22 Grade 10 Exam 2

1) **(22 points)** Given a population with a mean of 25 and a variance of 16.

a) A random sample of size 64 is selected, if $\bar{x}$ denotes the mean of the selected sample, find the probability that $\bar{x}$ will be greater than or equal to 25.

$\mu = 25$

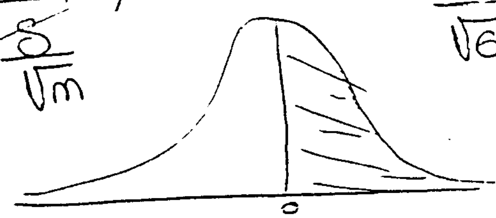
$\sigma^2 = 16$

$n = 64$

Since n is large, $\bar{x}$ has a normal distribution with mean $\mu_{\bar{x}} = \mu$ and st. dev. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$P(\bar{X} \geq 25) = P\left(Z \geq \frac{25 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z \geq \frac{25 - 25}{\frac{4}{\sqrt{64}}}\right)$

$= P(Z \geq 0) = \boxed{0.5}$



10

b) If 100 independent random samples each of size 64 were selected. Find the probability that at least half of them will have a mean $\bar{x}$ greater than or equal to 25.

Let X be a random variable that denotes the # of samples that have a mean $\bar{X} \geq 25$.

$X: 0, 1, 2, 3, 4, 5, --, 100$.

12

$P(X \geq 50)$?

$P(\bar{X} \geq 25) = 0.5$

Since $n = 100$ is large, X can be converted into a random variable Y that have a normal distribution with mean $= np = 100 \times 0.5 = 50$ and st. dev. $= \sqrt{npq}$

$= \sqrt{100(0.5)(0.5)} = \sqrt{25} = 5$.

$P(X \geq 50) = P(Y > 49.5) = P\left(Z > \frac{49.5 - 50}{5}\right)$

$P(Z > -0.1) = 0.5 + P(0 < Z < 0.1)$

20

2) (20 points) Copper produced by sintering (heating without melting) a powder under certain conditions is then measured for porosity (the volume fraction due to voids) in a certain laboratory. A sample of $n_1 = 4$ independent porosity measurements shows a mean of $\bar{x}_1 = 0.22$ and a variance of $s_1^2 = 0.0010$. A second laboratory repeats the same process on an identical powder and gets $n_2 = 5$ independent porosity measurements with $\bar{x}_2 = 0.17$ and $s_2^2 = 0.0020$. Estimate the true difference between the population means $(\mu_1 - \mu_2)$ for these two laboratories, with confidence coefficient 0.95. What assumptions are necessary for your answer to be valid?

$m_1 = 4$ $\quad$ $m_2 = 5$

$\bar{X}_1 = 0.22$ $\quad$ $\bar{X}_2 = 0.17$

$S_1^2 = 0.001$ $\quad$ $S_2^2 = 0.002$

Simce m_1 and m_2 are both small, $\bar{X}_1 - \bar{X}_2$ has a t-distribution with mean $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$ and st. dev $S_{\bar{X}_1 - \bar{X}_2} = S\sqrt{\frac{1}{m_1} + \frac{1}{m_2}}$ $\quad (V = m_1 + m_2 - 2 = 9 - 2 = 7)$

giren that $1 - \alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

From the table $t_{\frac{\alpha}{2}} = t_{0.025} = 2.365$

a $(1-\alpha)100\%$ conf. interval for $\mu_1 - \mu_2$ is giren by

$$\left[\bar{X}_1 - \bar{X}_2 - t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{m_1} + \frac{1}{m_2}},\ \bar{X}_1 - \bar{X}_2 + t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{m_1} + \frac{1}{m_2}}\right]$$

where $\bar{X}_1 - \bar{X}_2 = 0.22 - 0.17 = 0.05$

$t_{\frac{\alpha}{2}} = 2.365$

$$S_p = \sqrt{\frac{(m_1 - 1)S_1^2 + (m_2 - 1)S_2^2}{V}} = \sqrt{\frac{3(0.001) + 4(0.002)}{7}} = 0.0396$$

