For each of the following 17 questions circle the right answer. There is only <u>one</u> correct answer for each question. If you circle more than one answer per question, your answer would be considered incorrect

***If A and B are two independent events , with P(A) = 0.5, P(B)= 0.3, then $P(\bar{A} \cup \bar{B})$ equals to

a) 0.35 b) 0.15 c) 0.85 d) None of these

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

$P(A \cap B) = 0.5 \times 0.3 = 0.15$

*** If two events A and B are mutually exclusive with P(A) = 0.4 and P(B) = 0.5 , then we can say that A and B are

a) ~~Independent with P (A ∩ B) = 0.2~~
b) Dependent with p(A ∩ B) = 0
c) Independent with P(A ∩ B)=0
d) None of these

***A firm is placing three orders for supplies among five distributors. Each order is randomly assigned to one of the distributors, and a distributor can receive multiple orders. Find the probability that the three orders go to three different distributors

a) $\frac{12}{25}$ b) $\frac{15}{25}$ c) $\frac{2}{25}$ d) None of these

sample space = 5.5.5 = 5^3

$P(X) = \frac{C^3_1 C^2_1 C^1_1}{5^3} = \frac{3 \times 2}{5^3} = 0.048$

$= \frac{(1 \times 1 \times 1)5}{5^3}$

*** If 3 students were selected at random without replacement from a group of 5 students of which 3 are females and 2 are males, then P(exactly 2 of the 3 selected are females) equals

a) $\frac{6}{25}$ b) $\frac{6}{10}$ c) $\frac{6}{15}$ d) None of these

5 → 3 fe
→ 2 ma

$p(x) = \frac{C^3_2 C^2_1}{C^5_3} = \frac{3 \times 2}{10} =$

*** A construction job may not be completed on time. The probability that the job will be completed on time is 0.90 when there is no strike, and it is 0.6 when there is a strike. The probability that there will be a strike is only 0.4

$p(\bar{A}) = p(\bar{A}|B) \cdot p(B) + p(\bar{A}|\bar{B}) \cdot p(\bar{B})$
$= (1-0.6)(0.4) + (1-0.9)(0.6)$

Find the probability that there was no strike, knowing that the job was not completed on time

a) 0.27 b) 0.727 c) 0.22 d) None of these

success = 0.9 = 90% (no strike)
= 0.6 = 60% (strike)
p(strike) = 0.4

p(no strike) = 1 − 0.4 = 0.6

$p(\text{no strike} | \text{not on time}) = \frac{p(\text{no strike} \cap \text{not on time})}{p(\text{not on time})}$

$= \frac{(0.1)\times(0.6)}{(0.1)(0.6) + (0.4)(0.6)}$

$0.9 \times 0.6 + 0.6 \times 0.4 = 0.78$

$p(\bar{B}|\bar{A}) = \frac{(0.1)(0.6)}{(0.4)(0.4) + (0.1)(0.6)}$

1 − 0.78 = 0.22

$= \frac{}{0.3} = 0.2$

***The number of customers arrive at a certain checkout counter follows a Poisson distribution with λ= 16/hour. The opening time of the counter is 9:00 Am. What is the probability that no customers arrive before 9:15 Am?

$\lambda = \frac{4}{15}$/min

a) 1- e^{-4} b) e^{-16} c) e^{-4} d) None of these

$p(\text{no customers arrive within 15 min}) = \left(\frac{4}{15}\right)^0 e^{-\frac{4}{15}} =$

no cust
9 9:15

X = # cust per 15 min
X has a poisson with mean rate $= \frac{16}{4} =$ 4 cust/15 min

$p(x=0) = \frac{e^{-4} 4^0}{0!}$

*** If X is an exponential random variable with mean $= 5$ $=\beta$, and variance $= 25$ $=\beta^2$, then $P(0 \le X \le 15)$ equals to

a) At least 0.0 b) At least 0.75 c) 0.9502 d) None of these

$P(0 \le x \le 15) = \int_0^{15} \frac{1}{5} e^{-\frac{x}{5}} dx = \frac{5}{5}\left[-e^{-\frac{x}{5}}\right]_0^{15} = -\left[e^{-\frac{15}{5}} - e^0\right] = -e^{-3} + 1$

