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Notre Dame University
Faculty of Natural and Applied Sciences
Department of Mathematics and Statistics

MAT 326

Probability & Statistics for Engineers
Exam #1

Friday November 30, 2007

Duration: 55 minutes

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Grade: ____________________

Please note that you have 6 questions and 7 pages
Round all your answers to 3 digits after the decimal point.

1) **(12 points)** From a group of 7 students of which two are math students, a committee of 3 randomly selected students is needed. Let X be a random variable representing the number of math students among the selected committee. Find the probability distribution function of X. What is $P(X=2)$?

7 student, 2 math } 3 selected.

$X = 0, 1$ or 2.

Probability distribution function:

$F(x) = P(X \le x)$

$F(0) = P(X \le 0)$
$= P(X=0)$
$= \frac{C_2^5}{C_2^7} = \frac{10}{21}$ 35

b) $P(X=2) = P($2 math student are chosen$)$

$= \frac{C_2^2 C^5}{C_2^7}$

It's a bit messed-up

$C_2^7 = \frac{7!}{5!2!} = \frac{7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} 2!} = \cancel{21}$

$\boxed{P(X=2) = 1/21}$

$F(1) = P(X \le 1)$
$= P(X=0) + P(X=$
$= \frac{10}{21} + \frac{C_1^2 C_1^5}{21}$
$= \frac{10}{21} + \frac{10}{21}$
$= \frac{20}{21}$

$F(2) = P(X \le 2)$
$= 1 + \frac{20}{21}$
$= 1$

real value calculation mistake.

$\Rightarrow F(0) = \frac{C_3^5}{C_3^7}$

$F(1) = \frac{C_1^2 C_2^5}{35} + \frac{C_3^5}{C_3^7}$

I took 2 selected 1/7

$P(X=2) = \frac{C_2^2 C_1^5}{C_3^7}$

$\Rightarrow C_3^7 = \frac{7!}{4!\,3!}$
$= \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}\, 3!}$
$= 35$

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2) (16 points) Customers at a certain superstore pay their bills either by cash money with probability 0.8, or by their credit cards with probability 0.2.

a) For the 10 customers arriving at the superstore, find the probability that at least 8 of them pay by cash money.

$P(CH) = 0.8 \qquad P(CC) = 0.2$

This is a Binomial experiment with $n = 10$. $\boxed{p = 0.8}$
let Y be the r.v

$P(Y \geq 8) = P(Y=8) + P(Y=9) + P(Y=10)$

$= C_8^{10} p^8 (1-p)^2 + C_9^{10} p^9 (1-p)^1 + C_{10}^{10} p^{10} (1-p)^0$

$= \frac{10!}{8!2!} 0.8^8 0.2^2 + \frac{10!}{9!} 0.8^9 0.2 + 0.8^{10}$

$= \frac{10 \times 9}{2} 0.8^8 0.2^2 + 10 \times 0.8^9 \times 0.2 + 0.8^{10}$

$= 0.6777.$

7/7

b) Of the customers arriving at the superstore starting the opening time, find the probability that the 3rd customer pay by credit card is the 10th arrival.

This is a negative Binomial with $r = 3 \quad n = 10.$

here $\boxed{p = 0.2}$

$P(Y=10) = C_2^9 p^3 (1-p)^7$

$= \frac{9!}{7!2!} 0.2^3 \, 0.8^7$

$= \frac{9 \times 8}{2} \times 0.2^3 \times 0.8^7$

$= 0.06$

9/9

3) **(24 points)** The weekly demand "in thousands of liters" for kerosene at a certain supply station is a random variable X with probability density function

$$f(x)=\begin{cases} kx & \text{for} \quad 0\le x\le 2 \\ 2k & \text{for} \quad 2<x\le 4 \\ 0 & \text{elsewhere} \end{cases}$$

$\int_{-\infty}^{+\infty} f(x) = 1$ if it is a pdf.

a) Find k.

$$\int_0^2 kx + \int_2^4 2k = 1$$

3/3

$$\Rightarrow \left[k\frac{x^2}{2}\right]_0^2 + \left[2kx\right]_2^4 = 1.$$

$$2k + 8k - 4k = 1.$$

$$12k - 4k = 1 \Rightarrow \boxed{k = 1/6}$$

b) Find the probability distribution function $F(x)$.

8/8

$$F(x) = \begin{cases} 0 & x<0 \\ x^2/12 & 0\le x\le 2 \\ \frac{1}{3}(x-1) & 2\le x\le 4 \\ 1 & x>4 \end{cases}$$

for $0 \le x \le 2 \Rightarrow F(x) = \int_0^x f(x)\,dx = \int_0^x kx \; dx.$

$$= \left[k\frac{x^2}{2}\right]_0^x$$

$$= \frac{kx^2}{2} = \frac{x^2}{12}.$$

for $2 < x \le 4 \Rightarrow F(x) = \int_{-\infty}^x f(x)\,dx.$

$$= \int_0^2 kx\,dx + \int_2^x 2k\,dx$$

$$= \left[k\frac{x^2}{2}\right]_0^2 + \left[2kx\right]_2^x$$

$$= 2\times 1/6 + 2kx - 4k = \frac{1}{3} - \frac{2}{3} + \frac{1}{3}x$$

$$= 1/3 + \frac{1}{3}x - 4/6 = \frac{1}{3}$$

$$= -1/3 + 1/3\,x = \frac{1}{3}(x-1)$$

c) The weekly profit "in dollars" is a random variable Y given by $Y = 324X - 142$.
Find

i) $E(Y)$

$Y = 324X - 142$.

