18

Please note that you have 6 questions and 7 pages

Round all your answers to 4 digits after the decimal point.

1) **(18 points)** From a group of 6 people of which 4 are females, 3 are to be randomly selected. Let X be the random variable to represent the number of females being selected.

a) Find the probability distribution of X.

X: ~~0~~ 1, 2, 3.

$$P(X=1) = \frac{{}^{2}C_{2} \times {}^{4}C_{1}}{{}^{6}C_{3}} = \frac{4}{20} = \frac{1}{5} = 0.2$$

$$P(X=2) = \frac{{}^{4}C_{2} \times {}^{2}C_{1}}{{}^{6}C_{3}} = \frac{2 \times 6}{20} = \frac{12}{20} = \frac{3}{5} = 0.6$$

$$P(X=3) = \frac{{}^{4}C_{3} \times {}^{2}C_{0}}{{}^{6}C_{3}} = \frac{4}{20} = \frac{1}{5} = 0.2$$

b) Find $E(X)$.

$$E(X) = \sum_{1}^{3} x P(x) = 1 \times 0.2 + 2 \times 0.6 + 0.2 \times 3 = 2$$

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2) (20 points) The service times "in hours" at a teller window of a bank are exponentially distributed with a mean of $\frac{1}{4}$.

a) A customer arrives at 8:00 AM, and the service begins. What is the probability that he (she) will still be there at 8:20 AM?

$\beta = \frac{1}{4}$ hour = 15 min

$P(X > 20)$ = P (he will still be There at 8.20)

$= e^{-\frac{20}{15}} = 0.2636$

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b) Two customers arrive at 10:00 AM and the service begins. A third customer arrives at 10:45 AM.

i) What is the probability that he (she) has to wait before getting service?

let $Y = X_1 + X_2$.

Gamma distribution with $\alpha = 2$ and $\beta = \frac{1}{4}$ hour = 15 min

So $f(x) = \begin{cases} \frac{1}{15^2} x e^{-\frac{x}{15}} & \text{for } x > 0 \\ 0 & \text{elsewhere.} \end{cases}$

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P (he has To wait) $= P(Y > 45) = \int_{45}^{\infty} \frac{1}{225} x e^{-x/15} dx$

$\frac{1}{225}\left[[x \times (-15 e^{-x/15})]_{45}^{\infty} - \int_{45}^{\infty} -15 e^{-x/15} dx \right] = \frac{1}{225} [-15 x e^{-x/15}]_{45} + [15 \times 15 e^{-x/15}]_{45}^{\infty} \times \frac{1}{225}$

$= \frac{1}{15}[-15 \times 45] - \frac{15}{e^{45/15}} - e^{-45/15}$

$= \frac{3}{e^3} + \frac{1}{e^3} = 0.1991$

ii) What is the probability that he (she) has to wait at least 15 minutes before getting service?

$P(Y > 60 \text{ min}) = \int_{60}^{\infty} f(x)\,dx = 1 - P(Y < 60)$

$= 1 - \int^{60} \frac{1}{225} y e^{-\frac{y}{15}}$

$= \frac{1}{225}[-15 x e^{-x/15}]_{60}^{\infty}$

$\frac{3}{e^{60/15}} + \frac{1}{e^4} = \frac{3}{e^4} + \frac{1}{e^4} = \frac{4}{e^4} = 0.073 = 0.0916$

19.5

3) (20 points) In a certain firm that manufacturers insulation, it is known that 50% of the employees have positive indications of asbestos in their lungs.
If the firm is requested to send three employees who have positive indications of asbestos in their lungs to a medical center

a) Find the probability that six employees must be tested to find the required three employees.

X is a negative binomial case wal-

$P = 0.5$ and $r = 3$.

$P(X=6) = C^5_2 \, p^3 \cdot q^3 = 10 \times (0.5)^3 \times (0.5)^3$

$= 0.15625$

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b) If each test costs $10, find the expected value and the standard deviation of the total cost of conducting theses tests to locate the required three employees with positive indications of asbestos in their lungs.