*** If the cumulative distribution function F(x) of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & for\ x \le 1 \\ \frac{1}{44}(2x^2 - x - 1) & for\ 1 \le x \le 5 \\ 1 & for\ x > 5 \end{cases}$$

then $P(1 < X < 3)$ equals to

a) $\frac{34}{132}$ b) $\frac{14}{44}$ c) $\frac{30}{44}$ d) None of these

$P(1 < x < 3) = \frac{1}{44}\int_1^3 (2x^2 - x - 1)\, dx = \frac{1}{44}\left[\frac{2x^3}{3} - \frac{x^2}{2} - x\right]_1^3$

$= \frac{1}{44}\left[\frac{3^4}{2} - \frac{1}{2} - \frac{3^3}{3} + \frac{1}{3} - \frac{3^2}{2} + \frac{1}{2}\right) = \frac{41}{66}$

$= \frac{1}{44}\left[\frac{2x^3}{3} - \frac{x^2}{2} - x\right]_1^3 = \frac{1}{44}\left[\frac{2(3^3)}{3} - \frac{3^2}{2} - 3 - \frac{2}{3} + \frac{1}{2} + 1\right]$

$= \frac{1}{44}\left[18 - \frac{9}{2} - 3 - \frac{2}{3} + \frac{1}{2} + 1\right) = \frac{17}{66}$

*** The length of time " in years " to complete a certain construction project is a continuous random variable X whose probability density function is given by

$$f(x) = \begin{cases} \frac{1}{18}(10 - x) & for\ 4 \le x \le 10 \\ 0 & elsewhere \end{cases}$$

Find the expected time to complete this construction project.

a) 6 Years b) 7 Years c) 1 Year d) None of these

$E(X) = \frac{1}{18}\int_4^{10} 10x - x^2\, dx = \frac{1}{18}\left[\frac{10x^2}{2} - \frac{x^3}{3}\right]_4^{10} = \frac{1}{18}\left(\frac{10^3}{2} - \frac{10^3}{3} - \frac{10(4^2)}{2} + \frac{4^3}{3}\right)$

$= \frac{1}{18}\left[500 - \frac{1000}{3} - 80 + \frac{64}{3}\right) = 6$

*** The resistances of certain type of wires produced by company A are normal with a mean of 10 ohms and a variance of 0.25 . If three of these wires were randomly selected , find probability that at least one will have resistance less than or equal to 10 ohms.

a) 0.125 b) 0.875 c) 0.5 d) None of these

$\mu = 10$ $x \sim N(10, 0.25^2)$ $x = 3$ selected.
$V = 0.25$

$p(X \geq 1 \text{ resist} \leq 10) \rightarrow p(0 \leq R \leq 10) = p\left(\frac{0-10}{0.5} < Z < 0\right) = p(0 < Z < 20)$

$R \sim N(10, 0.25)$

$A = \{\text{at least 1 of 3 } R \leq 10\}$

$p(R \leq 10) = 0.5$; p(at least 1 success) = 1 - p(0 success)
$= 1 - C_0^3 \cdot p^0 \cdot 2^3 = 1/8$

-2	-1	0	1	2
$\frac{6}{20}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{+3}{20}$	$\frac{+2}{20}$

*** The probability distribution of a discrete random variable X is given by

$P(X = x) = \frac{|x-4|}{20}$ for x= -2, -1, 0, 1, 2

E(X) equals to

a) 0.0 b) - 1 c) $\frac{10}{20}$ d) None of these

$E(X) = (-2)\left(\frac{6}{20}\right) + (-1)\left(\frac{5}{20}\right) + 0 + 1\left(\frac{3}{20}\right) + 2\left(\frac{-2}{20}\right)$
$= -0.6 - 0.25 + 0.15 - 0.2 =$

$-2\left(\frac{6}{20}\right) + -1\left(\frac{5}{20}\right) + 1\left(\frac{+3}{20}\right) + \left(\frac{2}{20}\right)(2)$
$= -0.6 - 0.25 + 0.15 + 0.2 = -0.5$