$$\sqrt{\frac{1}{m_1} + \frac{1}{m_2}} = \sqrt{\frac{1}{4} + \frac{1}{5}} = 0.67$$

$$[0.05 - 2.365(0.0396)(0.67),\ 0.05 + 2.365(0.0396)(0.67)]$$

$$\boxed{[-0.012748,\ 0.112748]}$$

16

3) **(16 points)** A random sample of $n_1 = 10$ observations on breaking strengths of a type of glass gave $s_1^2 = 2.31$ (measurements were made in pounds per square inch). An independent random sample of $n_2 = 16$ measurements on a second machine, but with the same kind of glass, gave $s_2^2 = 3.68$. Estimate the true variance ratio, σ_2^2/σ_1^2, in 90% confidence interval. What assumptions are necessary for your answer to be valid?

Machine 1 $\begin{cases} n_1 = 10 \\ S_1^2 = 2.31 \end{cases}$ $\quad$ $\left.\begin{matrix} n_2 = 16 \\ S_2^2 = 3.68 \end{matrix}\right\}$ Machine 2

$\frac{S_2^2}{S_1^2}$ has an F distribution with $V_1 = n_1 - 1 = 10 - 1 = 9$ and

$V_2 = n_2 - 1 = 16 - 1 = 15$

given that $1 - \alpha = 0.9$

$\alpha = 0.1$

$\frac{\alpha}{2} = 0.05$.

A $(1-\alpha)100\%$ conf. interval for $\frac{\sigma_2^2}{\sigma_1^2}$ is given by:

$$\left[\frac{S_2^2}{S_1^2} F_{1-\frac{\alpha}{2}}(V_1, V_2),\ \frac{S_2^2}{S_1^2} F_{\frac{\alpha}{2}}(V_1, V_2)\right]$$

where $F_{\frac{\alpha}{2}}(V_1, V_2) = F_{0.05}(9, 15) = 2.59$.

$$F_{1-\frac{\alpha}{2}}(V_1, V_2) = F_{0.95}(9, 15) = \frac{1}{F_{0.05}(15, 9)} = \frac{1}{3.01} = 0.3322$$

$$\left[\frac{3.68}{2.31}(0.3322),\ \frac{3.68}{2.31}(2.59)\right]$$

$$\boxed{[0.529219,\ 4.126]}$$

The populations are assumed to have a normal distribution.

3/5

4) (20 points) The ratio of number of items produced in a factory by three shifts, first, second, and third, is 4 : 2 : 1, due primarily to the decreases number of employees on the later shifts. It is hypothesized that the number of defectives produced should follow this same ratio. A sample of 50 defective items was traced back to the shift that produced them, with the following results:

	Shift 1	Shift 2	Shift 3
Number of Defectives	20	16	14

Test the hypothesis indicated above, with $\alpha = 0.05$.

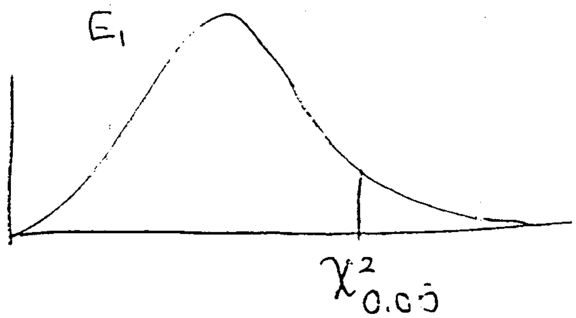
$k = 3$

shift	O_i	$E_i = np_i$
1	20	$\frac{200}{7} = 28.57$
2	16	$\frac{100}{7} = 14.28$
3	14	$\frac{50}{7} = 7.15$

