Notre Dame University
Faculty of Natural and Applied Sciences
Department of Mathematics and Statistics

MAT 326

Probability & Statistics for Engineers
Exam #2

Friday January 13, 2006

Duration: 55 minutes

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Section: TTH 9.30-11

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Grade: 60

Please note that you have 5 questions and 6 pages

Round all your answers to 4 digits after the decimal point.

1) **(20 points)** the grade point averages (GPA) of a large population of college students are approximately normally distributed with mean equal to 2.4 and standard deviation equal to 0.5.

a) If a random sample of size 36 is selected, find the probability that the sample GPA average will exceed 2.5.

$\mu = 2.4$

$\sigma = 0.5$

$n = 36$, n is large

$p(\bar{X} > 2.5) = p(\bar{x} > \frac{2.5}{36})$ $p(\bar{X} > 0.064)$

$= p(Z > \frac{2.5 - 2.4}{\frac{0.5}{6}})$

$P(\bar{X} > 2.5) = P(Z > \frac{2.5 - 2.4}{\frac{0.5}{\sqrt{36}}})$

$= P(Z >$

$= 0.5 - p(Z < 1.2)$

b) If 100 such samples are selected "each sample is of size 36", what is the probability that at least 15 of them each will have an average GPA larger than 2.5?

$0.5 - p(Z < \quad)$

P

$(100)($

$\mu = np$

$\sigma = \sqrt{npq}$

$P(X > 50)$

$P(X > 49.5)$

$P(X \geq 15) = P(Y > 14.5)$

$= P(Z > \frac{14.5 - 15}{\frac{\sigma}{\sqrt{100}}})$

$=$

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2) **(18 points)** A machine in a certain factory must be repaired if it produces more than 10% defectives among the large lot of items it produces in a day. A random sample of 100 items from the day's production contains 15 defectives, and the foreman says that the machine must be repaired.

a) Does the sample evidence support his decision at $\alpha = 0.01$?

n is large, the population is normal ; ~~then we ha~~

$n = 100$, $\alpha = 0.01$

$\hat{p} = \frac{15}{100} = 0.15$

$H_0 : p_0 \leq 0.1$

$H_a : p_0 > 0.1$, we have a right-sided test :

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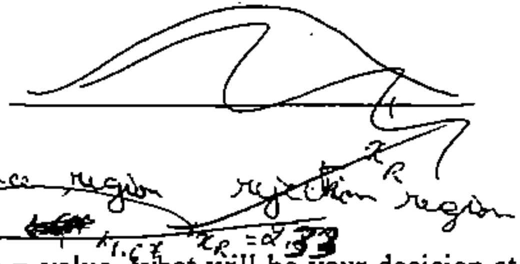
$x_R = Z_{\alpha} = Z_{0.01} = 2.33$

$Z_{\hat{p}} = O.V.TS = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.15 - 0.1}{\sqrt{\frac{(0.1)(0.9)}{100}}} = 1.67$

$Z_{\hat{p}}$ is in the acceptance region

we accept H_0

No need for the machine to be repaired



b) Find the p-value of the test. Based on this p-value, what will be your decision at $\alpha = 0.05$? Explain.

$p\text{-value} = p(Z > |O.V.T.S|) = p(Z > 1.67)$

$= 0.5 - p(Z < 1.67)$

$= 0.5 - 0.4525$

$= 0.0475$

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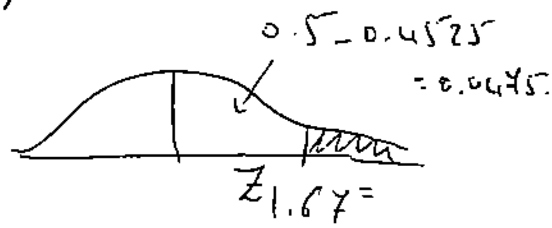

$\alpha = 0.05 > p\text{-value} = 0.0475$

we reject H_0 So?

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3) **(20 points)** A production process is supposed to be producing 10-ohm resistors. Assume resistance measurements are approximately normally distributed. A sample of 9 randomly selected resistors showed a sample mean of 9.8 ohms and a standard deviation of 0.2 ohm.

a) Are the specifications of the process being met at the 5% significance level?

n is small, we have a t- distribution.

$\alpha = 0.05$, $v = n - 1 = 9 - 1 = 8$.