$$E(x) = \int_{-\infty}^{+\infty} x f(x)\, dx$$

$$= \int_0^2 k x^2\, dx + \int_2^4 2kx\, dx$$

$$= \left[kx^3/3 \right]_0^2 + \left[kx^2 \right]_2^4$$

$$= \left[k \times \frac{8}{3} \right] + \left[16k - 4k \right]$$

$$= 8/3\, k + 12k = \frac{8}{18} + \frac{12}{6}$$

$$= \frac{4}{9} + 2$$

$$= \frac{22}{9} = 2.44$$

7/7

$E(Y) = 324 E(x) - 142$

$\boxed{E(Y) = 650\ \$}$

ii) $P(Y \le 749)$

$$P(Y \le 749) = P(324X - 142 \le 749)$$

$$= P(324X \le 891)$$

$$= P(X \le 2.75)$$

$$= F(X = 2.75) \text{ when } 2 \le x \le 4$$

$$= \frac{1}{3}(2.75 - 1)$$

$$= 0.583$$

6/6

4) **(24 points)** The number of customers arriving at a certain checkout counter is a random variable follows a Poisson distribution with average 3 customers/hour.

a) In a randomly selected hour, what is the probability that exactly 3 customers arrive?

pdf of poiss pdf : $f(x) = \left\{ \dfrac{e^{-\lambda} \lambda^{m}}{m!} \right.$ when m is the number of customer per hour.

$$P(X=3) = \frac{e^{-3}\, 3^{3}}{3!} = 4.5 \times e^{-3} = 0.224.$$

7/7

b) If a customer arrives at 9:00 AM, what is the probability that the next customer will arrive before 9:30 M?

9 ------- 9.30

$\lambda = 3$ customer/hour $\Rightarrow$ where T is the successive time between customers.

7/7

$\Rightarrow$ $e^{-\lambda t} = e^{-30/20}$

$P(T < 30 \text{ minutes})$

$= 1 - \underbrace{P(T > 30 \text{ minutes})}_{\text{no customer betw } (0,\, 30 \text{ minut})}$

$= 1 - e^{-30/20} = 1 - e^{-1.5} = 0.7768.$

c) If the opening time of the counter is 9:00 AM, what is the probability that the second customer will not arrive before 9:30 AM?

~~no customer in a period of 20 minutes.~~

9.00 —— 20 min. 1st custom. —— 9:30 am.

4/10

~~[illegible] the [illegible] opening time between the opening of the counter and arriving the~~

$$P(T > 30) = e^{-30/20} = e^{-1.5}$$

$= 0.2231.$

How about the 1st customer beyond that period. The question is about the 2nd customer arriving [illegible]

5) **(12 points)** A factory uses two machines to produce one type of tires. It is known that 5% of the tires produced by the first machine are defective, and of those produced by the second machine, 8% are defective. On a certain day 40% of the supply was produced by the first machine, one tire was randomly selected from the supply and found defective. What is the probability that it was produced by the second machine?

Machine A

$P(d/A) = 0.05$

$P(A) = 0.4$

Machine B

$P(d/B) = 0.08.$

$P(B) = 0.6.$

$$P(B/d) = \frac{P(d/B)\,P(B)}{P(d)}$$

$$\begin{aligned} P(d) &= P(d \cap A) + P(d \cap B) \\ &= P(d/A)P(A) + P(d/B)\,P(B) \\ &= 0.05 \times 0.4 + 0.08 \times 0.6 \\ &= 0.068. \end{aligned}$$

12/12

$$P(B/d) = \frac{0.08 \times 0.6}{0.068} = 0.705$$

6) **(12 points)** the life lengths 'measured in hundreds of hours" of a certain type of fuses are independent and normally distributed with a mean of 1000 hours and a standard deviation of 196 hours. Two such fuses are randomly selected. What is the probability that only one of them will operate for at least 1100 hours?

$X \simeq N(1000, 196)$ 2 are randomly selected.

$P(X \geq 1100)$

$\Rightarrow P(1100 \leq X < \infty)$

transform to standard normal distribution

$= P\left(\frac{1100-1000}{196} \leq Z < \infty\right)$

$= P(0.51 \leq Z < \infty)$

$= 0.5 - P(0 \leq Z \leq 0.51)$

$= 0.5 - 0.1950$

$P(X \geq 1100) = 0.305$.

The 2 are randomly selected $\Rightarrow$ only one will operate for at least 1100 hours.

It is a Binomial experiment
where $p = 0.305$ $n = 2$.

$P(\text{only 1 between 2}) = C_1^2 \, p^1 \, q^1$

$= 2 \times 0.305 \times 0.695$

$= 0.42395$.

12/12