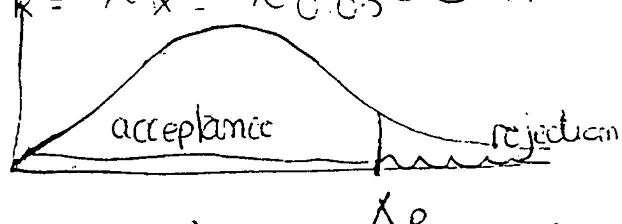
$H_0: p_1 = \frac{4}{7},\ p_2 = \frac{2}{7},\ p_3 = \frac{1}{7}$

H_1: at least one of the four values above is wrong

$\sum \frac{(E_i - O_i)^2}{E_i}$ has a χ^2 distribution with $v = k - 1 = 3 - 1 = 2$



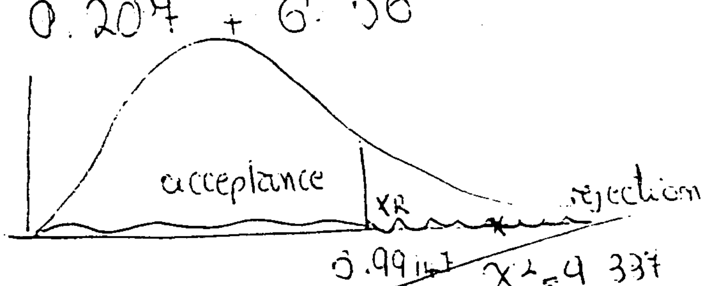
$X_R = \chi^2_\alpha = \chi^2_{0.05} = 5.99147$



O.V.T.S $= \chi^2 = \frac{(28.57 - 20)^2}{28.57} + \frac{(14.28 - 16)^2}{14.28} + \frac{(7.15 - 14)^2}{7.15}$

$= 2.57 + 0.207 + 6.56$

$\boxed{\chi^2 = 9.337}$



Since χ^2 is in the rejection rejection, H_0 is rejected

We have at least one of the three probabilities P_1, P_2, P_3 not true

22

6

5) (22 points) The mean breaking strength of cotton threads must be at least 215 grams in order for the thread to be used in a certain garment. A random variable sample of 50 measurements on a certain thread gave a mean breaking strength of 210 grams and a standard deviation of 18 grams..

a) Should this thread be used on the garments? Test at $\alpha = 0.05$.

$n = 50$

$\bar{x} = 210$

$s = 18$

$H_0 : \mu_0 \geq 215$

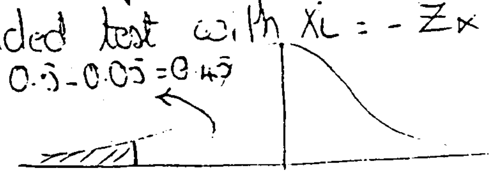
$H_1 . \mu_0 < 215$

Since n is large, $\bar{x}$ is normally distributed with mean $\mu_{\bar{x}} = \mu_0$

and st. dev $S_{\bar{x}} = \frac{S}{\sqrt{n}}$

We have a left sided test with $x_L = -Z_\alpha = -Z_{0.05} = -1.65$

$0.5 - 0.05 = 0.45$



rejection x_L acceptance

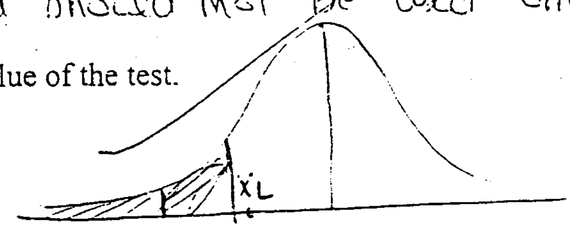
-1.96 -1.65

$$O.V.T.S = Z_{\bar{x}} = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{210 - 215}{\frac{18}{\sqrt{50}}} = -1.964$$

Since $Z_{\bar{x}}$ is in the rejection region, H_0 is rejected $\Rightarrow (\mu_0 < 215)$

No this thread should not be used on the garments

b) Find the *p*-value of the test.



$$p\text{-value} = P(Z < Z_{\bar{x}}) = 0.5 - P(0 < Z < 1.96)$$

$$= 0.5 - 0.4750$$

5/5

$\boxed{p\text{-value} = 0.025}$