$t_{\frac{\alpha}{2}} = t_{0.025}(8) = 2.306$

$\left[\bar{x} - t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}, \bar{x} + t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}\right]$

$\left[9.8 - (2.306)\frac{(0.2)}{3}, 9.8 + (2.306)\frac{(0.2)}{3}\right]$

$[9.64, 9.95]$

$H_0: \mu_0 = 10$

$a: \mu_0 \neq 10$

$x_L = -t_{\alpha/2} = -t_{0.025}(8) = -2.306$

$x_R = t_{0.025}(8) = 2.306$.

acceptance region

$x_L = -2.3$ $x_R =$

$O.V.T.S: t_{\bar{x}} = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} = \frac{9.8 - 10}{\frac{0.2}{\sqrt{9}}} = -3$ is in the rejection r

we reject $H_0 \Rightarrow$

b) Estimate the true variance of the resistance measurements in a 90% confidence interval.

Estimate σ^2, $1 - \alpha = 0.9$

$\Rightarrow \alpha = 0.1$

we have a χ^2 distribution with $v = n - 1 = 8$

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$\chi^2_{\alpha/2} = \chi^2_{0.05}(8) = 15.5073$

$\chi^2_{1-\alpha/2} = \chi^2_{0.95}(8) = 2.73264$

$\left[\frac{(n-1)S^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right] \Rightarrow \left[\frac{8(0.04)}{15.5073}, \right.$

zero

4) **(12 points)** Suppose X_1, X_2, X_3, X_4 denotes a random sample from some population with mean μ and variance δ^2. The following are suggested as estimators for μ:

$$\hat{\mu}_1 = X_1, \qquad \hat{\mu}_2 = \frac{1}{2}(X_1 + X_4), \qquad \hat{\mu}_3 = \frac{1}{2}(X_1 + 2X_4) \text{ and } \qquad \hat{\mu}_4 = \overline{X}$$

Which one among the above estimators is the best one to be used to estimate μ and why?

The best ~~one~~ estimator to use is :

$E(\hat{\mu}_1) = \mu$.

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5) (~~30~~ **points**) The average depth of bedrock at two possible construction sites is to be compared by driving piles at random locations within each site. The results, with depths in feet, are as follows:

Site A: $n_1 = 4$ $\quad \bar{x}_1 = 134$ $\quad s_1 = 14$

Site B: $n_2 = 8$ $\quad \bar{x}_2 = 142$ $\quad s_2 = 12$

a) Does the average depth of bedrock at site A seems significantly smaller than that at site B? Test at the 5% significance level. What assumption(s) if any are you making?

we have both n_1 and n_2 small

we have a t- distribut° with $\begin{cases} V = n_1 + n_2 - 2 = 10. \\ \alpha = 0.05 \end{cases}$

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$H_0: \mu_1 \geq \mu_2 \Rightarrow H_0: \mu_1 - \mu_2 \geq 0.$

$H_a: \mu_1 < \mu_2 \Rightarrow H_a: \mu_1 - \mu_2 < 0.$

We have a left sided test : $x_L = -t_\alpha = -t_{0.05}(10) = -1.812$

$O.V.T.S = \dfrac{\bar{x}_1 - \bar{x}_2 - \mu_0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ where : $\mu_0 = \mu_1 - \mu_2 = 0.$

$S_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$O.V.T.S_{\bar{x}_1-\bar{x}_2} = \dfrac{134-142}{(12.63)(0.612)} = -1.034$

$S_p = \sqrt{\dfrac{(3)(14)^2 + (7)(12)^2}{10}}$

$S_p = 12.63$

$\sqrt{\frac{1}{4}+\frac{1}{8}} = 0.612$

rejection region, acceptance region, -1.03, $x_L = -1.812.$

$t_{\bar{x}_1-\bar{x}_2}$ is in the acceptance region we accept H_0, we reject H_a

the average depth at site A is not smaller than that at site B

We ~~a~~ssumed the population is normally distributed.

If σ_1, σ_2 not given but equal

b) Is there sufficient evidence to say that the variances among depth measurements differ for the two sites? Test at the 10% significance level. What assumption(s) if any are you making? We assume the population is normally distributed.

$\boxed{\alpha = 0.1}$

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_a : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

We have a two sided test. We use the F-distribution

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$$x_L = F_{1-\alpha/2}(4,8) = F_{0.95}(4,8) = \frac{1}{F_{0.05}(8,4)} = \frac{1}{6.04} = 0.165$$

$$x_R = F_{\alpha/2}(4,8) = F_{0.05}(4,8) = 3.84$$

$$O.V.T.S = \frac{S_1^2}{S_2^2} = \frac{(14)^2}{(12)^2} = \frac{196}{144} = 1.36$$ is in the acceptance region. we accept H_0.

assumptions?

reject | accept 1.36 | reject

$x_L = 0.165$ $x_R = 3.84$

No there is not enough evidence to say that the depth measurements differ for the two sites