The cost is $Y = 10X$

$E(\text{Cost}) = E(Y) = E(10X) = 10E(X)$

$E(X) = \frac{r}{p} = \frac{3}{0.5} = 6$ $\quad$ $E(Y) = 60\$$.

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$V(X) = \frac{r \cdot q}{p^2} = \frac{3 \times 0.5}{0.5 \times 0.5} = 6.$

$V(Y) = 100\,V(X) = 100\,V(X) = 600\,\$$.

$\delta(Y) = 24.494$.

c) Approximate the probability that the cost of these tests will exceed $100.

Schebechuie: $\cancel{\mu} + K \sigma = 100$

$K = \frac{100 - 60}{24.494} = 1.63 \ldots$

5.5

$P(Y > 100)$ is at most $\frac{1}{K^2} = \frac{1}{2.63^2} = 0.3749$.

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4) (15 points) The time until the first failure of a brand of ink jet printer is normally distributed with a mean of 2000 hours and a standard deviation of 250 hours.

a) What should be the guarantee period "time in hours" for these printers, if the manufacturer wants not more than 5% of the printers to fail within the guarantee period?

$P(X < X_0) = \frac{5}{100} = 0.05.$

$P(Z < Z_0) = 0.05.$

$0.5 - P(Z_0 < Z < 0) = 0.05.$

$P(Z_0 < Z < 0) = 0.45$

So $Z_0 = -1.64.$

$Z_0 = \frac{X - 2000}{250} = -1.64.$

So $X = 1590$. (It is the period)

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b) If 5 such printers are sold today, what is the probability that at most one of them will fail before 2000 hours?

$P(\text{fail before } 2000) = P(X < 2000) = P(Z < 0) = 0.5$

It's a binomial case with $n = 5$ and $p = 0.5.$

$P(x \leq 1) = 1 - \cancel{P(x > 1) = 1 - P(x = 0) = 1 - C_0^5 p^0 q^5}$

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$= P(x = 0) + P(x = 1) = C_0^5 p^0 q^5 + C_1^5 p^1 q^4.$

$= 0.03125 + 0.15625 = 0.1875$

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5) (12 points) In a gambling game a woman is drawing randomly one card from an ordinary deck of 52 playing cards. She gets \$5 if she draws a jack, or a queen, or a king and she gets \$75 if she draws an ace. She pays \$$x$ if she draws any other card. Find the value of x that makes this game a fair game "By a fair game, we mean the expected profit of the woman is zero".

$E(X) = E(\text{profit}) = 0$

$P(\text{she gets } 5) = \frac{12}{52}$

Profit is a random variable which represent the profit

$P(\text{she gets } 75) = \frac{4}{52}$

$X: 5, 75, -x$

$P(\text{she pay } x) = \frac{36}{52}$

$E(\text{profit}) = 5 \times \frac{12}{52} + \frac{4 \times 75}{52} - x \times \frac{36}{52} = 0$

So $x = \frac{360}{36} = 10$

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6) (15 points) The length of time in minutes for an airplane to obtain clearance for takeoff at a certain airport is a random variable $Y = 2X + 5$ where X is a random variable with probability density function $f(x)$ given by $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$

a) What is the average time that an airplane takes to obtain clearance for takeoff?

$E(Y) = 2E(X) + 5$

X has an exponential density function with $\beta = 1$ because it has the form

$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

So $E(X) = \beta = 1$ min

So $E(Y) = 2 \times 1 + 5 = 7$ min

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b) What is the probability that, it takes more than 15 minutes for an airplane to obtain clearance for takeoff?

$P(Y > 15) = P(2X + 5 > 15) = P(X > 5) =$

$e^{-\frac{5}{1}} = \frac{1}{e^5} = 0.0067.$

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