*** Let X be a continuous positive random variable with mean = 8, and variance = 16. The upper bound of $P(X \geq 16)$ is

a) 0.75 b) 0.0228 c) 0.25 d) None of these

$\mu_x = 8$ $V(X) = 16$

$p(X \geq 16) = p(X - 8 \geq 8)$
$= p(|X - \mu| \geq 2\sigma)$

$\frac{16-8}{4} = 2$

$P(Z \geq 2) \overset{\text{or}}{=} P(0 < Z < 2) = 0.5 - 0.4772$

*** Suppose you are throwing a fair die. The probability of getting number one for the second time on the 3-rd trial is

a) $\frac{25}{108}$ b) $\frac{5}{108}$ c) $\frac{10}{108}$ d) None of these

$n = 3$ trials $x =$ getting # 1. $x \sim Nb(3, \frac{1}{6})$

$r = 2$

$\rightarrow p(\text{getting num 1 for second time}) = C^2_1 (\frac{1}{6})^2 (\frac{5}{6})^1 = 2(\frac{1}{36})(\frac{5}{6}) = \frac{10}{216}$

*** Let X be a geometric random variable. If P(X ≥ 3) = 0.16, then P (X= 3) equals to

a) 0.096 b) 0.96 c) 0.24 d) None of these

$p(X \geq 3) = 0.16$ $p(X = 3) = ?$

$p(X \geq 3) = 0.16 = 1 - p(X=2) - p(X=1) - p(X=0).$

$p(X \geq 3) = 0.16 \rightarrow p(X < 3) = 1 - 0.16 = 0.84$

$(\cancel{0.16})$ $(0.84)^3\, 0.16) = 0.1128$

$p(X \geq 3) = 0.16 = q^2 \Rightarrow q = 0.4$

$p(X \leq 2) = p(X=1) + p(X=2) = 1 - 0.16$

$p + (1-p)p = 0.84$

$p + p - p^2 = 0.84$

$\rightarrow p^2 - 2p + 0.84 = 0$

$\rightarrow (p - 0.6)(p - 1.4) = 0$

$p(X = 3) = (0.4)^2 \cdot 0.6$

$= (0.16)(0.6)$

$= 0.096$

*** In a certain mall it is known that 60% of the customers pay their bills by cash money. Five customers are waiting to pay their bills. What is the probability that the 3- rd one of them will pay cash money?

a) 0.035 b) 0.096 c) 0.3456 d) None of these

$\boxed{60\% = 0.6 = \text{Success} = p}$

$\boxed{n = 5}$ $r = 3$

$p(\text{3rd pays cash}) = C^5_3 (0.6)^3 (0.4)^2 = 10(0.6^3)(0.4^2) = 0.3456$

$\cancel{C^4_2 (0.6)^3 (0.4)^2 =}$

None

$E(X) = 6$ $\quad$ $\sigma(X) = 2 \rightarrow Var(X) = 4$

*** X is a random variable with mean = 6, and standard deviation = 2 If the random variable Y is given by $Y = 5X + 2X^2 - 100$, then E(Y) equals to

a) 10 $\qquad$ b) 62 $\qquad$ c) 3 8 $\qquad$ d) None of these

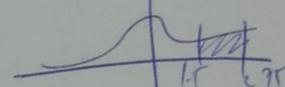
$E(Y) = 5E(X) + 2E(X^2) - 100$

$E(X) = 6$

$E(X^2) = ??$ $\quad Var(X) = 4 = E(X^2) - 36$

$\rightarrow E(X^2) = 40$

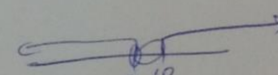
$\rightarrow E(Y) = 30 + 80 - 100 = 10$



*** X is a normal random variable with a mean 15, and a variance 16 , If P (X >10) = 0.8944, then P (X ≤ 9) equals to

a) 0.1056 $\qquad$ b) 0.0668 $\qquad$ c) 0.4332 $\qquad$ d) None of these

$X \sim N(15, 16)$ $\qquad P(X > 10) = 0.8944$



$P(X \le 9) = ??$

~~P(X≤9) =~~ $\quad P(X > 10) = 1 - P(X \le 10) = \boxed{1 - P(X=10) - P(X \le 9) = 0.8944}$

$P(X \le 9) = P(0 \le X < 9) = P\left(\frac{-15}{4} < X < \frac{9-15}{4}\right) = P(-3.75 < X < -1.5)$

$= P(1.5 < X < 3.75) = P(0 < X < 3.75) - P(0 < X < 1.5)$

$=$

$P(X \le 9) = 1 - P(X=10) - 0.8944$

$= 0.1056 - P(X=10)$