NDU

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 2 Hours.

Name: _____

Section: A

Instructor: Dr. Ishac Zoghbi

Grade: _____

1. (**5 points**) Solve the differential equation:

$$\frac{dy}{dx} = -\frac{(3x - 3y + 2)}{(2x - 2y + 1)}$$

(**Hint:** You can use the transformation z = x - y)

2. (14 points) Find the orthogonal trajectories of the family of curves $x^2 - cx + 4y = 0$.

3. (8 points) Find the general solution of the differential equation

$$y'' - 6y' + 9y = \frac{e^{3x}}{x-1}$$
, for $x > 1$.

4. (8 points) Find the general solution of the system

$$x' = 4x - 13y$$
$$y' = 2x - 6y$$

5. (6 points) Find the Laplace transform of

$$h(t) = \frac{\sin t}{t}$$

6. (6 points) Find the function f(t), which verifies the equation $f(t) = t^3 + \int_0^t \sin u \cdot f(t-u) du$.

7. (7 points) Given $F(s) = \ln\left(1 + \frac{1}{s^2}\right)$, find Laplace inverse of F(s).

8. (8 points) Use Laplace transform to solve the following initial – value problem $y'' + 2y' + y = \delta(t-1)$; with y(0) = 2, y'(0) = 3.

9. (18 points) Use Laplace transform to find solution of the given linear system that satisfies the given initial conditions

 $x' + 3x - 4y = e^{3t}$ 2x + y' - 3y = 0; with x(0) = 0, y(0) = 1 **10.** (20 points) Given the differential equation xy'' + (x-1)y' - y = 0. Show that $x_0 = 0$ is a regular singular point, then use the method of Frobenius to find solutions of the given equation in some interval 0 < x < R.