4}{100} \times A4 \leq 1200$$

$$\frac{0.7}{100} \times A1 + \frac{0.3}{100} \times A2 + \frac{0.7}{100} \times A3 + \frac{0.9}{100} \times A4 = 800$$

$$\frac{1.2}{100} \times A1 + \frac{1}{100} \times A2 + \frac{1.1}{100} \times A3 + \frac{0.7}{100} \times A4 = 1000$$

$$\frac{0.3}{100} \times A1 + \frac{0.8}{100} \times A2 + \frac{1.5}{100} \times A3 + \frac{2.2}{100} \times A4 = 1500$$

-1

b) Write this system in Matrix notation (Define each matrix and vector):

Coefficient of the unknowns in the 4 eq.: Vector unknowns · Value to the right side of the eq.:

$$A = \begin{vmatrix} 0.006 & 0.006 & 0.003 & 0.014 \\ 0.007 & 0.003 & 0.007 & 0.009 \\ 0.012 & 0.001 & 0.011 & 0.007 \\ 0.003 & 0.008 & 0.015 & 0.022 \end{vmatrix} \quad X = \begin{vmatrix} A1 \\ A2 \\ A3 \\ A4 \end{vmatrix} \quad B = \begin{vmatrix} 1200 \\ 800 \\ 1000 \\ 1500 \end{vmatrix}$$

— 1

c) What excel functions are used to solve this system of equations using matrix inversion? MINVERSE (to get A^{-1}) MMULT (to multiply $A^{-1}B$)

$X = A^{-1}B$

d) What other excel tool can be used to solve this system of equations? What is the additional equation you have to formulate to use this excel tool for solving the problem? We can use "Solver" in tools menu

If we name the 4 eq. f_1, f_2, f_3 & f_4 respectively

$y = f_1^2 + f_2^2 + f_3^2 + f_4^2$

4. The excel sheet below simulates throwing a pair of dice 15 times. Column A represents the values recorded after rolling dice 1 and column B represents the values recorded after rolling dice 2. Column C is the sum of the value from dice 1 and dice 2 (=sum(A2,B2)). Column D represents the outcome or result of each throw of the pair of dice depending on the sum of both dice.

D20

	A	B	C	D	E
1	Dice 1	Dice 2	Sum	Outcome	
2	5	6	11	You Win	
3	3	5	8	Inconclusive	
4	2	3	5	Inconclusive	
5	3	1	4	Inconclusive	
6	2	5	7	You Win	
7	3	3	6	Inconclusive	
8	4	6	10	Inconclusive	
9	3	6	9	Inconclusive	
10	6	6	12	You Lose	
11	4	5	9	Inconclusive	
12	2	4	6	Inconclusive	
13	1	1	2	You Lose	
14	2	1	3	You Lose	
15	5	3	8	Inconclusive	
16	4	2	6	Inconclusive	
17					

Using the excel sheet above write the excel formula that was entered into cell D2 and copied into cells D3 to D16 :

=IF(OR(C2=2, C2=3, C2=12), "YOU LOSE", IF(OR(C2=11, C2=7), "YOU WIN", "Inconclusive"))

5. Sporting Goods, Inc. makes two different types of tennis rackets: an adult model and a children's model. The firm has 900 hours of production time available in its molding and stringing department, 300 hours available in its finishing department, and 100 hours available in its packaging and shipping department. The production time requirements and the profit contribution per tennis racket are given in the following table.

	Production Time (Hours)			
Model	**Molding and Stringing**	**Finishing**	**Packaging and Shipping**	**Profit/ tennis racket**
Adult model	1	1/2	1/8	$5
Children's Model	3/2	1/3	1/4	$8

Assuming that the firm wants to get the most profit contribution, write the linear programming model for this problem:

Let X_1 be the # of adult racket produced

" X_2 " " " " children " "

Max Profit: $y = 5X_1 + 8X_2$ target cell

constraints $1 * X_1 + \frac{3}{2} * X_2 \leq 900$

$\frac{1}{2} * X_1 + \frac{1}{3} * X_2 \leq 300$

$\frac{1}{8} * X_1 + \frac{1}{4} * X_2 \leq 100$

BONUS:

Cell A1 : 30 (any initial given)

Cell A2 : 30

In cell A3: $= 5 * A1 + 8 * A2$

" A5: $= A1 + \frac{3}{2} * A2$

" A6: $= \frac{1}{2} * A1 + \frac{1}{3} * A2$

" A7: $= \frac{1}{8} * A1 + \frac{1}{4} * A2$

In Solver:

a) Target Cell A3

b) • MAX

c) By changing A1; A2

d) Constraints:

A5 ≤ 900

A6 ≤ 300

A7 ≤ 100

Solve ✓

Answers for X_1 & X_2 for max. profit will appear in cells A1 & A2 respectively